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Accounting for productivity growth in a small open economy: sector-specific technological change and relative prices of trade*

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Abstract

Many economies experienced a slowdown of measured productivity in the 2000s, coinciding with the commodity price boom. We use a multisector growth model for a small open economy to quantify the contribution of sector-specific technology and relative prices of trade to productivity slowdown. We show that the effective aggregate total factor productivity consists of two components: the weighted average of sector-specific technology, and the weighted averaged of domestic-export price ratios which reflect export costs. This extends the Domar aggregation result of Hulten (1978). When calibrated to the Canadian data, the model suggests that productivity slowdown was mainly attributed to two sectors: commodity; machinery and equipment. Cross-country data show that, in two thirds of countries that experienced productivity slowdown, slower productivity growth in sectors serving domestic market was a dominant factor, while in the other one third, reduced domestic-export price ratio played a major role.

Keywords: Productivity measurement, sector-specific technological change, input-output linkage, relative price.

JEL codes: O41, F43, D24, D57, E22.

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1 Introduction

In many economies, total factor productivity (TFP) boomed in the 1990s and then slowed down in the 2000s. For example, in Canada TFP accounted for about a quarter of the annual growth of gross domestic product (GDP) from 1981 to 2000. Starting in the early 2000s, TFP growth slowed down, and GDP growth was driven by capital deepening (i.e. increase in the capital to labor ratio). Productivity slowdown coincided with a global commodity price boom, and an improvement in the terms of trade (i.e., the export-import price ratio) in many countries. Do these movements of relative prices account for the stagnant growth of measured total factor productivity in the 2000s? Without productivity growth, what drives capital deepening?

Prices of trade relative to domestic prices can account for the growth of measured total factor productivity. While changes in relative prices shift the production frontier, they do not affect input mixes, for example, following a reduction in the cost of international trade. To examine the importance of relative prices of trade in accounting for the growth of measured total factor productivity, we use a canonical model of multiple-sector growth for a small open economy. The model takes into account sector-specific technological changes, input-output linkages, and trade.

Two results are established. First, the growth of measured aggregate total factor productivity (in producing gross domestic product) consists of two components: the weighted average growth of sector-specific productivity in producing real gross output (which is deflated by domestic price), where weights are ratios of sector-level gross output over aggregate gross domestic product (i.e., Domar weights); and the weighted average change in the domestic-export price ratio, where weights are shares of sectoral exports in aggregate gross domestic product. This result extends Hulten (1978), who derives the Domar aggregation of sector-level technological changes for a closed economy. The domestic-export price ratio in our model reflects the productivity of producing exported goods relative to that of producing domestic goods, it may also reflect the export cost. When this cost falls, the domestic-export price ratio becomes larger, equivalent to an improvement in aggregate productivity. Second, a reduction in import price has no first-order impact on the growth of aggregate total factor productivity. Rather, declined import price leads to a higher output through a larger capital-labor ratio.

These results confirm findings of previous studies that change in the terms of trade *per se* has no first-order impact on aggregate productivity growth. Changes in export price (relative to domestic price) account for aggregate total factor productivity growth, while changes in import price affect capital deepening. The magnitude of these effects differs between the export price and the import price. The effect of export price on aggregate productivity depends on the size of changes in relative export price and the share of individual exports in gross domestic product. On the other hand, the effect of import price on capital deepening depends on both the size of changes in import price and the share of individual imports in the composite intermediate input. If an improvement in the terms of trade is accompanied by a change in the domestic-export price ratio, the terms of trade may still be correlated with aggregate total factor productivity.

When calibrating the model to the Canadian economy, we find that the slowdown of aggregate total factor productivity in the 2000s was attributed largely to two sectors: commodity; and machinery and equipment. The sector producing machinery and equipment experienced a substantial increase in the domestic-export price ratio. So did the commodity sector but to a lesser extent. Therefore the contribution of these two sectors to productivity slowdown lies in the part of production serving the domestic market. Empirical evidence supports that increased domestic-export price ratio in the 2000s reflects a higher relative productivity in producing the exported goods, rather than an reduction in the export cost. Finally, the two periods, the 1980s and the 2000s, appear similar in that the slow productivity growth was associated with sectors or part of production serving the domestic market, such negative impact was offset by increases in the domestic-export price ratio.

The cross-country data based on the Penn World Table shows that the slowdown of productivity in producing gross output is the main factor leading to slower growth of aggregate productivity in the 2000s, as in Canada. On the other hand, half of the countries that experienced productivity slowdown also saw a decline in the domestic-export price ratio, and in more than dozens of countries, this decline was the dominant cause of productivity slowdown. These findings contribute to better understanding the roots of productivity growth slowdown in the 2000s for small open economies. Our model suggests that searching for causes of slower productivity growth in sectors serving the domestic market is constructive.

Our result regarding the decomposition of effective aggregate total factor productivity extends the Domar aggregation in Hulten (1978) to an open economy. We derive the decomposition from the long-run equilibrium of a multi-sector growth model, while Hulten (1978) uses the standard growth accounting framework in which the growth of primary inputs (capital and labor) is measured but not accounted for by exogenous factors. On the role of domestic-export price ratio, our result is closely related to Burstein and Cravino (2015). These authors show that, if changes in the export cost are reflected in the producer price, such changes then affect aggregate productivity. We generalize their result that, the domestic-export price ratio is a component of aggregate total factor productivity. This ratio can reflect both the export cost and the relative efficiency of producing exported goods.

Our paper complements to Diewert and Morrison (1986) and Kohli (2004) among others on measuring the effect of improvement in the terms of trade on real domestic income. In these studies, an aggregate production function is assumed to produce real gross domestic product. An improvement in the terms of trade is equivalent to an increase in total factor productivity. In their frameworks, imported goods are considered only as a negative component of real gross domestic product, an increase in imports does not affect quantities of final demands as such link is absent in the aggregate production function. In contrast, we show that the terms of trade have no direct first-order impact on the measured aggregate total factor productivity, partly because a change in import price does not affect value added when production functions display constant returns to scale.

Two papers, Kehoe and Ruhl (2008) and Kim (2014), also study the role of the terms of trade in productivity growth. Our results are different from theirs in two aspects. Both studies show that, in the presence of the import tariff, a change in the terms of trade has a first-order impact on measured productivity, because gross domestic product at the market price (expenditure plus tariff revenue) is used by these authors. However, we show that, if the value added on the producer side is used as a measure of gross domestic product, as is commonly practiced by statistical agencies to measure productivity, neither the terms of trade nor the import tariff can explain measured productivity. Further, both studies assume that export price and domestic price are the same and normalize them to be one, the terms of trade they use are in fact the import price, which does not explain productivity in the absence of import tariff.

Our quantitative analysis on the contribution of sectors to aggregate productivity is related to previous studies of investment-specific technological change. Greenwood et al. (1997) use a one-sector growth model and find that the declining relative price of machinery and equipment (reflecting faster investment-specific technological change) accounts for more than half of the post-war output growth in the United

States. In Ngai and Samaniego (2009), machinery and equipment is both an investment good and an intermediate input; they find that equipment-specific technological change accounts for most of the postwar output growth in the U.S. Our model extends Ngai and Samaniego (2009) to an open economy, taking into account the decline of relative prices of equipment produced both at home and in foreign countries. Different from these studies, our paper suggests that it is the relative productivity of producing exported equipment that contributes to aggregate productivity growth.

For the rest of the paper, in Section 2 we use a simple standard model to present our main result. In sections 3 and 4, we derive our main result from a multisector growth model with input-output linkages and trade. This is used for quantitative analysis. In Sections 5 and 6, we examine the decomposition of aggregate productivity in Canada and other countries. Finally, Section 7 concludes.

2 Productivity and relative prices revisited

We use a simple static model to present the main result.[1] There are two sectors in the economy, one produces the good for final uses (consumption and export) out of labor and intermediate input, the other produces the intermediate input using the imported good and the final good produced at home. Imports are financed with exports, so trade is balanced. Producers pay an import tariff, which is transferred to the household as a lump sum. Labor supply (L) is inelastic and there is no capital. To explicitly characterize the contribution of relative prices to measured productivity, we solve for the competitive equilibrium instead of the social planner's problem. The household's problem is to maximize utility $u(C) = \ln(C)$, subject to $C \le wL + \tau_f p_f m_f$, where τ_f is the import tariff, p_f and m_f are respectively the price and the quantity of the imported intermediate input. Note that consumption price is normalized to one.

The production function for the final good is $q = AL^{1-\alpha_m} M^{\alpha_m}$, where the composite intermediate input is combined using imported and locally produced goods, $M = \left(\frac{m_h}{1-\varphi}\right)^{1-\varphi} \left(\frac{m_f}{\varphi}\right)^{\varphi}$. The final good is used for consumption, export, and as an intermediate input:

$$p_q q = C + p_x X + m_h,$$

where p_q is the implicit price of the final good and p_x is the export price. Note that p_q can be different from the consumption price. The export is positive even if p_x is different from the domestic price (of consumption), because we can think that either the export is produced with a technology displaying a productivity different from that in the domestic sector, or exporting is costly. We will be clear on this later in the full model. On the income side, the zero-profit condition implies that

$$p_q q = wL + (1+\tau_f) p_f m_f + m_h.$$

In the presence of import tariff, the value added on the producer side, $wL = p_q q - (1+\tau_f) p_f m_f - m_h$, and the gross domestic product on the expenditure side, $C = wL + \tau_f p_f m_f$, are different. The latter can also be expressed as $C = p_q q - p_f m_f - m_h$, because of balanced trade ($p_x X = p_m m_f$). The difference lies in revenue from the import tariff being transferred to the household.

In measuring productivity, it is practiced by statistical agencies that gross domestic product at the basic price, instead of at the market price, should be used as the measure of output, see for example Statistics Canada

[1] I thank an anonymous referee for suggesting this section.

(2008) and Pilat and Schreyer (2001). Gross domestic product at the basic price represents the value of output received by the producer, in this case it equals to $wL = p_q q - (1+\tau_f) p_f m_f - m_h$. Gross domestic product at the market price is gross domestic product at the basic price plus taxes on products (both imported and produced at home). Tax on products in our case is the import tariff. Valuing gross domestic product using the basic price is appropriate for measuring productivity because such a measure of output reflects the income that goes to the services of primary input factors in production (in this case the labor supply L). This implies that productivity measured in this way may not be welfare relevant.

The real value of gross domestic product is to be defined. In national accounts and in measuring productivity, the real gross domestic product is measured as a chained quantity index, with either the Törnqvist index or the Fisher index. These indexes are a discrete-time approximation of the Divisia index. The latter is a weighted geometric mean of growth rates of components of real gross domestic product, where weights are current-price shares of individual final demands.[2] To obtain a definition of the real gross domestic product that is consistent with national accounts, we use the Divisia index.

We show that, if gross domestic product at the basic price (equal to the producer's value added) is used to measure total factor productivity, the import price does not account for productivity, regardless of the import tariff. The export price (relative to domestic price) can contribute to the measured productivity, which can be calculated if the implicit price of final good, p_q, is set to be the same as the domestic price (of consumption price in this case).

However, if gross domestic product is measured at the market price and the import tariff is non-zero, both the export and the import prices can account for measured productivity. The terms of trade itself does not fully capture the contribution of trade prices, even if trade is balanced.

2.1 Gross domestic product at the basic price

Using the optimal condition with respect to the composite intermediate input, we can write real gross output as $q = \alpha_m^{\frac{\alpha_m}{1-\alpha_m}} A^{\frac{1}{1-\alpha_m}} \left(\frac{p_q}{\overline{p}_f^{\varphi}}\right)^{\frac{\alpha_m}{1-\alpha_m}} L$, where $\overline{p}_f = (1+\tau_f)p_f$, and we have used the equilibrium price of the composite intermediate input $P_m = \overline{p}_f^{\varphi}$. Gross domestic product at the basic price equals the value added of final good producer, $p_q q - m_h - \overline{p}_f m_f$. For the real value added, we use the Divisia quantity index, aggregated from gross output q and the intermediate input M with the latter entering as a negative contribution to the real value added.[3] That is, $y = q^{s_q} M^{-s_m}$ with $s_q - s_m = 1$. We know that the optimal level of composite intermediate input is $M = \alpha_m \frac{p_q}{P_m} q$. This leads to $y = \left(\alpha_m \frac{p_q}{P_m}\right)^{-s_m} q$. Real gross domestic product at the basic price then becomes

$$y = \alpha_m^{\frac{\alpha_m}{1-\alpha_m} - s_m} A^{\frac{1}{1-\alpha_m}} \left(\frac{p_q}{\overline{p}_f^{\varphi}}\right)^{\frac{\alpha_m}{1-\alpha_m} - s_m} L.$$

We knew that $P_m M = \alpha_m p_q q$, then $s_m = \frac{\alpha_m}{1-\alpha_m}$. This shows that the import price has no impact on measured total factor productivity for the production of real value added, since $y = A^{\frac{1}{1-\alpha_m}} L$. The import tariff

[2] See Hulten (1973) for Divisia index numbers. Diewert (1976) shows that Törnqvist index is exact to the Divisia index if the aggregate production function is homogeneous translog.

