OCCASIONAL PAPER No. 8
2000

Young Children's Mathematics: A supporting document for the "Making Things Count" resource

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Series Editor: Carmen Dalli

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ISBN 0-475-20061-6

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INTRODUCTION

Making Things Count (Ministry of Education, 1999) is a teaching resource for early childhood educators. This paper expands on the ideas in *Making Things Count* and is intended as a resource for teachers, parents and caregivers who want to understand and enhance young children's mathematical thinking. It provides a framework for looking at children's mathematical thinking and explores why the development of a 'number sense' is so important. Annotated lists of books for children and further reading for adults are included, along with master copies of some games for young children. This is followed by a profile of a fictional early childhood centre where mathematical experiences occur as an integral and natural part of the daily programme. A summary of the early childhood teacher's role in providing mathematical experiences for children, and practical suggestions for mathematics in centres are presented at the end.

A FRAMEWORK FOR LOOKING AT EARLY MATHEMATICAL THINKING

Carr, Peters and Young-Loveridge (1994) suggest a framework for looking at early mathematical thinking, based on a sequence of levels developed by Herbert Ginsburg (Allardice & Ginsburg, 1983; Baroody & Ginsburg, 1990). These levels, described in *Making Things Count* (Ministry of Education, 1999), are considered in more detail below.

Informal and natural level of mathematical thinking

The search for order and to make sense of our world appears to be either innate, or develops very soon after birth, and research has shown that very young infants are sensitive to changes in number patterns (Antell & Keating, 1983). At this first level young children rely on intuitive concepts and techniques for solving mathematical problems. They are aware of numbers and quantities but do not yet use the number system and other mathematical tools of their culture. Early childhood educators can provide children with lots of opportunities to explore quantity using both discrete objects such as blocks, pebbles, shells, toys, and food, and continuous materials such as dough, water and sand. Adults and older children can also model the use of

counting and other mathematical concepts in their interactions with the infant, for example by counting the buttons on the child's coat as these are being done up, or commenting on the shapes in a sorting puzzle. Surrounding infants by a rich mathematical environment will facilitate their awareness of the mathematics of their culture.

Informal and cultural level of mathematical thinking

At this level children start to use the mathematics of their culture to solve problems informally. Whereas at the intuitive level mathematics is universal, at the informal and cultural level the tools used in mathematical thinking will be dependent on the culture, and will be influenced by the attitudes and behaviours of the people the child spends time with. Adults, siblings and peers can scaffold the young mathematician in his/her development of mathematical skills. Everyday life provides a rich source of mathematical opportunities and parents, whanau and early childhood educators who are sensitive to the many learning opportunities that surround them will be able to assist children in developing a good basis on which their future mathematical learning can build.

Learning to count

One important tool children develop during this stage is the ability to count. Counting is a complex skill that needs the co-ordination of several sub-skills and considerable practice. Initially children may count correctly on some occasions but not on others (Suggate, Aubrey & Pettitt, 1997).

While the counting words used vary from culture to culture, according to Gelman (1978) there are five principles that are involved in counting:

- 1-1 principle. This involves both keeping track of items (those counted and those not yet counted, called "partitioning") and producing distinct tags (number words) one at a time. Tagging and partitioning must stay together.
- Stable order principle. The sequence of number words has to be produced in a stable order.
- Cardinality. The last tag represents the whole set, e.g., if five objects have been counted "1, 2, 3, 4, 5" the last tag, "5" describes the whole set.
- There is no restriction on the number or type of items that can be counted.
- The order in which objects are counted is irrelevant.

Rote counting, where children practise the sequence of number words in songs and chants, helps to develop knowledge of the sequence but may not be related to the child's ability to count rationally (matching number words to objects following the 1:1 principle). Initially children may skip or double count objects. They may find it helpful to maintain a physical link by moving or touching items as they count. Providing lots of meaningful and enjoyable opportunities for rational counting will help children to develop this important skill.

Unglaub (1997) suggests that once rational counting has been mastered children can be encouraged to count by twos, threes, fives, tens, and so on as an aid to later work with addition and multiplication.

As children's counting develops, adults may notice that children learning to count in English may experience difficulties with the 'teens' words and the decade numbers (20, 30, etc.). In English, as in many European languages, the spoken numerals do not always reflect the elements of tens and ones that are contained in them. For example, the fact that 11 is composed of a single tens value and a single units value is not evident in the English number word *eleven* (Geary, Bow-Thomas, Liu & Siegler, 1997), and numbers like fourteen and forty are phonetically similar in English even though they represent very different amounts. In comparison, in many Asian languages the spoken numeral corresponds exactly to the implied quantity represented in the written form, e.g., fourteen is spoken as ten-four and forty as four-ten(s). Bell (1990) refers to this as a more 'transparent' number grammar. The difference in the structure of the number words appears to affect the ease with which children learn to count past ten and later their understanding of place value and their ability to solve number problems. This is suggested as one reason why Asian children tend to perform better in studies of children's mathematical ability than English-speaking American children (Geary et al., 1997; Miura, Kim, Chang & Okamoto, 1988). This implies that in Aotearoa/New Zealand children might benefit from learning to count in Maori as Maori shares the same explicit number grammar as the Asian languages.

Representing number

At the informal and cultural level of mathematical thinking, children also become more competent with representing numbers. Hughes (1986) and Leder (1989) describe different categories of responses children make when representing quantities of objects:

- Idiosyncratic – where there is no obvious link between the child's representation and the number of objects
- Pictographic – realistic drawings of the items
- Iconic – discrete marks such as tallies, circles, and so on to represent the items
- Symbolic – conventional symbols.

To prompt children to look at ways of representing quantity Hughes (1986) devised a simple game involving four identical closed tins, each containing a different number of objects (e.g., one, two and three blocks with the fourth tin being empty). The tins are shuffled around in front of the child, who is then asked, for example, to pick the tin with two blocks in. As the tins are identical the child can only tell by guesswork. Paper is then stuck to the lid of each tin and the child is provided with a pen and asked to 'put something on the paper' to show how many objects are in each tin. The tins are shuffled and the question asked again but this time the children have their representations to help them.

In replicating this activity we have found that three and four-year-olds relish the challenge and many are able to produce useable pictographic or iconic responses. Two examples are shown below.

Whitin (1997) goes further, looking at the sophisticated ways young children invent to represent data when posing their own, meaningful questions, such as "Can you tie your shoe?" and "Do you have a pet?". He states that "Even young children can breathe meaning and understanding into numbers and facts when they are given the opportunity to pose their own questions and represent data in their own way" (Whitin, 1997, p. 28) and suggests that teachers can supply clipboards to data gatherers (reserving clipboards for this task gives it an aura of importance) and provide unlined paper so that the children are not predisposed to record the data in a particular way.

Encouraging children to develop meaningful symbols of their own may be more profitable than pushing for early use of conventional symbols. Carr and Mannington (1996) found that once children had learned how to write the numerals and then used them to represent numbers they were reluctant to go back to invented symbols of their own. We have noticed this in our own work with four- and five-year-olds. Children who see mathematics in terms of producing a 'right answer' may focus on attempting to produce the conventional symbol even if it does not aid them in solving the problem.

Learning dispositions

Carr (1997) explored children's learning dispositions and noted that even by age four it appeared that children were making "quite firm decisions about whether it was appropriate to tackle difficulty and risk error" (p. 320). If, in our interactions with young children, we hope to foster an orientation towards learning goals, rather than a focus on performance goals, Carr's (1998a) 'Learning Stories' approach to assessment in early childhood provides a useful framework for looking at young children's mathematical learning.

A 'Learning Story' describes a package of five behaviours that children will engage in within a learning environment. These are:

- taking an interest in an activity or topic
- becoming involved in the activity over a sustained period
- persisting with difficulty, challenge or uncertainty
- expressing ideas or feelings to others in a range of ways
- taking responsibility to change the way things are, to teach others and to listen to another point of view (see Carr, 1998a, p.15).

In considering children's learning stories we can look at emerging dispositions, the accompanying people, places and things that make their emergence more likely, and consider how we can strengthen the dispositions (Carr, 1998a).

Mathematical purposes

In promoting an orientation to learning, as opposed to performance, it is important to consider the child's purpose for using mathematics. If the purpose for using mathematics is meaningful for the child she/he is likely to approach the task in a different way to the child who is presented with tasks that have been selected by an adult for teaching or testing purposes but have little meaning for the child. In the EMI-4s (Young-Loveridge, Carr & Peters, 1995) study we noticed children using number for a range of different purposes. These were:

1. *Ritualised number use*, such as chants, songs and rote counting.
2. *Status*, where numbers were used to establish status. For example, age, higher numbers being better, winners and losers in a game.
3. *Entitlement*, establishing rights and fairness, which included sharing materials, turn taking and rules.
4. *Timing*, counting sleeps or minutes to an event, counting for an ultimatum and timing an activity or game.
5. *Patterns*, repeating patterns for their own sake in a variety of contexts such as art work and construction.
6. *Orderliness*, checking things are in order, in the right place, matching, sorting and putting shapes together to form a whole (as in jigsaws).
7. *Labelling*, representing numbers with numerals, pictures, number words or dot patterns.
8. *Talking about Number for its own sake.*
9. *Exploring quantity to solve a particular problem.*

Extending children's informal and cultural mathematical thinking

In seeking to extend children's mathematical thinking we looked at extending not only their skills but also the purposes for which they were using mathematics and the contexts in which mathematics was used. For example, by actively encouraging and promoting mathematical thinking and problem solving through adult interactions and the introduction of materials in different areas of play (family corner, outside, and so on), the range of *contexts* can be extended. The nature of these interactions and materials can be used to extend the range of *purposes* for which mathematics is used. As children's skills develop, the level of challenge can be increased to promote more complex mathematical thinking, thus furthering the children's level of *skill*.

Categories of difficulty

Working with children in this way led to the identification of four categories of difficulty. These are described in Carr, Peters and Young-Loveridge (1994). A summary is provided here:

Practice. The skills and purposes of a mathematical activity are simple and easy to understand.

Unassisted challenge. Where a new context or purpose arrives for a well-practised skill, or an available skill needs altering or extending in a familiar context.

Assisted challenge. The activity is within the difficulty limit of the child if an expert (adult or more capable peer) is there to help. Vygotsky's notion of a zone of proximal development, and children performing more skilfully when they are together with others than when they are alone, is relevant here (see Smith, 1998). The expert provides support and scaffolding to allow the learner to complete the task.

Beyond the difficulty level. The mathematical activity is beyond the difficulty level of the child even with assistance. "If too many enterprises are beyond the difficulty limit, and they are recognised as mathematical, young children may begin to avoid or ignore all mathematical tasks" (Carr, et al., 1994, p. 272).

The teacher's role

While it is important to include opportunities for children to explore mathematics and refine their skills through practice and unassisted challenge, teachers also have an important role in fostering development through assisted challenge. Recent developmental literature supports a move towards a more proactive teacher role in scaffolding children's thinking (Smith, 1998). Where earlier theories gave teachers permission to stand back and wait for development to happen, current theory challenges teachers to support and extend cognition (Meade, 1998; Meade & Cubey, 1995; Smith, 1998). Jordan (1999) explores the teacher's role in scaffolding and encourages teachers to develop skills in really listening to children and to extend children's thinking through co-constructed challenges. Carr's (1997, 1998a, 1998b) descriptions of learning dispositions, and suggestions for using Learning Stories as a framework for assessment in early childhood (described briefly above) help to increase the focus on children's thinking in ways that are consistent with the principles of the early childhood curriculum, *Te Whäriki* (Ministry of Education, 1996).