[3] In practice, this is used by statistical agencies to measure the real value added, see Pilat and Schreyer (2001). Valued added defined this way is a general form of the double deflation in which gross output and intermediate input are deflated by their own implicit prices separately, see Bruno (1978).

affects the price of composite intermediate input, P_m. Thus, tariff does not affect measured total factor productivity (again for the production of gross domestic product). As the effective import price changes with tariff, import increases, leading to an increase in gross output, but the value added does not change, because increased gross output is offset by the increased import (used as an intermediate input).

The contribution of export price to measured productivity does not present either, it is already implicitly reflected in measured total factor productivity A. This is because the implicit price of gross output is an index aggregated from prices of its components (consumption, export, and intermediate input). If we use the Divisia index, we have $p_q = p_x^{s_x}$ where s_x is the share of export in the total amount of gross output.[4] If the implicit price p_q is set to be the same as domestic price (which in this case is one), the contribution of export price can then be explicitly measured.If we set the implicit price of gross output to be one (same as the consumption price), let the real value of gross output be $\tilde{q}$, the corresponding total factor productivity in the production function of $\tilde{q}$ is $\tilde{A}$. The current-price value is of course unchanged, $p_q q = \tilde{q}$. This suggests that $A = \frac{\tilde{A}}{p_x^{s_x}}$. Therefore, if p_q is set to be the same as the price of gross output sold in the domestic market, not only we can still properly measure the total factor productivity in the sense that it is consistent with that in the data, but also the contribution of export price is now explicitly measured. This is the main result of our paper.

2.2 Gross domestic product at the market price

Gross domestic product at the basic price is used in practice by statistical agencies to measure productivity. We now examine the productivity measurement if gross domestic product at the market price is used. In this case, the current-price gross domestic product is $p_q q - p_f m_f - m_h$. The Divisia quantity index for the real gross domestic product is defined as $y = q^{\tilde{s}_q} M^{-\tilde{s}_m}$ with $\tilde{s}_m = \frac{p_f m_f + m_h}{p_q q - p_f m_f - m_h}$ and $\tilde{s}_q - \tilde{s}_m = 1$. Using again optimal conditions with respect to intermediate inputs, real gross domestic product at the market price is then expressed as[5]

$$y = \alpha_m^{\frac{\alpha_m}{1-\alpha_m} - \tilde{s}_m} A^{\frac{1}{1-\alpha_m}} \left(\frac{p_q}{\overline{p}_f^{\varphi}} \right)^{\frac{\alpha_m}{1-\alpha_m} - \tilde{s}_m} L,$$

where again we have used $P_m = \overline{p}_f^{\varphi}$. The share of intermediate input (net of tariff) is $\tilde{s}_m = \frac{\left(1 - \frac{\varphi \tau_f}{1+\tau_f}\right)\alpha_m}{1 - \left(1 - \frac{\varphi \tau_f}{1+\tau_f}\right)\alpha_m}$, which is smaller than $\frac{\alpha_m}{1-\alpha_m}$ unless $\tau_f = 0$. Clearly, measured productivity for the real gross domestic product includes both the total factor productivity of producing gross output and the relative price of the imported good. We can show that $\frac{\alpha_m}{1-\alpha_m} - \tilde{s}_m = \frac{\alpha_m \varphi \tau_f / (1+\tau_f)}{(1-\alpha_m)\left[1 - (1 - \varphi \tau_f / (1+\tau_f))\alpha_m\right]}$,

Further, if the implicit price of gross output is set as $p_q = p_x^{s_x}$, real gross domestic product at the market price becomes

$$y = \alpha_m^{\frac{\alpha_m}{1-\alpha_m} - \tilde{s}_m} A^{\frac{1}{1-\alpha_m}} \left(\frac{p_x^{s_x}}{\overline{p}_f^{\varphi}} \right)^{\frac{\alpha_m}{1-\alpha_m} - \tilde{s}_m} L.$$

Here, $s_x = \frac{p_x X}{p_q q} = \frac{\varphi \alpha_m}{1+\tau_f}$. Now use domestic price (equal to one) to deflate gross output, we showed that

[4] In fact, the price of gross output is $p_q = p_x^{s_x} p_d^{1-s_x}$ where p_d is the domestic price (aggregated from consumption and intermediate input). We normalized $p_d = 1$.

[5] Note that the optimal choice of intermediate input, both imported and produced at home, is made by the producer, hence the same as in the case of gross domestic product at the basic price.

$A = \frac{\tilde{A}}{p_x^{s_x}}$, use it in the above value added production function, we have

$$y = \alpha_m^{\frac{\alpha_m}{1-\alpha_m} - \tilde{s}_m} \tilde{A}^{\frac{1}{1-\alpha_m}} \left(\frac{p_x^{-1}}{\overline{p}_f^{\frac{\varphi \tau_f}{1-\alpha_m}}} \right)^{\frac{\varphi \alpha_m}{(1+\tau_f)\left[1-\left(1-\varphi\tau_f/(1+\tau_f)\right)\alpha_m\right]}} L.$$

Total factor productivity consists of two components: efficiency in the production of gross output and relative prices of trade. A change in the terms of trade can be associated with changes in measured productivity, if gross domestic product is measured at the market price, even when trade is balanced. However, such relationship is ambiguous. It is the export price and the import price, respectively relative to domestic prices, that can account for measured aggregate productivity.

2.3 Comparison with the literature

The above results extend Burstein and Cravino (2015), who find that the trading technology (export cost) can account for measured productivity if such technology is reflected in the producer price. We showed that it is the relative export price that can contribute to measured productivity growth. The export cost is but does not necessarily reflected in the export price, the latter also reflects the production technology of producing the exported good. A reduction in export cost and an increase in the neutral technology of producing the exported good are equivalent, one of which needs to be observed or measured in order to differentiate their relative contribution.

Our results are also different from those in Kehoe and Ruhl (2008) and Kim (2014). Both studies show that, in the presence of import tariff, a change in the terms of trade has a first-order impact on measured productivity, because gross domestic product at the market price (expenditure plus tariff revenue) is used for this result. We also show that the terms of trade can account for measured productivity. However, if the value added on the producer side is used as a measure of gross domestic product, neither the terms of trade nor the import tariff can explain measured productivity. Further, both studies normalize the export price to be one and assume that it is the terms of trade that matters for analysis. Our result suggests that, in fact both the export and the import prices can account for measured productivity, not the terms of trade per se.

3 A multi-sector growth model

We now use a multiple-sector growth model for a small open economy to formally derive the decomposition of aggregate total factor productivity, which is then used for quantitative analysis of global productivity slowdown in the 2000s. The model takes into account both the sector-specific technological change and trade. There are four sectors in the model economy, producing respectively machinery and equipment, structures, natural commodity (crude energy and mining products), and the rest of goods. We categorize sectors in this way because relative prices across these sectors exhibit salient movements in the data, and equipment-specific technological change was shown previously to be important in aggregate productivity. Each sector produces one single good, which can be used for investment, export, final consumption, and as an intermediate input. In the domestic market, equipment is used for consumption, investment and as an intermediate input. Structures are used only for investment. Commodity is used as an intermediate input. The rest of goods is an aggregate of outputs not belonging to the first three groups, which can be used as an intermediate input and for final consumption. We often call this last sector The Rest. Goods other than structures can be exported and imported.

3.1 Production in sectors

We start with notation. Let sector $j = 1,2,3,4$ produce respectively equipment, structures, commodity, and the rest of goods. Let p_j be the price of domestic output produced by sector j, and let $p_{fj}, j = 1,2,3,4$ be the price of imported good j. Producing a good requires capital (equipment $K_{e,j}$ and structures $K_{s,j}$), labor L_j and a composite intermediate input M_j. Let r_e and r_s be the rental price of respectively machinery and structures, w be the wage rate of labor input, P_e be the machinery price, P_s be the price of structures, and P_m be the price of composite intermediate input. Note that p_1 and P_e are different because equipment investment is assembled using both domestically produced and imported machinery.[6]

Neutral technological efficiency in sector j is $A_{jt+1} = a_j A_{jt}$, where a_j is the gross rate of growth, and $a_i \neq a_j$ for $i \neq j$. Production function of gross output in sector j is given by

$$q_j = A_j \left(K_{e,j}^{\alpha_{ej}} K_{s,j}^{\alpha_{sj}} L_j^{\alpha_{lj}} \right)^{1-\alpha_{mj}} M_j^{\alpha_{mj}}. \tag{1}$$

Factor shares differ across sectors, but satisfy the assumption of constant returns to scale, i.e., $\alpha_{ej} + \alpha_{sj} + \alpha_{lj} = 1$ and $\alpha_{mj} \in (0,1)$ for all j.

In the above Cobb-Douglas production function, the composite intermediate input is substitute for, not complementary with, capital and labor. This function form allows us to characterize the stationary growth path, and is also a reasonable approximation, as the share of intermediate input in total input cost is roughly constant over time in the Canada and some other countries.

Heterogeneity in factor shares is meant to be realistic. In the input-output table of some countries, such as Canada, properly calculated factor shares are not significantly different among the sectors 1, 2, and 4, specifically if leased capital is considered as a capital cost, instead of an intermediate input (services as in the current national accounts). The commodity sector, however, is different, where capital share is significantly larger than in other sectors. One implication of heterogeneous factor shares is that relative output prices reflect not only the relative level of sector-specific technological efficiency but also differences in factor shares across sectors. Greenwood et al. (1997) show that in a reasonably calibrated model, differences in capital intensity in production are not enough to generate the observed ratio of equipment to real gross domestic product.[7]

3.2 Uses of sector-level output

In general, output produced by sector j can be used as an input to produce the composite intermediate input, and for final uses (consumption, investment and export). Market clearing condition for the gross output of sector j is given by

$$q_j = c_{hj} + m_{hj} + v_{e,hj} + v_{sj} + \tau_{xj} x_j, \tag{2}$$

where on the right hand side are consumption, intermediate input, investment in machinery, investment in structures, and export. Here, $\tau_{xj} > 0$ reflects the export cost, for each unit of export, the producer needs

[6] We omit the time subscription, if without creating confusion.