Early childhood teachers also have an important role to play in alerting parents and whanau to the vital contribution they make to their children's development of mathematical thinking. Young-Loveridge (1989) found that children with high levels of number knowledge had been exposed to a wide range of experiences involving number and came from homes where there was a strong orientation towards numeracy. These children had also watched their mothers solving everyday problems of their own. On the other hand, children with low number knowledge had few number experiences and came from homes where there was an orientation to literacy but not to numeracy, where there was little opportunity to watch their mothers using numbers to solve problems of their own, and where there were relatively low family expectations for the children's mastery of skills. The current "Feed the Mind" campaign in Aotearoa/New Zealand aims to show families that they can help children learn with simple, fun, everyday activities. Teachers can provide more information about such activities through noticeboards, newsletters and workshops.

With appropriate interactions and experiences at the informal and cultural level children are able to develop competence in the mathematics of their culture and are able to use it effectively to solve everyday problems. Competence in informal and cultural mathematical thinking provides important skills for life and also provides the basis on which the formal study of mathematics can build.

Formal and cultural level of mathematical thinking

The final level of mathematical thinking in the framework described by Carr, et al., (1994) is the formal and cultural level of mathematical thinking. This is where children begin the formal study of the mathematics of their culture, usually at school. Hughes (1986) looks at the difficulties children may experience in learning formal mathematics and highlights the importance of making links between the informal and cultural level of thinking and the more abstract formal and cultural thinking.

The New Zealand Early Childhood Curriculum, *Te Whäriki* (Ministry of Education, 1996), provides a framework for mathematics development in the early years, and shows clear links to the essential learning areas and essential skills of the school curriculum documents. Early childhood educators may also find it helpful to look at the *Mathematics in the New Zealand Curriculum* document (Ministry of Education, 1992). This describes both skills of mathematical processes, such as problem solving, reasoning, and communicating mathematical ideas, as well as the areas of knowledge and skill of number, measurement, geometry, algebra and statistics that form the basis of the school curriculum. Consideration of the suggested activities for the early levels of school learning will help early childhood educators recognise and articulate the mathematics children encounter in early childhood, and how this relates to the study of mathematics at the formal and cultural level. For example, a conversation in which children are classifying events from their experiences as certain, possible or impossible is an early investigation of probability (statistics) while a game involving children giving, or following, a sequence of instructions related to movement and position is exploring shape and space (geometry).

The *Making Things Count* resource (Ministry of Education, 1999) focuses largely on numeracy, which is only one aspect of mathematics. With the current focus on literacy and numeracy it is important not to overlook the other aspects of

mathematics. At the same time it is important to be aware of why numeracy has been seen as so important.

WHY IS NUMBER IMPORTANT IN THE EARLY CHILDHOOD YEARS?

Research has shown that there is substantial variability in levels of mathematics understanding, especially in relation to number, in early childhood and at the beginning school level in New Zealand (Gilmour, 1998; Yeoman, 1987; Young-Loveridge, 1987, 1993; Young-Loveridge et al., 1995) and elsewhere (Aubrey, 1993; Fuson, 1988; Hughes, 1981; Suggate et al., 1997). These early differences in children's understanding about number have been found to influence the children's level of achievement in mathematics at school. Children who are initially ahead tend to stay ahead, while those who are behind tend to stay behind. A study by Young-Loveridge (1991) showed that the problem was particularly acute for girls. Girls who started school with low levels of number knowledge fell increasingly behind their peers between the ages of five and nine, and looked likely to remain low achievers in mathematics unless specific steps were taken to address the problem.

More recently Wylie and Thompson (1998) found that children's scores at age five in Mathematics, and to a lesser extent, Literacy and Logical Problem-Solving, predicted their scores at age six (even though in most other subjects measures taken at five were not good predictors of later performance). The stability of mathematical performance in relation to early number knowledge raises questions about the nature of school programmes that do little to ameliorate early difference, and also draws attention to the important role early childhood education can play in influencing children's dispositions towards mathematical learning.

School Entry Assessment

In Aotearoa/New Zealand School Entry Assessment (SEA) activities have been developed in Numeracy, Literacy and Oral Language to help new entrant teachers identify children's skills on entry to school so that programmes can be tailored to the individual child (Ministry of Education, 1997). Testing at the new entrant level is not new, many teachers had developed their own assessments prior to SEA being provided (see Thackery, Syme & Hendry, 1992) and some continue to utilise their

own tests alongside SEA (Williams & Dixon, 1998) but the introduction of SEA has perhaps led to greater awareness of assessment procedures on entry to school, and stimulated debate on this issue.

The numeracy component of SEA called *Checkout/Rapua* was developed by Young-Loveridge, based on the skills identified by her 1991 research which showed correlations between skills children had on entry to school and children's later success in mathematics. Concern has been expressed that such a test may become a self-fulfilling prophecy, with children identified as competent being given more challenges, while those with low scores may "put at risk dispositions to be courageous, mindful, persistent and responsible" (Carr, 1997, p. 325). Carr (personal communication) adds that in her opinion it is not early assessment *per se* that might put these dispositions at risk because they might narrow teacher and parent expectation. Rather, the risk is that test measures which quantify the acquisition of basic skills become the only valued early assessment so that the outcomes of more complex and authentic projects, which can document the diversity, and some of the strengths, of a child's early understandings, may be lost from view (see James & Gipps, 1998). It would be unfortunate if this were to occur since the intention of the assessment is to identify the skills and understandings children bring to school, so that appropriate programmes can be planned. The assessment is also intended to inform school management about the children entering the school; if schools wish to return the results to the Ministry of Education, this information also contributes to national policy and the allocation of resources (Ministry of Education, 1997). Greater awareness of the skills children bring to school will hopefully avoid situations like the one identified by Young-Loveridge (1987) where teachers underestimated children's ability in mathematics and spent much of the first year teaching concepts the children already knew on entry to school. However, in the end, the advantages or disadvantages of School Entry Assessment will be determined by what the new entrant teachers do with the information.