[7] Herrendorf et al. (2013), using data for the United States, find that common factor shares with different neutral technological changes between sectors capture well the main forces behind the structural transformation in the U.S.

to ship τ_{xj} amount of sector-j exported good. The export cost is $(\tau_{xj}-1)$ per unit of export. Alternatively, $\frac{1}{\tau_{xj}}$ can be interpreted as the total factor productivity in producing the exported good in sector j, and the production function is a linear technology using sector j's output as the input. Let q_{xj} be the output of sector j that is used to produced export of sector j, export of sector j is produced with a linear technology $x_j = \frac{q_{xj}}{\tau_{xj}}$. This measure of total factor productivity in producing export j is of course relative to that of the production in sector j. In the data, we rely on empirical evidences to separate the two interpretations.[8]

Equation (2) is a general form. Specific uses of gross output differ across sectors. Output in sector 1 (machinery) is used for consumption (c_{h1}), domestic investment ($v_{e,h}$), as an intermediate input (m_{h1}), and for exporting (x_1). Output in sector 2 (structures) is used for investment (v_s) in structures. Output in sector 3 (commodity) is used as an intermediate input (m_{h3}) and for exporting (x_3). Output in sector 4 (the rest) is used as an intermediate input (m_{h4}), for exporting (x_4), and for final consumption (c_{h4}). Final uses are summarized as follows

$$\begin{aligned}
q_1 &= m_{h1} + c_{h1} + v_{e,h} + \tau_{x1} x_1, \\
q_2 &= v_s, \\
q_3 &= m_{h3} + \tau_{x3} x_3, \\
q_4 &= m_{h4} + c_{h4} + \tau_{x4} x_4.
\end{aligned}$$

3.3 Intermediate input

The composite intermediate input is produced using both domestic outputs and imported goods, given by the following production function

$$M = \prod_{i=1}^{4} \left(\frac{m_{hi}}{\lambda_{hi}}\right)^{\lambda_{hi}} \prod_{i=1}^{4} \left(\frac{m_{fi}}{\lambda_{fi}}\right)^{\lambda_{fi}}. \tag{3}$$

We assume that this production function displays constant returns to scale, or $\sum_{i=1}^{4}(\lambda_{hi}+\lambda_{fi}) = 1$. Structures are not used as an intermediate input ($\lambda_{h2} = 0$) and not traded ($\lambda_{f2} = 0$). To import the sector i good incurs a tariff τ_{fi}. The optimal amounts of domestic and imported intermediate inputs of type i satisfy

$$p_i m_{hi} = \lambda_{hi} P_m M, \;\; (1+\tau_{fi}) p_{fi} m_{fi} = \lambda_{fi} P_m M, \;\; i = 1,3,4. \tag{4}$$

Substituting optimal choices of the other sectors (domestic and imported) in the first-order condition for one input, we can obtain the equilibrium price of the composite intermediate input as $P_m = \prod_{i=1}^{4} p_i^{\lambda_{hi}} [(1+\tau_{fi}) p_{fi}]^{\lambda_{fi}}$. To simplify the notation, let $\overline{p}_{fi} = (1+\tau_{fi}) p_{fi}$.

3.4 Investment

Investment in structures is produced by sector 2. Capital accumulation for non-residential structure is $K'_s = (1-\delta_s) K_s + v_s$, and the investment price of structures is the output price of sector 2, $P_s = p_2$.[9]

[8] The wedge τ_{xj} can also be linked to other factors. Cao et al. (2015), using micro-level data and in the context of exchange rate pass through, find that changes in the domestic-export price ratio is associated with the currency of invoice in domestic and foreign markets.

[9] A capital adjustment cost can be added to capital accumulation, with which stationary growth exists. For example, let $K' = (1-\delta)K + x - \frac{\gamma}{2} \cdot \frac{x^2}{K}$. If capital grows at rate of g_k, then $(g_k - 1 + \delta)K = x - \frac{\gamma}{2} \cdot \frac{x^2}{K}$. Solving for x, we obtain $\frac{x}{K} = \frac{1 \pm \sqrt{1-2\gamma(g_k+\delta-1)}}{\gamma}$,

Capital accumulation for machinery and equipment is $K'_e = (1-\delta_e)K_e + v_e$. Investment in machinery and equipment is assembled from domestic and imported machines with a Cobb-Douglas production function, as follows

$$v_e = \left(\frac{v_{e,h}}{1-\psi}\right)^{(1-\psi)} \left(\frac{v_{e,f}}{\psi}\right)^{\psi}. \tag{5}$$

By the first-order conditions with respect to $v_{e,h}$ and $v_{e,f}$, we have $p_{e,h}v_{e,h} = (1-\psi)P_e v_e$ and $(1+\tau_{f1})p_{e,f}v_{e,f} = \psi P_e v_e$. Note that here $p_{e,h} = p_1$ and $p_{e,f} = p_{f1}$. On the stationary growth path, gross growth rate of the investment good equals the product of the growth rate of relative price of imported investment goods and the growth rate of imported investment good. Market-clearing price of the machinery investment is obtained as $P_e = p_1^{(1-\psi)}[(1+\tau_{f1})p_{f1}]^{\psi}$, or $P_e = p_1^{(1-\psi)}\overline{p}_{f1}^{\psi}$.

3.5 Export and import

The structure of production in the rest of the world is similar to the domestic economy, but is simplified. Export prices, p_{xj}'s, are determined in a competitive world market. Suppose that the aggregate production function in the rest of the world is given by

$$q^* = A^* F(K_e^*, K_s^*, L^*, m_h^*)^{(1-\lambda_f^*)} \cdot X^{\lambda_f^*},$$

where X is the composite intermediate input produced by our home country. It is aggregated from a Cobb-Douglas technology $X = \prod_{j=1}^4 \left(\frac{x_j}{\tilde{\lambda}_{fj}^*}\right)^{\tilde{\lambda}_{fj}^*}$ with $\tilde{\lambda}_{fj}^* = \frac{\lambda_{fj}^*}{\lambda_f^*}$ and $\sum_{j=1}^4 \tilde{\lambda}_{fj}^* = 1$. The aggregate demand for export is $X = (\lambda_f^* P^* A^*)^{\frac{1}{1-\lambda_f^*}} F(\cdot) P_x^{-\frac{1}{1-\lambda_f^*}}$. The price elasticity of demand for exports is $-\frac{1}{1-\lambda_f^*}$. The optimal quantity X, given prices, is obtained from the following optimization problem

$$\max_{\{x_j\}_{j=1,3,4}} P_x \prod_{j=1}^4 \left(\frac{x_j}{\tilde{\lambda}_{fj}^*}\right)^{\tilde{\lambda}_{fj}^*} - \sum_{j=1}^4 p_{xj}x_j.$$

The export of sector j domestic good satisfies $p_{xj}x_j = \tilde{\lambda}_{fj}^* P_x X$. The aggregate export price satisfies $P_x = \prod_{j=1,3,4} p_{xj}^{\tilde{\lambda}_{fj}^*}$.[10] The export price and the domestic price, by no-arbitrage condition, satisfy $p_{xj} = \tau_{xj}p_j$ for all j. The export-domestic price ratio then reflects the export cost.

We impose that trade is balanced at the aggregate level, which is a reasonable approximation since in this paper we focus on the stationary growth path. Imports need to be financed with exports, as follows[11]

$$P_x X = p_{f1}c_{f1} + p_{f4}c_{f4} + p_{f1}v_{e,f} + \sum_{i=1,3,4} p_{fi}m_{fi}. \tag{6}$$

Import tariffs are paid to the domestic government, so are not financed from export revenue.

which exists if $\gamma(g_k + \delta - 1) < \frac{1}{2}$.

[10] This is obtained by solving for $x_j, j = 3,4$ from their first-order conditions, and plug them into the first-order condition for x_1.

[11] In this equation, price of imported machinery is the same, irrespective the use. Import price can be different according to the use of imported good, since anyway import prices are exogenous.

3.6 Final consumption good

Final consumption good is produced using domestic consumption goods (c_{h1} and c_{h4}) and imported consumer goods (c_{f1} and c_{f4}) with the following aggregation technology,

$$C = \left(\frac{c_{h1}}{\mu_{h1}}\right)^{\mu_{h1}} \left(\frac{c_{h4}}{\mu_{h4}}\right)^{\mu_{h4}} \left(\frac{c_{f1}}{\mu_{f1}}\right)^{\mu_{f1}} \left(\frac{c_{f4}}{\mu_{f4}}\right)^{\mu_{f4}}. \tag{7}$$

By first-order conditions, shares of sector-level domestic and imported consumption goods are constant, that is, $\frac{p_i c_i}{P_C C} = \mu_i$ for $i = h1, h4, f1, f4$. The price of final consumption is given by $P_c = p_1^{\mu_{h1}} p_4^{\mu_{h4}} [(1+\tau_{f1})p_{f1}]^{\mu_{f1}} [(1+\tau_{f4})p_{f4}]^{\mu_{f4}}$, or $P_c = p_1^{\mu_{h1}} p_4^{\mu_{h4}} \overline{p}_{f1}^{\mu_{f1}} \overline{p}_{f4}^{\mu_{f4}}$. In this case, if the balanced growth path exists, growth of aggregate consumption is constant and equals to the product of the growth of import price (relative to domestic consumption price) and the growth of imported consumption goods. If the relative price is constant in equilibrium, growths of aggregate final consumption and imported consumption goods are equal; Otherwise, the growth of imported consumption good can be different from the growth of domestic final consumption good.

3.7 Household

A representative household purchases the final consumption good (C_t), supplies labor input (L_t), and makes investment decisions. Labor quality Q_t, measuring the efficiency of hours worked, grows at an exogenous gross rate. Flow of utility is $u(C_t, L_t) = \theta \ln C_t + (1-\theta)\ln\left(N_t - \frac{L_t}{Q_t}\right)$. The household maximizes $\sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$ subject to

$$P_{ct}C_t + P_{et}v_{et} + P_{st}v_{st} = (1-\tau_l)w_t L_t + (1-\tau_k)r_{et}P_{et}K_{et} + (1-\tau_k)r_{st}P_{st}K_{st} + T_t. \tag{8}$$

On the left hand side of the budget constraint are final consumption and investments in equipment and structures. On the right hand side are labor income and capital incomes. The lump-sum transfer is $T_t = \tau_l w_t L_t + \tau_k r_{et} P_{et} K_{et} + \tau_k r_{st} P_{st} K_{st} + \tau_{f1} p_{f1}(v_{e,f} + c_{f1}) + \tau_{f4} p_{f4} c_{f4} + \sum_{i=1}^{4} \tau_{fi} p_{fi} m_{fi}$. The budget constraint can also be written as

$$P_{ct}C_t + P_{et}K_{et+1} + P_{st}K_{st+1} = (1-\tau_l)w_t L_t + R_{et}P_{et}K_{et} + R_{st}P_{st}K_{st} + T_t.$$

Gross capital interest rates are $R_{et} = [1 - \delta_e + (1-\tau_k)r_{et}]$ and $R_{st} = [1 - \delta_s + (1-\tau_k)r_{st}]$.

3.8 Competitive equilibrium

State variables include sector-level total factor productivity $A_t = \{A_{jt},\ j = 1,2,3,4\}$, labor quality Q_t, and aggregate capital stocks K_{et} and K_{st}. Foreign demand of exports and imports are exogenous. Domestic commodity price p_3 is also assumed to be exogenous. Endogenous prices are P_{ct}, P_{et}, P_{st}, P_{mt}, w_t, R_{et}, and R_{st}, as well as sector output prices p_{jt}, $j = 1,2,4$. The competitive equilibrium is defined as a series of prices as functions of state variables and exogenous variables, such that

(i) The household utility is maximized, by optimal choices of consumption, labor and investments.

(ii) Firms in all sectors maximize the profit, by optimal choices of labor input and capital rentals.

(iii) Resource constraints for inputs among sectors are given by

$$K_e = \sum_{j=1}^{4} K_{e,j}, \quad K_s = \sum_{j=1}^{4} K_{s,j}, \quad L = \sum_{j=1}^{4} L_j, \quad M = \sum_{j=1}^{4} M_j.$$

(iv) Output markets clear for all sectors.

(v) Trade is balanced, by assumption.

The stationary growth path is defined as a stationary equilibrium in which

(i) Sector-level gross outputs grow at constant but different rates.

(ii) capital rental rates are constant, but different between the two types of capital.

(iii) Relative prices of sector-level outputs change at constant rates.

(iv) Shares of current-price inputs in total inputs are constant.

(v) Shares of sector-level current-price gross outputs in total outputs (or aggregate value added) are constant.

The existence of stationary growth path requires that i) relative prices of imported goods change at constant rates, which are assumed to hold; and ii) sector-level productivity grows at constant rates.

4 Effective aggregation production function

Our main interest is to map sector-specific technology and relative prices into the effective aggregate productivity, by constructing an effective aggregate production function for real gross domestic product. One important question is to what extent relative prices of trade can account for the growth of measured aggregate total factor productivity in a small open economy.[12]

In the presence of import tariff, collected from producers and transferred to the household, the current-price gross domestic product equals the sum of sector-level value added in current prices and the revenue from import tariff. Aggregating optimal conditions with respect to primary factors among sectors, we know that $wL + r_e P_e K_e + r_s P_s K_s = \mathbf{1}'\mathbf{p_y y}$, the aggregate value added on the producer side. From the budget constraint of the representative household, we also know that in equilibrium $wL + r_e P_e K_e + r_s P_s K_s + \sum_{j=1}^{4} \tau_{fj} p_{fj} q_{fj} = P_c C + P_e v_e + P_s v_s$, the aggregate final demand (noting trade is balanced). This suggests that the sum of final demands net of tariff equals the sum of sector-level value added. In national accounts, the aggregate final demand is called gross domestic product at the market price, while the aggregate final demand net of taxes is called gross domestic product at the basic price. Whether relative prices of trade and tariff affect measured productivity depends upon the choice of real gross domestic product.