Intervention studies

Several New Zealand studies have looked at ways of addressing early differences in number knowledge. The EMI-5s study explored the effectiveness of two kinds of

intervention procedures for improving the number knowledge of five-year-olds. Intervention A was school-based and involved withdrawing pairs of children to work with a teacher/researcher, using number books and games. Intervention B was home-based and involved a researcher visiting parents at home, five weekly parent workshops on supporting children's numeracy, and providing a library of books and games for parents to borrow to use at home with their children. Both interventions were successful but children in Intervention A made the highest gains and this intervention was successful for both boys and girls. Intervention B was more effective for boys, particularly when their parents showed a high level of commitment to the programme, while girls in Intervention B made gains that were similar to those made by children who were not involved in interventions (Young-Loveridge, 1993).

In a follow up to the EMI-5s study Peters (1991) monitored the effects of combining aspects of both the school-based and the home-based intervention programmes. Parents were invited to come into the classroom and play mathematical games and read number books to small groups of children. The effects of this intervention were large and persisted over the eight months in which the intervention took place. It was equally effective for both boys and girls. However, when a similar method was used with seven-year-olds it was more difficult to get parents to participate and the results were less impressive (Peters, 1994). The results of the two studies showed that games appeared to be most effective as a way of enhancing children's learning when a sensitive adult was available to support and extend children's learning as they played (Peters, 1998). Examples of some of these games have been included in this resource. Others are available in Young-Loveridge and Peters (1994).

The EMI-4s Study

A different approach was taken in the EMI-4s study, which looked at the mathematical understanding of four-year-olds at kindergarten and ways to enhance this (Young-Loveridge et al., 1995). Detailed observations of children's activities during the ongoing programme in four kindergartens revealed a relatively low incidence of numeracy occurring in the kindergarten context, even for children who were identified as 'expert' mathematicians. Following the observations, teachers in three of the kindergartens became actively involved in constructing resources and working with children to enhance their mathematical thinking, while those in the

fourth kindergarten continued with their existing programme. Many of the ideas developed during this study are incorporated in the resource *Making Things Count* (Ministry of Education, 1999). The exact approach taken in each of the three kindergartens differed but all included a greater focus on the mathematics involved in everyday routines and activities (e.g., carpentry, collage, baking, checking the roll, tidying up), exploring the mathematical opportunities in the areas and activities children were already engaged in (i.e. challenging a child's mathematical thinking in the context of the family play he/she was engaged in rather than inviting a child to "come and do some maths"), utilising number books, magnetic stories, games, songs and so on. Through observation and parent interview children's interests were identified and incorporated into the resources and activities that were developed. Analysis of children's mathematical skills before and after the intervention period showed a significant gain for the children attending the participating kindergartens that compared to those who attended the contrast kindergarten (Young-Loveridge et al., 1995).

The results of the EMI-4s study have exciting implications for teachers at the early childhood level because they showed that significant gains could be achieved by having an 'orientation towards numeracy' in their programme. The study also revealed how, through engaging in activities and conversations with the children, the teachers were able to develop an awareness of each child's current level of skill and therefore to offer activities which provided a suitable level of challenge.

The following sections provide resources and ideas for parents and teachers wishing to enhance young children's mathematical thinking.

NUMBER BOOKS FOR CHILDREN

Books provide an enjoyable and familiar context for developing and practising mathematical skills. Don't limit your choices to counting books. There is a wide range of books where mathematics is included as a meaningful part of the story. *A Handbook of Number Games and Books* (Young-Loveridge & Peters, 1994) contains an extensive annotated bibliography of children's number books and books for

parents and teachers. The following list provides information on additional titles not included in Young-Loveridge and Peters (1994). The same coding system for number skills included in each title has been used:

S – Sequence Number words in order (Sa = sequence starting above one; Sb = backwards sequence)
P - Process of counting objects is part of the story
C - Cardinality Pictures correspond to number words/numerals in text
N - Numerals
O - Operations Adding and/or subtracting is part of the story

Adding your own books to a list like this provides a useful reference of the number skills children may be using while enjoying books. You may wish to extend the categories to capture a broader range of mathematical ideas.

Barber, Shirley: *Count with Me!* (Victoria, Australia: Noble Park, 1998)
A counting book with numbers from one to ten. Large, detailed illustrations provide lots of opportunities for counting. The simple text encourages exploration. For example "I can count 4 dolls playing on the beach. Can you see 4 of anything else?" The illustration provides more than twenty other sets of four, e.g. four boats, four birds, four crabs, four spades, four of each type of shell, four starfish, and so on. (S, P, C, N)

Micklethwait, Lucy:*I Spy Numbers in Art* (London: Harper Collins, 1993)
This book is designed to introduce children to art. Nineteen paintings and one Japanese woodblock (dating from the 15th century to the present day) are beautifully reproduced and the children are encouraged to study each picture and play 'I spy' looking for particular objects, e.g. "I spy ten hens", "I spy eighteen cherries". Numerals and number words are shown for numbers one to 20. (S, P, C, N)

Moore, Inga: *Six Dinner Sid* (New York: Aladdin, 1991)
A delightful story about a cat who lives in six houses and has six dinners, six beds, six different ways of behaving. All is well until he has to visit the vet and is taken by his six different owners and so receives six doses of medicine. When his owners discover

what has been happening they agree that Sid should only get one dinner a day, so Sid decides to move….. (S, P, C(6), N)

Mr Bear's Apple Tree: A magic Counting Book (Noble Park, Australia: The Five Mile Press, 1997)
Textured foam apples and cutaway pages provide a novel dimension. The first page shows ten apples on Mr Bear's tree and ten naughty bees coming to steal the apples. The number of bees and apples decreases by one each time a page is turned until finally there is only one apple left for Mr Bear to eat. Some sequences and cardinality are given but the main focus of the short rhyming text is to encourage the reader to count the apples and see what is happening. (S, Sb, P, C)

O'Leary, John: *Ten on a Train: A cut-out counting book* (London: Frances Lincoln, 1995)
Clever cut-outs provide a 3D effect. Ten animal friends are going for a ride. As problems arise with each mode of transport (train, bus, boat, go-cart, plane, etc.) one friend is left behind with each turn of the page (showing simple subtraction as well as sequence), until the last page where all the animals are united in time for tea. (Sb, P, C, N, O)

1 2 3 (Woodbrige, England: Funfax, 1998)
A chunky little 'first word' counting book, ideal for babies and toddlers. The numbers go from one to twelve. Each shiny cardboard page shows an attractive photograph of everyday objects and the corresponding numeral and number word. (S, P, C, N)

Potiki, Tahu, Rendall, Jenny & Huria, Gabrielle: *Te Kete a Rakaihautu.* (Christchurch: Ngai Tahu Development Corporation, 1998).
A short story with Maori text. Quantities from one to ten are illustrated, to show the objects in the kete. Illustrations are bold and clear and the number word is incorporated as part of a sentence, e.g., "E ono ka tuna". (S, P, C)

Sturges, Philemon: *Ten Flashing Fireflies* (New York: North-South Books, 1995)
Ten flashing fireflies are burning in the night sky. One by one they are caught and added to the children's jar. The pictures clearly show the decomposition of ten, e.g. two in the jar, eight in the sky, three in the jar, seven in the sky, until all ten are in the jar. When the fireflies' lights start to dim the children decide to let them go. (S, Sb, P, C, O)

Suzy's 123 count with me (Auckland: David Bateman, 1997)
A counting book with a New Zealand focus. Each double page shows a number from one to ten as a set of objects, a set of fingers, a numeral and the number word in both English and Maori. The illustrations feature children's television presenter Suzy Cato and familiar New Zealand wildlife such as tuatara, tui and kereru. The final page asks the reader to name the numerals from one to ten and identify the creatures that have been shown in the book. (S, P, C, N)

Voce, Louise: *Over in the Meadow: A counting rhyme* (London: Walker Books, 1995)
The catchy rhyming text is delightfully illustrated. The numbers increase from one to ten, with the pictures corresponding to the number words in the rhyme. For example "Over in the meadow in a snug beehive, Lived an old mother bee and her little bees FIVE." (S, P, C)

NUMBER GAMES

A Handbook of Number Games and Books (Young-Loveridge & Peters, 1994) also contains instructions and templates for making and playing more than 25 different games. Three of these have been reproduced here in Appendix A. The templates can be photocopied onto coloured card/paper or onto white card/paper and coloured in. Laminating the game boards will help to keep them in good condition. Some games can be enlarged onto old sheets, or drawn onto concrete with chalk or paint. With enlarged board games children move their bodies around the tracks (for games like Snakes and Ladders) instead of using counters. This provides active involvement and reduces the demand for fine motor skills. Illustrated fabric could be made into bean bags to use instead of counters for a larger version of the Dinosaur Game.

The games provided in *A Handbook of Number Games and Books* (Young-Loveridge & Peters, 1994) are just a starting point. The more you make and play games the easier it becomes to develop your own variations. Often these can be tailored to the children's interests or to the materials you have on hand.

When designing games for young children we have found it helpful to keep the rules very simple (or let children negotiate their own). Games can be played competitively or co-operatively, depending on the preferences of those involved (for some children competition provides motivation, others are distressed by it. Too much emphasis on winning and losing can distract from the mathematics.)

Games can be adapted to suit the needs of novice mathematicians (e.g., making dice that only have the numbers from one to three on the faces for a child who is just learning to count). This can be extended as the child's skills develop (e.g., replacing the dice with one that goes from one to six, using numerals instead of dots, including poly-dice with seven, ten or more faces, adding a second dice so that the scores can be added or subtracted to determine a move in a game).

We have found children also enjoy designing their own games. Counters, dice, boards, spinners and so on can be made available for children to develop their own activities.

The handbook of books and games can be ordered from:

Dr Jenny Young-Loveridge
Department of Education Studies
University of Waikato
Private Bag 3105
Hamilton
New Zealand

Sally Peters
Department of Education Studies
University of Waikato
Private Bag 3105
Hamilton
New Zealand

At the time of going to print the cost was $25 including postage.

RECOMMENDED READING FOR ADULTS

Resources for the early childhood centre.

Baker, A. & J. (1991). *Raps and rhymes and maths.* Armadale, Vic.: Eleanor Curtain Publishing

A collection of traditional and modern rhymes, riddles and stories with mathematical themes.

Griffiths, R. & Clyne, M. (1988). *Books you can count on: Linking mathematics and literature.* Portsmouth, NH: Heinemann

Although this is a resource for primary school teachers, early childhood teachers will find the concepts described useful as a guide for encouraging discussion of mathematical ideas.

Young Children. Washington DC: NAEYC

This publication regularly includes articles about mathematics, including numeracy. There are often reviews of recent research and practical suggestions from teachers.

Young-Loveridge, J. M. & Peters, S. (1994). *A handbook of number books and games*: From the EMI-5s study. Hamilton, New Zealand: University of Waikato.

There are four sections to this handbook: a summary of findings from the EMI-5s Study; an annotated bibliography of number stories; collections of number rhymes and songs and books for parents; a description of a range of number games and instructions on how to make the games and dice. There is also a description of the characteristics and significance of specific number skills and these specific number skills are identified with the individual number games.

Parent workshops

Griffin, R. & Coles, J. (1992). Assisting parents to develop their preschool children's mathematics. *SAMEpapers,* 81-89

Workshops for parents about mathematics for young children can result in parents both recognising the mathematical possibilities in everyday events and using these everyday events to enhance their children's mathematical understanding.

Young-Loveridge, J. M. (1993). The relationship between children's home experiences and their mathematical skills on entry to school, *Early Child Development and Care,* 43, 43-59.