[12] Details of solving the model and the derivation of the aggregate production function can be obtained from the author through email.

4.1 Gross domestic product at the basic price

In deriving the aggregate production function, we use the aggregate value added on the producer side as the measure of gross domestic product, or $P_c C + P_e v_e + P_s v_s + P_x X - P_f q_f - \sum_{j=1}^{4} \tau_{fj} p_{fj} q_{fj}$ which equals $\mathbf{1}'\mathbf{p_y y}$. We also provide the result of using gross domestic product at the market price as the aggregate output measure. Import price does not affect measured aggregate productivity for gross domestic product at the basic price, but it does for gross domestic product at the market price.

The real value of gross domestic product is to be defined. In national accounts and in measuring productivity, real gross domestic product is measured as a chained quantity index, with either the Törnqvist index or the Fisher index. These indexes are discrete-time approximation of the Divisia index which is weighted geometric mean of growth rates of components of gross domestic product, where weights are current-price shares of individual final demands. To obtain a definition of real gross domestic product at the basic price that is consistent with national accounts, we use the Divisia index, given by

$$g_y = g_c^{\omega_c} g_e^{\omega_e} g_s^{\omega_s} g_x^{\omega_x} g_f^{-\eta_f},$$

where $g_y = \frac{y_{t+1}}{y_t}$ is the gross growth rate of the real gross domestic product, and so on. On the right hand side are gross growth rates of the real value of final demands, in order consumption, investment in machinery, investment in structures, export, and import. Exponent ω_c is the share of current-price consumption in gross domestic product at the basic price, and so on. These shares have been obtained on the stationery growth path. Here, η_f is the share of imports and tariffs in the aggregate value added, clearly $\eta_f = \sum_{i=1}^{4}(1+\tau_{fi})\omega_{fi}$ with ω_{fi} the share of imported good i in the aggregate value added. Using the derived shares, it is straightforward to show that $\omega_c + \omega_e + \omega_s + \omega_x - \eta_f = 1$. It is of note that the implicit import price of the imported good i is $\overline{p}_{fi} = (1+\tau_{fi})p_{fi}$, not p_{fi}. The above Divisia index implies that one representation of the level of real gross domestic product can be given by[13]

$$Y = \left(\frac{C}{\omega_c}\right)^{\omega_c} \left(\frac{v_e}{\omega_e}\right)^{\omega_e} \left(\frac{v_s}{\omega_s}\right)^{\omega_s} \left(\frac{X}{\omega_x}\right)^{\omega_x} \left(\frac{q_f}{\eta_f}\right)^{-\eta_f}, \tag{9}$$

in which q_f is the real value of the aggregate import.

In equilibrium, individual final demands are constant proportions of both value added and gross output, allowing us to substitute final demands for the production functions of sector-level gross output. In doing that, real gross domestic product at the basic price becomes a function of the aggregate primary factors. Let $\mathbf{s} = [s_j]_{4\times 1}$ be a vector, s_j is the share of sector j's value added in gross domestic product at the basic price. Let d_j be the Domar aggregation weight of sector j and $\tilde{\lambda}^*_{fj}\omega_x$ be the share of sector j's export in aggregate value added. These shares are derived an appendix, available upon request. The following proposition is our main result.

Proposition 1. *If production technology is Cobb-Douglas and trade is balanced at the aggregate level, on the stationery growth path, there exists an effective production function of the real gross domestic product at the basic price, given by*

$$Y = Z K_e^{\tilde{\alpha}_e} K_s^{\tilde{\alpha}_s} L^{\tilde{\alpha}_l}, \tag{10}$$

[13] In the discrete-time approximation, The two-period average of shares is used, suggested in Hulten (1973). In current paper, shares are constant in equilibrium since production functions are Cobb-Douglas.

with $\tilde{\alpha}_e = \alpha_{\mathbf{e}}{}'\mathbf{s}$, $\tilde{\alpha}_s = \alpha_{\mathbf{s}}{}'\mathbf{s}$ and $\tilde{\alpha}_l = \alpha_{\mathbf{l}}{}'\mathbf{s}$. The effective economy-wide total factor productivity Z is given by

$$Z = const. \cdot \prod_{j=1}^{4} A_j^{d_j} \cdot \prod_{j=1}^{4} \left(\frac{1}{\tau_{xj}}\right)^{\tilde{\lambda}^*_{fj}\omega_x}. \tag{11}$$

To remind ourselves, in Equation (11) $\tilde{\lambda}^*_{fj}$ is the share of sector j's export in aggregate export, and ω_x is the share of aggregate export in aggregate value added at the basic price. Also, $\tau_{xj} = \frac{p_{xj}}{p_j}$. Proof of this proposition is in an appendix, available upon request. Summing the first-order optimal conditions with respect to primary factors, we can obtain that $wL = \tilde{\alpha}_l \cdot \mathbf{1}'\mathbf{p_y y}$ and so on. This suggests that the aggregate production function exhibits constant returns to scale, $\tilde{\alpha}_e + \tilde{\alpha}_s + \tilde{\alpha}_l = 1$. Therefore, the effective aggregate production technology in Equation (10) is equivalent to our multiple-sector production network, in producing final demands at the basic price.

Proposition 1 states that, in a small open economy, the growth of effective aggregate total factor productivity is accounted for by changes in sector-specific technology of producing gross output and changes in export prices relative to domestic prices. The magnitude of the contribution by each sector depends on the Domar aggregation weights which in turn is determined by the connectedness among sectors. The importance of relative export price depends on the share of export in the aggregate value added, a measure of international connectedness.

4.2 Implicit price of gross output

The effective economy-wide total factor productivity given by Equation (11) is consistent with measured aggregate total factor productivity in growth accounting data, even though the implicit price of gross output in our model is set to be the same as the domestic price while in practice they are an index aggregated from both domestic and export prices. We show that, our treatment of the price of sector-level gross output is not only consistent with the data, but also allows us to account for the relative contribution of the domestic sector and the export sector to aggregate productivity growth. Let p_{qj} be the implicit price of the sector-j gross output, and let s_{xj} be the share of export in gross output in sector j. Again, we can use the Divisia index for p_{qj}, aggregated from domestic and export prices, then $p_{qj} = p_j^{1-s_{xj}} p_{xj}^{s_{xj}}$. The no-arbitrage condition suggests that in equilibrium $p_{xj} = \tau_{xj} p_j$, which implies that $p_{qj} = \tau_{xj}^{s_{xj}} p_j$.

Let the measured total factor productivity be $\tilde{A}_j$ if the implicit price of gross output is p_{qj}. In current prices, we have $p_{qj}\tilde{q}_j = p_j q_j$, suggesting that $\tilde{q}_j = \frac{p_j}{p_{qj}} q_j$, or $\tilde{q}_j = \frac{1}{\tau_{xj}^{s_{xj}}} q_j$. This in turn implies that $\tilde{A}_j = \left(\frac{1}{\tau_{xj}}\right)^{s_{xj}} A_j$, since the production function is Cobb-Douglas. Note that $\tilde{\lambda}^*_{fj}\omega_x = s_{xj} d_j$, Equation (11) becomes $Z = \text{const.} \cdot \prod_{j=1}^4 \tilde{A}_j^{d_j}$, which is the result of Hulten (1978). Therefore, the effective aggregate productivity when using domestic price p_j as the implicit price to deflate gross output of sector j is consistent with the productivity measured in the data. Moreover, using the price index p_{qj} may not separate the contribution of the exporting sector to productivity even in the model we allow for that export price differs from domestic price.

The above result confirms our findings from the simple model, which is different from the baseline model of Burstein and Cravino (2015). Measured productivity partly reflects export price relative to domestic price. This relative price can arise from both trading technology operated at home country and productivity of producing the exported good. In the absence of data on the trade cost, the impact of this cost on aggregate productivity can be measured only when the implicit price of gross output is the same as the domestic price, even if this cost is operated in the home economy.

4.3 Terms of trade and measured productivity

Equation (11) suggests that the aggregate terms of trade, $\frac{P_x}{P_f}$, itself does not explain the effective aggregate productivity. This confirms the result of Kehoe and Ruhl (2008), who show that shocks to the terms of trade have no first-order impact on total factor productivity while in data the two are correlated. The export price in our model does affect measured productivity, which is absent from their paper. In both Kehoe and Ruhl (2008) and Kim (2014), the export price is assumed to be the same as the domestic price and is normalized to one. This has two implications. First, their models abstract from differences between the trading technology and the production technology of the exported good, two factors that can be reflected in the export price and measured productivity. Second, their measure of the terms of trade is essentially the import price, which has no effect on the effective aggregate productivity.

As we saw in Section 2, import price does not affect measured productivity because a change in import price leads to changes in both the gross output and the intermediate input. On net, the real value added is unaffected when production function displays a constant return to scale. We can obtain this from a slightly general argument. Let q be the amount of gross output, produced with a constant-return-to-scale technology $q = AF(K, L, M)$ and M be the real import used as an intermediate input. Real value added is defined as $y = q^{s_q} M^{-s_m}$ with $s_q - s_m = 1$ and $s_m = \frac{p_m M}{pq - p_m M}$, this is one representation of y when y is a Divisia quantity index. It can be shown that

$$\frac{\partial y}{\partial p_m} = q^{s_q} M^{-s_m - 1} \frac{\partial M}{\partial p_m} \left(s_q \frac{p_m M}{pq} - s_m \right).$$

Terms in the bracket reduce to zero under the constant-return-to-scale technology, hence import price has no effect on aggregate total factor productivity. This result does not require the assumption of balanced trade. It also holds in the presence of import tariff because tariff is reflected in the effective import price.

4.4 Gross domestic product at the market price

If we use gross domestic product at the market price as the output measure, import price can affect productivity when producers pay an *ad valorem* tax on imported goods. This has been shown in Kehoe and Ruhl (2008) and Kim (2014). In practice, gross domestic product at the market price is not recommended as an output measure for the purpose of productivity measurement, because it is the gross domestic product at the basic price that measures the amount of revenue retained by producers.

If gross domestic product at the market price is used, we can show that both the export price and the import price relative to domestic prices affect measured aggregate total factor productivity. Let $\tilde{Y}$ be the real value of gross domestic product at the market price. It can be shown that $\tilde{Y} = G(\tau_{fi_{\{i=1,2,3,4\}}}, p_{fi_{\{i=1,2,3,4\}}}; P_Y) \cdot Y$, where $G(\cdot)$ is a function of import tariff, import price, and P_Y, the implicit price of real gross domestic product at the basic price. If import tariff equals zero, $G(\cdot) = 1$. Apparently, in this case, both export prices and import prices affect the effective aggregate productivity.

5 Quantitative analysis for Canada

Our main interest is to quantify to what extent relative export price can account for the productivity slowdown in Canada and globally. The case of Canada is relevant because, in the 2000s, it experienced falling productivity growth, a large drop in the price of machinery and equipment relative to consumption price, increased commodity export due to commodity price boom, and a significant improvement in the terms of trade. We calibrate our model to the Canadian economy. Later on, we also present evidence from the cross-country data.

5.1 Recent trends of productivity and relative prices in Canada

Output and productivity growth in Canadian business sector are measured by Diewert and Yu (2012) and updated by Cao and Kozicki (2015) to reflect revisions to national accounts. Details of the data are described in Appendix A.1. Total factor productivity grew at an average annual rate of 0.27 percent from 1981 to 2015, accounting for close to a quarter of labor productivity growth. After a rapid growth of 1.16 percent in the 1990s, productivity growth was at -0.33 percent since 2001, see Figure B.1 for this recent trend.