Describes three home influences on young children's knowledge of number concepts: exposure to a wide range of experiences involving numbers, a strong orientation towards numeracy by members of their families, and the opportunity to observe their mothers using numbers to solve everyday problems of their own. Young-Loveridge stresses the importance firstly of increasing parents' awareness about the importance of numeracy and secondly of providing examples of how everyday experiences can help children learn number concepts. She notes many similarities with literacy development.

Young-Loveridge, J. M. (1996), The number language used by preschool children and their mothers in the context of cooking, *Australian Journal of Early Childhood,* 22, 16-20.

Describes cooking as an example of many familiar contexts in the home and the early childhood centre which are rich in mathematical activity and can be capitalised on to encourage and support children's mathematical learning. Adults can be most effective if they recognise the potential for mathematical learning and engage in reciprocal interactions supporting the children's learning.

Further reading about early childhood mathematics

Carr, M., Peters, S., & Young-Loveridge, J.M. (1994). Early childhood mathematics: a framework. In J. Neyland (Ed) *Mathematics education: A handbook for teachers*, Vol. 1. Wellington, New Zealand: Wellington College of Education. pp. 262-270.

If teachers recognise the levels of mathematical learning of individual children they can provide the appropriate support in consolidating this understanding and in moving from one level to the next. This can be achieved by a combination of provision of open-ended activities and adult scaffolding of children's understanding.

Carr, M., Peters, S., & Young-Loveridge, J.M. (1994). Early childhood mathematics: finding the right level of challenge. In J. Neyland (Ed.), *Mathematics education: A*

handbook for teachers, Vol. 1. Wellington, New Zealand: Wellington College of Education, pp271-282
Mathematical activities should be within young children's difficulty level. This level is a combination of the familiarity of the level of context, the level of understanding of the purpose, and the level of complexity of the mathematical input. Teachers should provide open-ended activities where the difficulty level can be manipulated by individual children and teachers can provide appropriate support.

Hughes, M. (1986). *Children and number*. Oxford: Basil Blackwell
This book argues for more recognition of young children's natural abilities to think about and use number. Using fascinating anecdotes, Hughes contrasts these early abilities with the difficulties that many children experience when faced with the more formal mathematics of school. Practical approaches (including games) are outlined to help young children acquire a better understanding of number and then make the link between concrete and abstract mathematics.

Hughes, M. (1986). Bridge that gap, *Child Education*, February, 13-15.
This article summarises research implemented by Hughes (see above) on young children's knowledge about number, providing excellent examples of this knowledge and children's use of a variety of strategies to count and represent number. Experiences for children that support and extend these understandings and strategies will ensure a sound foundation for the next challenge.

Price, G. G. (1989). Mathematics in early childhood, *Young Children*, May, pp. 53-58.
How recent research has influenced understanding of young children's mathematical development. Information processing theory emphasises concepts such as working memory and familiarity. Cognitive science examines the strategies and knowledge structures of individuals, with a focus on differences between novices and experts. The connection between skill at counting and strategies for simple word problems is emphasised.

Young-Loveridge, J. M. (1989). The development of number knowledge in five to eight-year-olds, *Delta, 42*, 25-37

A concise explanation about the importance of numeracy for young children in developing mathematical understanding. Previous emphasis on the work of Piaget is balanced by more recent research.

Young-Loveridge, J.M., Carr, M., & Peters, S. (1995). *Enhancing the mathematics of four-year-olds: The EMI-4s study*. Hamilton, New Zealand: University of Waikato.

An "orientation towards numeracy" in the centre programme describes a greater focus on the mathematics involved in everyday routines and activities. Highlights how identifying children's interests through observation and parent interview, and then incorporating these interests into the centre activities at an appropriate level of challeng, is effective in increasing the mathematical skills of young children.

MATHEMATICS (INCLUDING NUMERACY) IN AN EARLY CHILDHOOD CENTRE

Imagine an early childhood centre with a very effective, committed management committee, dedicated teachers who are either in training or trained, appreciative and supportive parents, and a well-planned and equipped centre environment. At this fictional centre there will be much mathematical learning occurring for young children.

Working with Parents

Let us imagine that several weeks ago parents at this centre expressed concern that their children needed more formal teaching in mathematics, to ensure success later at school. The head teacher finds it easy to understand the causes of parents' concerns. For example, she is aware that until fairly recently intelligence tests were relied on for decisions about school and career opportunities. These tests emphasised mathematical ability, so a person's ability at mathematics was considered very important. She is also aware that many parents remember their difficulties in understanding the relevance of school mathematics and wants to draw on the emphasis of the new school mathematics curriculum on using mathematics for everyday living.

The early childhood centre staff know the community well and are aware that several families have children already at school and/or children almost ready for school. The early childhood teaching team meet regularly with the local teachers of new entrants. In response to the parents' concerns, a joint meeting is organised for interested parents to explain the development of young children's learning from practical exploration of the physical world to the ability to problem solve without handling objects. This workshop also emphasises the role of parents in supporting this mathematical development and results in parents and teachers agreeing to purchase and construct equipment which would encourage mathematics outdoors.

Since there is less information available for parents about mathematics in early childhood and school compared, for example, to information about reading, the

teachers also decide to distribute a leaflet about mathematics in the home. The leaflet describes the potential for mathematical learning during events such as:

(i) when getting dressed: sorting out clothes to wear, matching socks and shoes, button to button-hole;

(ii) going on trips: the shapes of buildings, signs; discussing the distance to travel, estimating the time it will take, how much petrol is needed, how much is left;

(iii) when shopping: what do we need, how much do we need;

(iv) when cooking: weighing quantities, measuring amounts, making biscuit shapes, dividing quantities;

(v) when setting the table: one-to-one correspondence as a knife, fork and spoon is placed at each setting, working out how many places are needed;

(vi) when folding the washing: matching clothes to different family members, folding;

(vii) when tidying up: where do different toys belong, what is missing.