Accompanying the slowdown of total factor productivity growth were significant changes in relative prices. First, the terms of trade improved by about 20 percent starting 2001, and was flat before that, as shown in Figure B.1. This extraordinary large increase lies mainly in The Rest sector. The terms of trade for machinery and equipment displays a stable upward trend, in total gained 22 percent from 1981 to 2015. Commodity price relative to non-investment GDP price, highly correlated with the terms of trade, also increased after 2000. Second, the price of machinery and equipment relative to the price of non-machinery gross domestic product displays a long-run trend, and declined at a faster pace in the 2000s. This trend is reflected in both the prices of sector-level gross output and the prices of gross domestic product components. Figure B.2 plots two relative prices of machinery and equipment, one from the industry productivity data and the other based on gross domestic product components. Both relative prices declined since 1981. Relative investment price of machinery dropped about 90 percent from 1981 to 2015, on average 2.6 percent per year. Because investment is produced at home and also imported, this decline arises from the decline of two components: relative gross output price and relative price of imported machinery. Relative gross output price of machinery, dropped 35 percent from 1981 to 2012, about 1 percent per year. Price of imported machinery relative to price of imported consumer goods declined by 54 percent from 1981 to 2015, an average drop of 1.5 percent per year.

5.2 Model calibration

Sectors in the model are defined as in Appendix A.2. Of particular note, sector 3 (commodity) includes sectors/outputs corresponding roughly to the commodity basket used to calculate Bank of Canada commodity price index (BCPI). The definition of sectors across different data sources may slightly differ. Long-run trends suggest that the Cobb-Douglas function form of production approximates well the actual data.[14]

Three main data sources are used for calibration: growth accounting data based on the modified Diewert-Yu productivity estimates; World Input-output Database (WIOD) developed by Timmer et al. (2015); and the industry productivity data (KLEMS) by Statistics Canada. Some parameters are drawn directly from the data and others are calibrated to target the growth accounting data. The model has the following parameters:

[14] Shares of sector-level gross output in aggregate gross output are not constant, the share of sector 4 output displays a small upward trend (from 57% in 1981 to 63% in 2012). Output shares of other sectors together display a downward trend, but individually they do not display a strong trend. The output share of sector 3 (commodity) declined slightly until around 2000, and increased since then. Shares of sector-level value added display similar trends as gross output, in particular share of valued added of sector 4 increased from 1981 to 1991, and overall stayed flat after 1991.

Preference: θ and β.

Growth of productivity, population, and labor quality: a_j, $j = 1,2,3,4$; g_l.

Sector production technology: α_{ej}, α_{sj}, α_{mj}, $j = 1,2,3,4$.

Intermediate input, imports and exports: λ_{hi}, λ_{fi}, and $\tilde{\lambda}^*_{fi}$, $i = 1,2,3,4$.

Investment and consumption goods production: ψ, μ_{h1}, μ_{h4}, μ_{f1}, μ_{f4}.

Export cost: τ_{xj}, $j = 1,2,3,4$.

Capital depreciation rates and tax rates: δ_e, δ_s, τ_l and τ_k.

We first obtain the Domar weights and shares of sector-level value added. We then obtain the sector-level total factor productivity to target growth rates of gross outputs.

5.2.1 Domar weights

Domar weights are determined by parameters regarding sector-level outputs used for various purposes as a share of total uses, for example, sector-level output used as intermediate inputs as a share of composite intermediate input. These shares are obtained from data of KLEMS and WIOD.

Factor shares in production. We use factor shares in the KLEMS data. In some sectors, total cost does not equal to total revenue. We impose constant returns to scale in each sector, and assume that labor income is less subject to measurement error than capital income. Therefore, the share of capital is a residual, calculated using shares of labor cost and intermediate inputs. Table 1 reports values of factor shares, the aggregate shares are calculated from the aggregated series of inputs and outputs. Labor share in value added in sector 3 (commodity) is 0.39, significantly lower than in other sectors.[15]

Table 1: Factor shares, based on KLEMS 1981-2012

Sector	α_{mj}	α_{lj}	$\alpha_{ej} + \alpha_{sj}$
Machinery (1)	0.655	0.662	0.338
Structures (2)	0.559	0.724	0.276
Commodity (3)	0.539	0.389	0.611
The Rest (4)	0.475	0.615	0.385
Aggregate	0.516	0.594	0.406

Factor shares in composite intermediate input. The WIOD 1995-2012 data are used to obtain intermediate inputs supplied by sectors as a share of total composite intermediate input, as well as shares of imported intermediate inputs in total composite intermediate input. Average values of shares over this period are used, reported in Table 2. The share of intermediate input supplied by sector 2 in total intermediate input is, $\lambda_{h2} = 0.027$, while in the model this parameter is zero. We move it to the sector 4, making $\lambda_{h4} = 0.596$. The share of intermediate input produced at home in composite intermediate input is $\lambda_h = 0.78$, and the share of imported intermediate input in composite intermediate input is $\lambda_f = 0.22$.

[15] Labor shares in value added can be under-measured because leased and rented capital is counted as an intermediate input (service) in national accounts, not as capital expenditure. Wang and Moussaly (2014) document that rented capital accounts for 3.6% of value added in the construction sector.

Table 2: Intermediate inputs produced by sectors as shares of composite intermediate input

Sector	Domestic (λ_{hi})	Imported (λ_{fi})
Machinery (1)	0.052	0.080
Structures (2)	0.027	0.000
Commodity (3)	0.128	0.057
The rest (4)	0.569	0.086
Sum	0.776	0.224

Production functions of final consumption and investment. The use of domestic and foreign goods is obtained from the WIOD data. Following values are chosen first: $\mu_{h1} = 0.03$, $\mu_{f1} = 0.08$, and $\mu_{f4} = 0.09$. This suggests that $\mu_{h4} = 0.80$. For investment in machinery, the imported machinery accounts for 14 percent of total investment, suggesting that $\psi = 0.143/(1-0.607) = 0.36$. About 40 percent of investment in machinery and equipment is imported.

Shares of individual exports. Shares of individual exports in total export are obtained from national accounts. This gives $\tilde{\lambda}^*_{f1} = 0.351$, $\tilde{\lambda}^*_{f3} = 0.352$, and $\tilde{\lambda}^*_{f4} = 0.297$. These shares before and after 2000 are slightly different, the share of exported machinery declined to 0.31 in the period after 2000.

Import tariff. We follow Diewert and Yu (2012) and assume that imports of energy, automobile (including parts) and services are not taxed at the custom, and the tariff rate is the same for the rest of imports. We then calculate the effective ad valorem tariff rate for each sector. First, we obtain an aggregate effective tariff rate as the ratio of the aggregate custom import duty over the aggregate import (excluding energy, automobile and services). We then apply this rate to sectors to obtain the effective import duty for each sector. The effective tariff rate at the sector level is calculated as the sector-level import duty divided by the sector-level import. The obtained average effective tariff rates are shown in Table 3. These rates declined from 1981 to 2000, and stayed flat since then.

Table 3: Average effective tariff rates (%)

	Sector 1	Sector 3	Sector 4
1981 to 2012	1.42	2.28	1.51
1981 to 1999	1.96	3.24	2.09
2000 to 2012	0.64	0.88	0.67

Domar weights. With the shares of above, we calculate the Domar weights by solving the system of equations, $[\mathbf{I}-\mathbf{X}]\,\mathbf{d} = \mathbf{b}$. Here, $[\mathbf{I}-\mathbf{X}]^{-1}$ is the Leontief inverse matrix, and $\mathbf{b}$ is a vector with the jth element the share of final demand supplied by sector j in the aggregate value added. Elements of $\mathbf{X}$ and $\mathbf{b}$ are functions of factor shares and tariff rates, all obtained as above. The predicted Domar weights are $\mathbf{d} = [0.286, 0.078, 0.277, 1.406]$. These values are fairly close to the KLEMS data. Sector 2 in our model is the non-residential structure, which is used entirely for investment, accounting for 7.8 percent of business sector GDP, therefore $d_2 = 0.078$. The sector Construction in the KLEMS does not map to Sector 2 in the model.

If we force the Domar weight for the construction sector to be 0.078 in the KLEMS data, we can obtain modified Domar weights in the data for sectors 1, 3 and 4, which respectively are 0.30, 0.37, and 1.48.

5.2.2 Sector-specific total factor productivity

We obtain growth rates of sector-specific total factor productivity by matching growth rates of gross output at the sector level that are predicted by the model with those in the KLEMS data. The model predicts that gross growth rates, in logarithm, are linear functions of sector-specific technological change, changes in import prices, and the growth of labor quality.

Capital depreciation rates. They are set as $\delta_e = 0.217$ and $\delta_s = 0.080$, estimated as in the modified Diewert-Yu productivity data.

Hours worked and labor quality. The growth rate of the labor input is a product of growth rates of hours worked and the labor quality, i.e. $g_l = n \cdot g_Q$. The growth of hours worked in steady state is set to be the same as the labor force growth. The growth of hours worked in business sector is on average 1.28% per year from 1982 to 2012 in the growth accounting data, which is virtually the same as in Statistics Canada productivity account data. This leads to $n = 1.0129$.[16] Labor quality in the growth accounting data grew at 0.39% per year, or, $g_Q = 1.004$.

Labor income tax. It is set at $\tau_l = 0.24$, calculated as government revenue from household income taxes as a fraction of compensation of employees (wages and salaries). This rate is in line with values calculated in Mendoza et al. (1994).

Export cost. The export cost per unit of export is $(\tau_{xj} - 1)$. In equilibrium, relative price is given by $\frac{p_{xj}}{p_j} = \tau_{xj}$. We use the changes in the export-domestic price ratio to measure changes in τ_{xj}. Of course, our model does not distinguish between the export cost and the relative productivity of the exporting sector.

Calibrating β and τ_k. Equilibrium conditions are used to calibrate them. Following moments are used

- Average share of current-price investment in structures in business-sector GDP is 7.83%, and average share of current-price equipment investment in business-sector GDP is 10.12%, both from the modified Diewert-Yu productivity data 1981-2012.
- Average annual growth rate (log-difference) of non-investment real GDP (business-sector real GDP excluding investments in equipment and structures) is 2.18%, from the modified Diewert-Yu productivity data 1982-2012.[17]
- Average after-tax rate of return to structures is assumed to be 7.0%. This rate is slightly larger than the average yield of long-term government bond, but lower than the after-tax rate of return to capital estimated in the modified Diewert-Yu productivity data. With this rate, the overall rate of return to capital predicted by the model is lower than but close to that in growth accounting data.

[16] Growth rates in the data are the exponential of the average log-differences. This ensures that definitions in model are consistent with those in data.

[17] In the Diewert-Yu approach of measuring productivity growth, imputed rent from owned housing is excluded from GDP. In national accounts, the average share of imputed rent in GDP is 7.6% (see Cansim Tables 380-0064 and 380-0085).

- Average growth rate of price of non-investment GDP relative to the price of investment in structures, -0.09%.

Discount factor β is calibrated using the Euler equation for investment in structures, $\beta = \frac{g_c}{R_s} \cdot \frac{g_{P_c}}{g_{P_2}} = \frac{1.022}{1.07} \cdot 0.999 = 0.954$.

Next, Euler equations regarding investments are used to obtain τ_k and $\tilde{\alpha}_e$. The Euler equation with respect to equipment investment can be re-written as

$$\frac{g_c}{\beta g_{\tilde{P}_{ec}}} = 1 - \delta_e + (1-\tau_k)\tilde{\alpha}_e(g_e + \delta_e - 1)\frac{\mathbf{1}'\mathbf{p_y y}}{P_e v_e},$$

where $g_{\tilde{P}_{ec}}$ is the gross growth rate of $\frac{P_e}{P_c}$. On the right hand side is the after-tax gross rate of return net of depreciation, R_e, in which the share of equipment investment in aggregate value added ($\frac{P_e v_e}{p^y y}$) is applied to replace r_e, as implied by the model. Similarly, the Euler equation with respect to investment in structures is given by

$$\frac{g_2}{\beta g_{\tilde{P}_{sc}}} = 1 - \delta_s + (1-\tau_k)\tilde{\alpha}_s(g_s + \delta_s - 1)\frac{\mathbf{1}'\mathbf{p_y y}}{P_s v_s},$$

where $g_{\tilde{P}_{sc}}$ is the gross growth rate of $\frac{P_s}{P_c}$. Solving these two equations leads to $\tau_k = 0.405$ and $\tilde{\alpha}_e = 0.211$. This implies that $\tilde{\alpha}_s = 0.194$.

Shares of machinery in production. To obtain values of α_{ej}'s (or α_{sj}'s), we use the first-order conditions with respect to machinery and structures, e.g., $\frac{r_e P_e K_{ej}}{\alpha_{ej}} = \frac{r_s P_s K_{sj}}{\alpha_{sj}}$. Ratios of capital costs between machinery and structures can be obtained from calibrated interests rates and capital stocks in the data. Table 4 reports the calibrated values.

Table 4: Sector-specific capital shares

Sector	Machinery (α_{ej})	Structures (α_{sj})
Machinery (1)	0.258	0.080
Structures (2)	0.232	0.044
Commodity (3)	0.303	0.309
The Rest (4)	0.188	0.197

Sector-specific total factor productivity. We first obtain growth rates of real gross output where the implicit price of domestic sales is used to deflate gross output. We then use the model-predicted gross output growth as follows

$$\mathbf{J}\widehat{\mathbf{g}} = [\mathbf{I}-\mathbf{M}]\widehat{\mathbf{a}} - [\mathbf{I}-\mathbf{M}]\Theta_{\mathbf{f}}\widehat{\mathbf{g}}_{\overline{\mathbf{p}}_{\mathbf{f}}} + [\mathbf{I}-\mathbf{M}]\Lambda_{\mathbf{l}}\widehat{g}_l.$$

The derivation of this equation is straightforward. Vector variables with a hat are growth rates in log-difference, $\widehat{\mathbf{g}}$, $\widehat{\mathbf{a}}$, $\widehat{\mathbf{g}}_{\overline{\mathbf{p}}_{\mathbf{f}}}$, and $\widehat{g}_l$, are respectively the growth rate of gross output, sector-specific total factor productivity, import price, and labor input. The coefficient matrices $\mathbf{J}$, $\mathbf{M}$, $\Theta_{\mathbf{f}}$, and vector $\Lambda_{\mathbf{l}}$ are functions of factor shares in production and aggregation.

In the model, we have assumed that labor quality grows at the same rate for all sectors, and growth rates of labor input and hours worked are equal across sectors on balanced growth path. In the KLEMS data, these growth rates are rather different across sectors. With the Cobb-Douglas production function, this implies that sector-specific total factor productivity reflects not only technological change, but also the heterogeneity in labor quality growth (or sector-specific labor-augmenting technological change) across sectors. We want to separate this heterogeneity from sector-specific total factor productivity. We obtain sector-specific labor quality growth as the difference between the observed sectoral labor input growth and the model-predicted growth of hours worked.

The obtained gross growth rates of sector-specific total factor productivity are respectively, $a_1 = 0.997$, $a_2 = 1.000$, $a_3 = 0.998$, and $a_4 = 1.003$. Multifactor productivity in the KLEMS data (calculated using gross output deflated by prices of domestic sales) grows at rates of respectively, $0.989, 0.999, 0.994$, and 1.001.

Finally, for growth accounting, we do not need to know the value of θ. Nevertheless, it can be obtained from the equilibrium rate of labor force participation, which can be re-written as,

$$\frac{1}{\xi_t} = 1 + \frac{1-\theta}{\theta(1-\tau_l)} \cdot \frac{1}{\tilde{\alpha}_l} \left(1 - \tilde{\alpha}_e \frac{\delta_e + g_e - 1}{r_e} - \tilde{\alpha}_s \frac{\delta_s + g_s - 1}{r_s} + \sum_{i=1}^{4} \tau_{fi} \omega_{fi} \right).$$

The term in the big bracket on the right hand side is the share of consumption in gross domestic product. Average annual hours worked per worker are assumed to be the same for all workers, so that hours worked as a fraction of hours available equal labor force participation rate, that is, $\xi_t = 0.66$ (from the Labor Force Survey, 1981 to 2012).

In the above equation, the second term in the bracket is the share of investment in machinery in gross domestic product, and the last term is the share of structures investment in gross domestic product. These two shares are respectively, 10.12% and 7.83% in growth accounting data. Solving the above equation gives $\theta = 0.784$.[18]

5.3 Growth accounting: data and model

With the calibrated model, both aggregate labor productivity growth and aggregate total factor productivity growth can be decomposed into the contribution from sector-specific productivity and relative export prices. Aggregate total factor productivity, calculated using Equation (11) and shown in Table 5, grows faster than that obtained by aggregating sector-specific in the KLEMS data, as well as that measured using the Diewert-Yu approach. All of them are faster than the multifactor productivity growth rate measured by Statistics Canada's productivity program.[19]

One important difference between the model and the data is that inputs grow at the same rates across all sectors in the model, while in the data they can be different. Therefore, the calibrated total factor productivity may capture both the sector-specific technological change and factors that lead to differentials in input growth. In the calibrated total factor productivity growth, we have taken into account heterogeneity in labor input growth across sectors.

[18] This is rather different from the calibrated value in Greenwood et al. (1997). The difference arises from the definition of L_t/N_t, we define it as the fraction of working population, assuming every one works for given fixed hours. In Greenwood et al. (1997), they assume that L_t/N_t is the fraction of hours worked over total available hours, and they set $L_t/N_t = 0.24$. There is no population growth in their paper.

[19] Gu (2012) and Diewert (2012) show that the main source of difference is measured capital services which grow at different rates between Diewert-Yu and Statistics Canada.

Table 5: Annual growth of aggregate TFP (%)

Statistics Canada MFP	0.10
Diewert-Yu-Cao-Kozicki	0.27
KLEMS, Domar aggregation	0.23
Model	0.54

Aggregate total factor productivity growth predicted by the model is mainly driven by reduction in the export cost (reflected in the export-domestic price ratio), as well as the total factor productivity in Sector 4, as seen in Table 6. Sector-specific technological change on net is negative. In particular, neutral technology in the machinery sector changes at a negative rate, which is in sharp contrast to its significant positive growth in the U.S. as shown in Greenwood et al. (1997). The overall contribution of the machinery sector serving the domestic economy appear large, given its size relative to The Rest sector.

Table 6: Contribution to total factor productivity growth (percentage points)

Sector	Tech. change	Export cost reduction	Total
M & E (1)	-0.08	0.29	0.20
Structures (2)	-0.00	n/a	-0.00
Commodity (3)	-0.06	0.00	-0.06
The Rest (4)	0.38	0.01	0.39
Total	0.24	0.30	0.54

Standard but model-based growth accounting can be established, in which labor productivity growth is decomposed into the contribution from total factor productivity growth, capital deepening, and labor quality improvement. Using the effective aggregate production function, Equation (10), we can decompose the labor productivity growth as follows

$$\ln \frac{g_y}{n} = \ln g_z + \tilde{\alpha}_e \ln \frac{g_e}{g_l} + \alpha_s \ln \frac{g_s}{g_l} + \ln g_Q.$$

Table 7 shows the decomposition result, as compared with data. Growth accounting in the data is based on the productivity estimates with the approach by Diewert and Yu (2012) and Cao and Kozicki (2015), which is slightly different from the KLEMS data, while the latter is used for most part of the calibration.

Aggregate GDP growth predicted by the model is higher than in the data, due to both faster capital deepening and faster growth of labor quality in the model. Capital deepening of machinery and structures together 0.58 percentage points to labor productivity growth, or 38 percent of the labor productivity growth. In the data, capital deepening accounts for 0.41 percentage points (38 percent) of labor productivity growth. Faster capital deepening in the model arises from two differences between the model prediction and the data. First, the share of capital service for equipment in aggregate GDP is 0.21 in the model, larger than the 0.17 in the data. Second, in the model, by assumption the growth rate of capital service is the same as the growth rate of investment, which is 4.1 percent for machinery and equipment. This growth is faster than the 3.4 percent in the data. Capital service of machinery and equipment in the data grows at a lower rate than investment because depreciation rate increases over time.[20]

[20] Capital depreciation rate of machinery and equipment, which increased from 15.3% in 1981 to the peak value of 26.5% in 2012.

Table 7: Growth accounting: model and data (in percentage)

	Data (1982-2012)	Model
Output	2.36	2.80
Hours worked	1.28	1.28
Labor productivity	1.08	1.52
Total factor productivity	0.27	0.54
Capital deepening, machine	0.29	0.49
Capital deepening, structure	0.12	0.09
Labor quality	0.40	0.40

Import prices do not account for the growth of measured total factor productivity, but changes in these prices affect the growth of real gross domestic product through capital deepening. In the data, the growth of import prices for sectors 1, 3, and 4 grows respectively at an average rate of -0.2%, 1.9%, and 1.6%. Had these growth rates been zero, the production of investment in machinery and equipment will use more domestic outputs, and it is relatively cheaper to import intermediate inputs. The growth of gross output becomes faster in all sectors, by as much as 32 percent in Sector 1 and 14 percent in Sector 4. The growth of average labor productivity at the aggregate level is also faster, by 0.14 percentage points (or close to 13 percent of the baseline growth).

In summary, the calibrated model suggests that the export price (relative to domestic price) accounts for about half of the aggregate total factor productivity growth in the Canadian economy. Import prices affect the labor productivity growth through its impact on capital deepening. Both measured total factor productivity and labor productivity are associated with prices of trade, but the terms of trade per se have no first-order impact on productivity.

5.4 Accounting for historical aggregate TFP growth

We now examine the contribution of relative export prices to historical growth of aggregate total factor productivity, in particular its slowdown in the 2000s. The balanced growth in the model concerns average growth rates, it may not fit certain episodes when the growth of relative prices varies. For historical analysis, we rely solely on Equation (11) and decompose the effective aggregate productivity into contributions from sectors and relative prices. We have calculated the sector-specific total factor productivity based on the real gross output using the price of domestic sales as the deflator. We also obtained ratios of export to domestic prices. The Domar weights are calculated from the calibrated model, not from the KLEMS data, since the Sector 2 (structures) can not be separated from the construction sector in the KLEMS data. Shares of exports in aggregate gross domestic product are calculated from the national accounts and they vary over time. Figure B.3 shows that the calculated effective aggregate total factor productivity index is fairly close to the estimates using the national accounts data based on the Diewert-Yu approach. Sector-level total factor productivity in the production of gross output and relative domestic-export prices are plotted respectively in Figure B.4 and Figure B.5. All series are normalized to 1 in the beginning year. These figures clearly show that productivity growth in sector 4 (the Rest) and increase in relative domestic-export price in sector 1 (machinery), overall, experienced the most rapid growth.

Over different episodes, the contribution of relative domestic-export price differs significantly (Figure B.6). In the 1990s, sector-level productivity was the main source of aggregate productivity growth. Coming to the 2000s, sector-level productivity growth slowed down (making a negative contribution of 1.50 percentage points), increase in relative domestic-export price became the main driver of aggregate productivity growth (making a positive contribution of 1.05 percentage points). On net, aggregate productivity dropped 0.45 percent per year over the period of 2001 to 2012. The decomposition by sector, as shown in Figure B.7, suggests that in the 2000s the negative contribution of sector-level productivity mostly arises from the commodity sector and machinery sector serving the domestic economy, while productivity in the rest sector observed a positive growth. In the machinery and equipment sector, the significant increase in relative domestic-export price was offset a sharp drop in total factor productivity, on net, making a negative contribution to the aggregate productivity.

So far we have labeled the ratio of export price to domestic price as the export cost, though in the model this ratio can also reflect the total factor productivity of the exporting sector relative to the domestic sector. Was the drop in the export-domestic price ratio in the 2000s a drop in the export cost or an increase in the relative productivity in the exporting sector? Empirical evidence seems to suggest that the export cost has increased in the post-2001 period. The ESCAP-World Bank International Trade Costs data show that the Canada-U.S. bilateral trade cost has been rising since 2000.[21] This is consistent with empirical evidence by Brown (2015), who finds that the cost of cross-border shipping relative to that of domestic shipping for the same distance increased after 2001, due to increased border compliance costs among other factors. This suggests that the export cost may have increased since 2001. The drop in the export to domestic price ratio is then likely attributed to an increase of the relative productivity in the exporting sector. There is lack of evidence regarding the export cost by sector, we therefore cannot preclude that export cost could have dropped in some sectors.

6 Productivity growth slowdown across countries

The slowdown of aggregate total factor productivity in Canada in the 2000s was mainly accounted for by the commodity sector, and this sector accounts for a significant share of the export growth. Productivity growth also slowed down in many other countries over the same period. We are interested in investigating to what extent the relative export price is quantitatively important for other open economies for productivity slowdown. To this end, we use two set of data sources and decompose the aggregate total factor productivity according to our model prediction.