The leaflet also describes home activities involving *emergent numeracy*, such as board games, using the telephone, reading out numbers from letter boxes and signs, counting money, recognising patterns, discussing time in relation to the clock and calendar, activities which may include representation of number in numerals and words and, as children show interest, adding, subtracting, or multiplying.

Outdoors

At the entrance to this imaginary centre there is a whiteboard which welcomes parents and children to the centre and describes the daily programme. Today the news is that new outdoor equipment has arrived. Parents are thanked for their part in fund-raising and for their help in making outdoor game kits. Attached to the whiteboard are leaflets for parents describing the potential mathematical learning for children as they use the equipment and giving examples of how parents can support the mathematical learning of young children outdoors.

A challenge course has been set up with both commercial equipment, such as a nylon collapsible tunnel, climbing frame and rope webbing and recycled material such as tyres, car cases, old sheets and hessian, cable drums and planks. A 4-year-old has

organised the children into a line, describing them as being first, second, third, and so on, and using a large egg timer to make sure each child has equal time. A teacher suggests that the children could write their names with chalk on the asphalt path nearby and then put a mark by their names each time they complete a circuit. Although the teacher had carefully organised the course so as to encourage climbing, crawling, balancing and jumping skills, she is delighted when some children independently co-operate to adapt the circuit. She discusses their ideas, assists with moving the heavy pieces, and only intervenes when safety is at risk.

A rug and cushions have been placed next to the challenge course and one of several parents, who regularly assist at the centre is reading as children wait their turn on the equipment. The books such as *Bears in the Night, Going on a Bear Hunt, Rosie's Walk* and *My Cat Likes to Hide in Boxes*, have been chosen because of the mathematical concepts and language: *through, under, over, far, apart/close to, up, down, below, above* which reinforce the learning of the challenge course.

The teacher recognises that the language of young children may not be as advanced as their mathematical understanding and that adults are important in providing the relevant precise vocabulary. She provides this language as she assists children in completing the circuit. In this activity the main focus of the children appears to be on developing their physical skills and the teacher acknowledges their sense of achievement in this. At the same time children are developing spatial awareness (awareness of the space they take up and the space they fit into in relation to other things), ideas of size, distance, proximity, direction, and boundaries, contrasting spatial relationships, with vocabulary for concepts such as *up/down, high/low, in/out, over/under, above, high/low, on/off, far/near, inside/outside, upside down/right way up, top/bottom, behind/in front of, around, next to/beside, between.* They count the steps across a plank and the time to complete the circuit.

Nearby in the sandpit, there are children working with real purpose, estimating how much water should be added to maintain a shape, counting the number of containers of water to fill a bucket, how much sand needs to be patted into a bucket and how much sand needs to be shifted to make a roadway. The vocabulary and concepts involved include: *big/little, up/down, empty/full, holds more than/less than, not*

enough, around, under/on top of, higher than. Teachers at this centre have developed that special sense which tells them when to describe in precise language what the children are doing, when to supply more equipment to extend their play, when to ask questions about and/or suggest different solutions to problems, when to be physically involved, and when to stand back, observe and record the mathematical learning taking place.

The teacher could be observing in what ways a child attempts to make meaning from mathematical experiences. For example, a child might be talking to others to describe and analyse actions, to justify selected criteria, to clarify ideas, to experiment with language. Similarly, a child might be trying alternative solutions, estimating, checking, transferring knowledge to new situations, selecting appropriate materials, recognising the reasonable nature of results, becoming more accurate. Another aspect to observe could be how a child investigates. For example a child might investigate individually, requesting adult assistance, co-operatively, persistently, creatively, collaboratively, discussing and sharing information, selecting appropriate materials. The teacher also looks for how a child shows development of positive attitudes towards mathematics, for example, shows enjoyment or interest, is willing to experiment, shares discoveries and achievements, follows up interests.

The teachers know children need many experiences exploring the same materials, to help them develop basic ideas of position, shape, size, quantity, classification, order and space and they need varied experiences with different kinds of materials, before they can learn to generalise about the properties they are repeatedly exploring.

At the water play there are several groups of children engrossed in different types of play. This centre has plenty of space to allow a combination of activities to take place in each play area. On a low table there are several baby baths. Children are in various stages of washing dolls and washing dolls' clothes and hanging them on a low temporary clothesline nearby. Perhaps most of the focus is on dramatic play as "mothers" dress and undress their "babies", tell them not to cry or that they are naughty for getting so dirty. But there is also mathematical learning from filling and emptying the baths, discussing how many towels are needed, working out what size garments will fit different sized dolls. The vocabulary includes: *full/empty, float/sink,*

more than/less than, not enough/too much, heavy/light, fill up, too big/too small/right size. Some practical problem-solving occurs as when the line breaks and children discuss the difference to the weight of the washed clothes when the wet clothes are wrung out.

Nearby, several children become excited about how far the water from squeezed plastic bottles will go and then look for moving (preferably squealing) targets. A quick-thinking adult diverts them to targets painted on the fence, extending the play with plastic bags filled with water, balls and beanbags. The children enjoy practising their throwing skills as well as discussing why some objects go further than others, why there are different sounds on impact. The vocabulary includes: *force, further than/not as far as, high/low, distance, heavy/light* .

In the shelter of the veranda, there is a trolley containing a variety of interesting objects to sort into different compartments on the nearby trays. Objects to be sorted include plastic animals, buttons, cut up colour charts, small containers and matching tops, shapes such as squares, circles, triangles, and oblongs, and natural materials. Three children are working side by side each sorting into sets. The sets can be sorted in a variety of ways: colour, size, weight, shape, length, and relationship to one another. The adult nearby watches carefully before discussing with the children the properties of the objects and how the children are sorting.