6.1 Evidence from WIOD and KLEMS

We build a 4-sector industry growth accounting data from the WIOD, the EU KLEMS, and the Penn World Table (PWT) data.[22] Our primary data source is the WIOD, it does not have prices of capital service and labor input for which we use series from the EU KLEMS data. We build the sector-level price and quantity of exports and imports using the UN Comtrade data constructed by Feenstra and Romalis (2014). The aggregate total factor productivity is constructed using Equation (11). The Domar weights are calculated using the WIOD and the EU KLEMS data. Shares of individual exports in total export are calculated from

[21] See Arvis et al. (2016) for the data set, which is based on methods by Novy (2013). The United States is the predominant destination of Canadian exports.

[22] For Korea, we use the Asian KLEMS data, and for Japan we use the database at the Research Institute of Economy, Trade and Industry (RIETI), Japan.

the Feenstra-Romalis data, and the share of total export in GDP is from the Penn World Table. We are able to build the growth accounting data for eight countries.

Figure B shows the aggregate total factor productivity, the contribution of sector-level productivity, and the contribution of relative domestic-export price, all normalized to one at the beginning of the year at which data are available. The trend of measured aggregate total factor productivity is overall close to those in the Penn World Table data, except for Korea and Sweden. For these two countries, the aggregate total factor productivity was declining in much of the period of 2000s, while it was increasing in the PWT data. This difference is most likely because the trends of relative export price in our data are different from those in the PWT data. The plots suggest that, irrespective of the fall in the contribution of domestic-export price ratio, four European countries (Finland, France, Germany, and Sweden) experienced positive growth of aggregate total factor productivity. For Spain and Italy, the aggregate total factor productivity observed negative growth in the 2000s, largely due to the negative contribution of domestic-export price ratio, and the sector-specific productivity was actually edging up. For the two Asian countries, the role of relative export price differs. In Japan, the aggregate total factor productivity follows closely the sector-specific productivity, suggesting a small contribution from the domestic-export price ratio. For Korea, the aggregate productivity slowed down in the 2000s, dragged by the falling contribution of domestic-export price ratio.

Obviously, the relative export price has played an important role in the slowdown of aggregate productivity for some countries. For the three countries (Italy, Korea and Spain) where the relative export price was important in accounting for the slowdown, the ESCAP-World Bank Trade Cost data show that the trade cost with their respective top trade partners has not gone up. This suggests that the fall of domestic-export price may arise from falling relative productivity of the exporting sector in these countries.

The decomposition into contribution by sectors, as shown in Figure B, suggest that Sector 4 (The Rest) is the main contributor to the slowdown of aggregate total factor productivity (except for Korea), and the commodity sector in these countries is quantitatively unimportant.

6.2 Aggregate TFP and aggregate relative price across countries

The decomposition of aggregate total factor productivity requires sector-level data, which are not available for a broader set of countries. But, we can examine the role of domestic-export price ratio at the aggregate level, based on the simple model of Section 2, and use the PWT data developed by Feenstra et al. (2015). Recall that, if gross output is deflated by domestic price, measured total factor productivity (of aggregate gross domestic product at the basic price) consists of two components, $Z = \tilde{A}^{\frac{1}{1-\alpha_m}} \cdot \left(\frac{p_d}{p_x}\right)^{\frac{s_x}{1-\alpha_m}}$, where again $\tilde{A}$ is the total factor productivity of gross output (deflated by domestic price) and $\frac{p_d}{p_x}$ is the ratio of domestic price (of gross output) over export price. The share s_x is the ratio of export over gross output, and $1-\alpha_m$ is the ratio of value added over gross output. Clearly, $\frac{s_x}{1-\alpha_m}$ is the share of export in aggregate gross domestic product. At the aggregate level, domestic final demand (consumption, investment and government spending) includes structures, which are not trade-able in our model. Thus, domestic price is for both domestic sales of traded goods and non-traded goods and services.

The above decomposition of total factor productivity holds if gross domestic product at the basic price is used as the measure of output. In the PWT data, only gross domestic product at the market price is available. If the import tariff rate and the share of import in gross domestic product are small, the impact of import price and tariffs on measure productivity is small, we expect that our decomposition approximately holds. Regardless, we still show the decomposition results. We also present results based on panel regressions to demonstrate that domestic-export price is significant in explaining variations of aggregate productivity.

We calculate domestic price from prices and quantities of consumption, investment and government spending. We follow the approach taken by Feenstra et al. (2015) to calculate total factor productivity using measures of inputs and output in national currency. Using these information, we calculate the contribution of domestic-export price ratio to aggregate productivity growth, hence also infer the contribution of productivity (again of producing gross output). The share of intermediate input in gross output, α_m, is not needed to know. At the end, we obtain data of productivity and relative price for 113 countries which have data for the period from the 1990s to 2014.

We compare the average annual productivity growth in two sub-periods: 1991 to 2000 and 2001 to 2014. Among the 113 countries, 52 observed a slowdown in aggregate productivity growth after 2000 which include most advanced economies in Americas and Europe, as well as some countries where commodity is a major export. Countries that did not experience productivity slowdown are mostly in Africa, Southern America, Asia, and Eastern Europe. First focusing on countries with productivity slowdown, 32 of those 52 countries also observed decreased domestic-export ratio, and this decrease in price ratio was the main cause of aggregate productivity slowdown in 15 countries (listed in Table A.6). Most of these 15 countries experienced both an improvement in the terms of trade (TofT) and a faster growth of in the productivity of producing gross output. This is consistent with our canonical model, it is the domestic-export relative price, not the terms of trade, that can be an important factor accounting for productivity growth. In the other 37 countries that experienced a slowdown in aggregate productivity, the falling productivity of producing gross output was the dominant cause of the slowdown (list in Table A.7). This includes three countries we studied using WIOD and KLEMS data (Germany, Italy, Spain and Sweden).

The above decomposition does not preclude that the slower growth of measured productivity was caused by other factors, such as the import price and the import tariff. But, we can estimate the relationship between productivity and relative prices, to see whether they are consistent with the model prediction. For country i in year t, the model suggests the following empirical equation

$$\ln A_{it} = \frac{1}{1-\alpha_m} \ln \tilde{A}_{it} + b_1 \left[\ln \left(\frac{p_d}{p_x} \right)_{it} + b_2 \ln p_{f,it} \right],$$

where $b_1 > 0$ and $b_2 < 0$. Data for the productivity $\tilde{A}_{it}$ of producing gross output are not available, we therefore cannot estimate α_m. Considering that $\tilde{A}_{it}$ is possibly serially correlated, we estimate the first differencing of the above equation, as follows

$$\Delta \ln A_{it} = b_0 + b_1 \left[\Delta \ln \left(\frac{p_d}{p_x} \right)_{it} + b_2 \Delta \ln p_{f,it} \right] + \varepsilon_{it}. \tag{12}$$

We use the generalized method of moment (GMM) to estimate the above equation. We use the lagged growth rates of independent variables and of the real GDP as instruments. The import price in one country can be the export price in another, we therefore let error terms to be clustered by region, allowing for the intra-region correlation.

Table 8 reports two sets of estimation, the first two columns for Equation (12). In the last two columns, the independent variable is the growth rate of the terms of trade. The estimates is consistent with our model prediction. The coefficient estimates for domestic-export price ratio are positive and statistically significant for both sub-samples as expected. The estimate for import price is negative using the longer sample but

Table 8: Total factor productivity and relative prices

	1990-2014	2000-2014	1990-2014	2000-2014
b_0	0.626	0.450	0.580**	0.930***
	(0.457)	(0.379)	(0.232)	(0.307)
b_1	0.730***	0.562***		
	(0.275)	(0.208)		
b_2	-0.084*	0.159		
	(0.050)	(0.145)		
$\Delta \ln \text{TofT}_{it}$			-0.117	-0.270
			(0.138)	(0.326)
Obs.	2,747	1,726	2,752	1,726

Dependent variable is TFP growth rate.
Robust standard errors in parentheses.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

statistically insignificant for the sample from 2000 and onward, as appears to be consistent with the overall declining import tariff and increased trade liberalization globally.

The second set of estimation suggests that the correlation between measured total factor productivity and the terms of trade is weak and negative for a broad set of countries. An improvement in the terms of trade might slightly reduce measured productivity, instead of raising it, though for some individual countries that correlation can be strong and positive.

7 Conclusions

In this paper we have extended the Domar aggregation of Hulten (1978) to a small open economy, measured aggregate total factor productivity consists of two components: the Domar aggregation of sector-level productivity of producing gross output, and the weighted average of domestic-export price ratios. These ratios reflect the relative productivity of the exporting sector, as well as export cost operated at home country. Import price does not explain changes in measured productivity perfect competition and the assumption of constant return to scale of production function. When calibrated to the Canadian data, the model suggests that productivity slowdown in the 2000s can be attributed to the commodity sector and the domestic sector producing machinery and equipment. The contribution of domestic-export price ratio in this period increased, making a positive contribution to productivity growth.

When applying our decomposition to cross-country data, we find that productivity slowed down globally in the 2000s. This slowdown in most countries can be explained by a slower growth of productivity in the domestic sector, while the domestic-export price ratio played a dominant role in about one third of countries. We also find that the correlation between measured productivity and the terms of trade is weak.

The result that lowered efficiency in sectors serving the domestic market suggest that identifying factors contributing to this efficiency loss is constructive to further understand productivity slowdown, as will be informative for policies intended to boost productivity growth. One potential source is distortions in the domestic economy that behavior like a negative productivity as shown in Jones (2013). These distortions are amplified in an economy like ours with input-output linkages.

The results of this paper crucially rely on assumptions: perfect competition and constant return to scale of production. These assumptions are usually maintained by statistical agencies to measure productivity, as could explain the insignificant coefficient estimates for import price in the cross-country data. Relaxing these assumptions can allow for a role played by import price on productivity growth, for example, as in Gopinath and Neiman (2014).

Finally, it should be noted that productivity growth slowdown in an open economy does not necessarily lead to a lower living standard. In the case of Canada, the improved terms of trade in the 2000s raises the purchase power of the domestic economy, leading to consumption growth in spite of a poor record of productivity.

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A Data sources

A.1 Growth accounting data

Sources of growth in Canadian business sector are calculated using the Diewert-Yu estimates of total factor productivity which were updated by Cao and Kozicki (2015) to reflect revisions to the national accounts. Total factor productivity growth estimated by the Diewert-Yu top-down approach is faster than estimated by Statistics Canada. Over the period of 1982 to 2015, the average annual total factor productivity growth in Statistics Canada data is 0.16%, much slower than 0.30% in the modified Diewert-Yu estimates. This gap lies primarily in the periods before 2001. See Gu (2012) and Diewert (2012) for detailed comparison between the two methods.

For our purpose, we modified the original Diewert-Yu estimates by excluding two types of inputs: business inventory and land. In the Diewert-Yu estimates, production inputs consist of machinery and equipment, non-residential structure, intellectual property products, business inventory, non-agricultural business land, and agricultural land. Business inventory and lands, together accounting for 9.5% of the total cost, are excluded because this paper does not model inventory and land. Excluding land and inventory affects the quantitative contribution of other capital assets in that the share of labor in total income is unchanged by the exclusion.

Another modification that has no effect on productivity measurement is to re-group the assets. We group capital into two categories: machinery and non-residential structures. The new national accounts introduced a new asset type, intellectual property products, consisting of software, mineral exploration and evaluation, and research and development. In the old national accounts, software was a component of machinery, mineral exploration and evaluation was a component of non-residential structures, and research and development was an intermediate input. These assets on average make small contribution to the gross domestic income, together at 2.5%. We put software back to machinery, and mineral exploration and evaluation back to structures. We also combine research and development with machinery. However, price of investment research and development displays a similar trend as the price of structures, which is different from the price of machinery. Research and development accounts for only 0.9% of total income, bundling it with machinery would not change main findings of this paper.

A.2 Definition of sectors

Sectors in the model are defined by taking into account both the definition of machinery and equipment in the literature of investment-specific technological change such as Cummins and Violante (2002) and the definitions of sectors in OECD STAN input-output tables. Sectors are defined as follows (using categories in the OECD STAN input-output table).