The teacher may be observing what materials a child selects for exploration, how a child uses the materials: with enjoyment, with great or little interest, spontaneously talks about activity, experiments, matches, sorts, patterns, orders, builds, designs, predicts, hypothesises; how a child sorts objects: using one criterion (property), more than one criterion, imaginatively. Does the child sort, group and subgroup in relation to colour, size, shape, number, use, length, width, surface, texture, mass? Does the child choose his/her own criteria for classification? Do the child's patterns show relationships of number, size, shape? When/where does the child approximate, estimate and/or compare number, size, length? Does the child count, order, group, associate names with groups, match one to one?

Indoors

Because it is a fine day most of the children are outside, but inside there are many opportunities for mathematical learning besides the dough, clay, collage, painting, blocks, and puzzles.

Children can independently use recipes that combine diagrams, numbers and writing. Lockers and coathooks are labelled with numbers as well as names. Teachers model the use of number in checking the roll, writing a shopping list, allocating adults to children for a walk, sharing food, estimating pieces of equipment needed for an activity.

Teachers and parents know many songs, finger plays, rhymes and stories involving number. Songs may include both rote counting (*one, two, three four five, once I caught a fish alive*) and rational counting (*five currant buns in the baker's shop*). Attractive song charts in the centre have a combination of pictures, words, and numerals so children can select songs. Acting out the songs reinforces the counting and children are encouraged to use their fingers to help with counting. Hand and finger puppets, magnetic and flannel board characters provide effective variety. A number jacket with interesting objects, books or stories in numbered pockets sets the stage for activities which may focus on the related number. Parents and teachers vary the stories and activities as they build up their repertoire.

The dramatic play area is regularly supplemented with specific materials for different themes which include numeracy, for example, a doctor's surgery, with measurement for height, weight, and temperature, telephone book and appointment book; a restaurant: with money, table places, menus with prices; an airport, with flight numbers and times; scales for weighing baggage, and tickets.

Books include counting books, rhymes with counting, and books in which counting is integral to the story. Books have been made in the centre about children's interests and recent events. Both numerals and words have been introduced to represent numbers. In the resulting discussions, adults respond to the attraction young children may have to particular numbers, a sense of ownership because of their age, their address, telephone number, the number or their place in their family, their birthday.

Numbers can be related to location: where in the street would an address be, why some phone numbers begin with same number.

Board games are within easy access. Adults provide a variety of activities, model the required skills, encourage children's active participation and provide the language associated with the skills involved. Many of the games are designed to encourage co-operation rather than competition. Adults adapt the games to provide the appropriate level of challenge for different children, for example a dice with numbers to 3 for a novice, two dice for an expert where the numbers may be added or subtracted. The progress of results is recorded to encourage literacy and representation of the numbers.

Teachers

At this centre the mathematical learning of young children has been supported by the experiences offered, the active participation of teachers and parents, and the resulting discussion.

The teachers at this centre understand that young children's mathematical learning should not be separated from other learning in a child's life. They recognise that children who are involved in purposeful play can be supported in their mathematical learning by an adult who is aware of children's interests and knows about the progression of mathematical learning, from active exploration with concrete objects to manipulation of abstract numbers.

They strive to establish a partnership with parents/whanau which will provide the most effective support in young children's mathematical learning, sharing an understanding of individual children's interests, abilities and experiences and how to build on these with active participation and reciprocal discussion.

The teachers have learnt from research that the confidence, interests and skills of parents and family influence children's understandings of, and skills in, numeracy. They know that unless teachers are proactive, some children will have little interaction with numbers at an early childhood centre, focusing on other experiences available. If the home is providing this support the children will be fine. If the home

isn't, they may begin school behind other children and, because of the large numbers of children in a typical primary school class, these children often slip further behind. The early childhood centre needs to support all children's emerging numeracy.

A SUMMARY OF THE EARLY CHILDHOOD TEACHER'S ROLE IN PROVIDING MATHEMATICAL EXPERIENCES FOR CHILDREN.

At the early childhood level the teacher's role is to support and scaffold children as they develop a number sense. By exploring number concepts in the context of their own experience and the world around them, children are empowered to become users of the cultural tools about mathematics that exist around them.

The following is a summary of how teachers work to achieve this outcome:

Development of a number sense with groups of children

Teachers work for children to :

- discover these things called numbers;
- acquire the names for numbers;
- find numbers can be fun, in songs, rhymes, and jingles;
- develop an understanding that numbers can represent objects, ideas;
- understand numbers are useful in naming or representing objects, for sorting, taking turns, sharing, creating order and seeing patterns;
- realise that other children and adults can share the same understanding of number and may have different understandings;
- become involved in cooperative play;
- problem solve together.

Development of a number sense with individual children

Teachers:

- observe individual children to learn about their interests, their level of understanding of number, and why and how they use numbers;
- introduce number to the activities individual children are interested in, linking with their interest, purpose and goals at the time.

Development of a number sense: planning and acting at the centre level.

Teachers:

- look at the centre programme and environment to see what opportunities exist for children to explore, enjoy and use number, and how these opportunities could be increased, varied or made more explicit (an orientation to numeracy);
- provide many experiences exploring the same materials, and varied experiences with different kinds of materials to explore ideas about number and develop their understanding of how numbers can be fun and useful;
- seek opportunities to provide the names, order and uses of numbers;
- use and talk about numbers themselves during the routines of the centre day;
- share the enjoyment of numbers in stories, songs, rhymes, and jingles;
- provide books, puzzles, and games that include number;
- provide tools and equipment involving number and model their uses especially in the literacy centre and dramatic play area.

Development of a number sense: working with parents

Teachers:

- share research about the development of young children's mathematical understanding;
- share information about experiences provided in the centre, home and community which involve number and how children demonstrate their understanding of and use number;
- are enthusiastic with parents about their shared roles in supporting children's developing understanding of number;

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Appendix A

Templates and instructions for the following three games taken from *A Handbook of Number Games and Books* (Young-Loveridge & Peters, 1994). See p. 16 for further details.