Commodity includes 1) Agriculture, hunting, forestry and fishing; 2) Mining and quarrying (energy); 3) Mining and quarrying (non-energy); 4) Wood and products of wood and cork; 5) Pulp, paper, paper products, printing and publishing; 6) Iron and steel ; 7) Non-ferrous metals; and 8) Fabricated metal products, except machinery and equipment. These categories are similar to the Bank of Canada Commodity Price Index (BCPI) composition. The last group, fabricated metal, is listed as a commodity because, basic metal and fabricated metal are listed as one group in the EU KLEMS data,

Machinery and equipment consists of 1) Machinery and equipment, n.e.c.; 2) Office, accounting and computing machinery; 3) Electrical machinery and apparatus, n.e.c.; 4) Radio, television and communication equipment; 5) Medical, precision and optical instruments; 6) Motor vehicles, trailers and semi-trailers; 7) Building and repairing of ships and boats; 8) Aircraft and spacecraft; 9) Railroad equipment and transport equip n.e.c.; 10) Manufacturing n.e.c.; recycling (include Furniture); 11) Research and development.

Structures correspond to the construction sector in the data, from which it is not to distinguish between residential and non-residential structures. For model calibration, we rely on as much as possible other data sources to impute or infer parameters regarding this sector.

The Rest includes categories other than the above ones. It covers sectors producing manufactured non-equipment intermediate inputs, consumption goods, and services. We exclude education, health services and public administration from this sector, as they are considered as non-business sectors in the data.

We use the World Input-Output Database (WIOD) to obtain the shares of intermediate inputs by each sector in total intermediate input. Categories of industry are similar between WIOD and the OECD STAN databases, but WIOD categories are less detailed than the latter. We modify categories in WIOD tables so that sectors in data match more accurately to sectors in the model. First, as in new national accounts, we treat research and development as a type of equipment asset, because our model does not have the asset type intellectual property products (again including software, research and development, and mineral exploration). In fact, in the OECD STAN input-output table, output values of research and development are zero, while in WIOD there is research and development but it is not a stand-alone sector.

Software (of general purposes) in Canadian national accounts is treated as an asset type, while in OECD STAN input-output table and WIOD it is treated as a commodity (listed under paper pulp, paper, printing and publishing). We use Canadian input-output table at the detailed level (Cansim 381-0022) to separate software and printed products from the paper category, and assign software to sector 1 and printed products to sector 4. What is left is paper pulp, paper, and newsprint, which is consistent with the BCPI composition.

A.3 Factor shares

Share of intermediate input in gross output. Table A.1 reports the factor shares in output. Except in the construction sector, the sum of factor shares is close to 1, justifying the assumption of constant returns to scale in production at the sector level. Capital cost as a share of output in construction is small, likely due to measurement errors (e.g., related to how labor and capital costs are allocated to this sector in the input-output table for certain construction activities). In calibration, we impose the constant returns to scale in each sector's production, and calculate the capital share as the total share net of the labor share and the share of intermediate input.

Table A.1: Factor shares in output, KLEMS 1981-2012

	In gross output			in GDP	
Sector	Intermediate input	Labor	Capital services	labor	Capital services
1	0.655	0.229	0.106	0.662	0.308
2	0.559	0.319	0.040	0.724	0.091
3	0.539	0.177	0.287	0.389	0.621
4	0.475	0.323	0.202	0.615	0.385
Aggregate	0.516	0.288	0.187	0.593	0.387

In the WIOD data, the average shares of intermediate input in gross output from 1995 to 2012 are respectively, 0.67, 0.51, 0.48, and 0.46, respectively for sector 1, 2, 3, and 4. These shares are fairly close to those in Table A.1.

We obtain the shares of sector output used for consumption in total consumption, as well as that for investment and export, all from the WIOD 1995 to 2012, as in Table A.2.

Table A.2: Shares of final demands produced by sectors in total final demands, WIOD 1995-2012

Sector	Final consumption	Investment	Export
Domestic, 1	0.025	0.078	0.311
Domestic, 2	0.005	0.607	0.003
Domestic, 3	0.031	0.019	0.329
Domestic, 4	0.764	0.147	0.357
Imported, 1	0.076	0.143	n/a
Imported, 2	0.000	0.000	n/a
Imported, 3	0.008	0.003	n/a
Imported, 4	0.091	0.004	n/a
Total	1.00	1.00	1.00

Shares of imports and exports in total. From CANSIM 380-0070, shares of exports in total export and imports in total imports are calculated in Table A.3.

Table A.3: Shares of sector-level exports and imports in totals, National Accounts

Sector	1982-2012	2001-2012
	Exports ($\tilde{\lambda}^*_{fi}$)	
Machinery (1)	0.351	0.309
Commodity (3)	0.352	0.353
The Rest (4)	0.297	0.338
	Imports	
Machinery (1)	0.439	0.394
Commodity (3)	0.163	0.177
The Rest (4)	0.398	0.429

From the WIOD, we also obtain the uses of sector i outputs as a share of sector i outputs, as well as the uses of sector i imports as a share of sector i imports. These are summarized in Table A.4.

Parameters $(1-\psi)$, μ_{f1}, and μ_{f4}. Table A.2 shows that the share of imported machinery in total investment in machinery is $\psi = 0.364$ (=0.143/(1.0-0.607)). An alternative calculation is to use Table A.4 and imports from national accounts. Table A.4 shows that 30% of imported machinery is used for investment. From national accounts we know the aggregate machinery imports. We then can obtain the value of investment from imported machinery, dividing which by non-structures investment gives values of 40% or 50%, depending on the definition of non-structures investment. This suggests that the value of ψ is fairly different between the WIOD and national accounts. We choose to set $\psi = 0.40$.

Similar calculation can be done for μ_{f1}. Table A.4 shows that 30.3% of imported machinery is used for final consumption. This share, together with the imported machinery from national accounts, gives the value

Table A.4: The use of sector-level outputs and sector-level imports

Sector	Intermediate input	Final consumption	Investment	Export	Total
Uses of domestic goods, as share of sector-level outputs					
Machinery (1)	0.221	0.073	0.086	0.620	1.00
Structures (2)	0.146	0.017	0.830	0.007	1.00
Commodity (3)	0.394	0.069	0.016	0.520	1.00
The Rest (4)	0.441	0.400	0.029	0.130	1.00
Uses of imported goods, as share of sector-level imports					
Machinery (1)	0.484	0.303	0.212	n/a	1.0
Commodity (3)	0.916	0.073	0.011	n/a	1.0
The Rest (4)	0.575	0.419	0.007	n/a	1.0

of final consumption from imported machinery. Dividing this imported machinery used for consumption by the aggregate non-investment GDP leads to $\mu_{f1} = 0.071$,[23] which is close to the value of 0.076 in Table A.2. We set $\mu_{f1} = 0.076$.

Use the similar calculation, we obtain imported sector-4 good for final consumption as a share of total non-housing consumption, 0.089.[24] This value is close to the WIOD data 0.09 in Table A.2. We set $\mu_{f4} = 0.090$.

A.4 Shares of sector outputs

We use the KLEMS data to calculate both the share of sector-level output in aggregate gross output, and the Domar aggregation weights (sector-level gross output divided by gross domestic product). It is noted that Sector 2 (construction) in Table A.5 includes both residential buildings, non-residential building, non-residential engineering, as well as construction repairs. From data of GDP at basic prices (Cansim 379-0031), ratio of non-residential construction over total construction is about 0.66.

Table A.5: Output shares, KLEMS 1981-2012

	Sector			
	1	2	3	4
Share of gross output	0.127	0.099	0.156	0.618
Share of value added	0.090	0.090	0.150	0.670
Domar weight	0.263	0.204	0.323	1.276

[23] If non-housing consumption is used as the denominator, the implied μ_{f1} is 0.10.

[24] If non-housing consumption is used, this number would be 0.13.

A.5 Cross-country data

Table A.6: TFP slowdown due to reduced domestic-export price (percentage points, annual)

	1991-2000			2001-2014			
Country	TFP	Relative price	Domar Agg.	TFP	Relative price	Domar Agg	TofT Improved
Bahrain	0.68	-0.80	1.48	-0.66	-2.78	2.12	no
Netherlands	0.93	1.02	-0.09	-0.05	0.48	-0.53	no
Portugal	0.17	0.56	-0.40	-0.16	0.24	-0.40	no
Uruguay	1.72	0.83	0.88	1.52	0.08	1.44	no
Argentina	1.81	0.10	1.71	0.68	-0.76	1.45	yes
Chile	1.25	0.78	0.47	-0.12	-1.00	0.89	yes
Kuwait	3.84	0.00	3.84	-1.63	-3.28	1.65	yes
Mauritania	-0.43	-0.17	-0.26	-0.94	-3.91	2.97	yes
Mexico	-0.41	0.84	-1.25	-1.09	0.04	-1.13	yes
Namibia	1.14	1.23	-0.09	-0.30	-1.03	0.73	yes
Poland	2.31	1.32	0.99	1.32	-0.27	1.59	yes
South Korea	1.49	1.40	0.10	1.10	0.92	0.18	yes
Saudi Arabia	0.07	0.32	-0.25	-1.55	-2.85	1.30	yes
Sudan (Former)	-0.27	-0.17	-0.11	-1.50	-3.18	1.67	yes
Tunisia	0.53	0.14	0.39	0.19	-0.64	0.82	yes

Source: author's calculation based on the Penn World Table.

Table A.7: TFP slowdown due to falling productivity of producing gross output (percentage points, annual)

	1991-2000			2001-2014			
Country	TFP	Relative price	Domar Agg.	TFP	Relative price	Domar Agg.	TofT Improved
Armenia	7.24	0.13	7.11	5.61	0.39	5.22	no
Belgium	0.61	0.95	-0.34	-0.39	0.68	-1.07	no
Central African Republic	0.54	0.02	0.52	-2.41	0.47	-2.89	no
Costa Rica	0.47	0.36	0.10	0.42	1.18	-0.75	no
Egypt	0.23	0.01	0.22	-1.33	1.11	-2.44	no
Estonia	3.27	0.79	2.49	1.60	0.81	0.80	no
Finland	2.01	0.24	1.77	-0.29	0.90	-1.20	no
Germany	1.40	0.25	1.15	0.41	0.33	0.08	no
Greece	0.50	0.23	0.26	-0.74	-0.11	-0.63	no
Iran	-0.17	-1.15	0.99	-0.42	-0.73	0.32	no
Iraq	4.30	0.00	4.30	-0.66	0.00	-0.66	no
Ireland	3.47	1.36	2.11	0.15	0.93	-0.78	no
Israel	0.43	0.47	-0.03	0.18	0.38	-0.21	no
Jamaica	0.09	0.00	0.09	-0.44	0.31	-0.75	no
Malta	1.51	-3.89	5.40	-0.37	0.56	-0.92	no
Mauritius	1.17	0.53	0.64	0.54	0.73	-0.19	no
Norway	1.98	-0.51	2.49	-0.54	-0.35	-0.19	no
Slovenia	3.14	1.57	1.57	0.56	0.45	0.11	no
Spain	-0.25	0.13	-0.38	-0.46	0.18	-0.64	no
United Kingdom	1.25	0.34	0.91	0.39	0.07	0.32	no
United States	1.23	0.19	1.03	0.74	0.03	0.71	no
Australia	1.50	0.08	1.42	0.51	0.02	0.49	yes
Austria	0.97	0.42	0.56	0.06	0.42	-0.35	yes
Benin	1.14	0.33	0.81	0.17	-0.04	0.20	yes
Canada	0.87	-0.19	1.06	-0.23	0.27	-0.50	yes
China	4.83	0.58	4.26	3.74	0.27	3.47	yes
Croatia	4.09	-0.30	4.39	-0.34	-0.09	-0.25	yes
Cyprus	1.80	0.99	0.81	-0.50	0.26	-0.75	yes
Denmark	1.14	0.21	0.93	-0.21	0.24	-0.45	yes
France	1.03	0.43	0.60	-0.13	0.27	-0.40	yes
Italy	0.32	0.13	0.20	-1.08	0.16	-1.24	yes
Luxembourg	1.58	-1.47	3.05	-1.10	-1.61	0.51	yes
Mozambique	2.18	0.23	1.95	0.91	0.14	0.77	yes
New Zealand	0.88	-0.11	0.99	0.27	0.38	-0.11	yes
Qatar	3.06	-2.15	5.21	-3.30	-2.16	-1.15	yes
Senegal	0.13	-0.70	0.84	-0.25	0.06	-0.31	yes
Sweden	1.75	0.49	1.25	0.70	0.56	0.14	yes

Source: author's calculation based on the Penn World Table.

B Figures

Figure B.1: Terms of trade, commodity price, and productivity

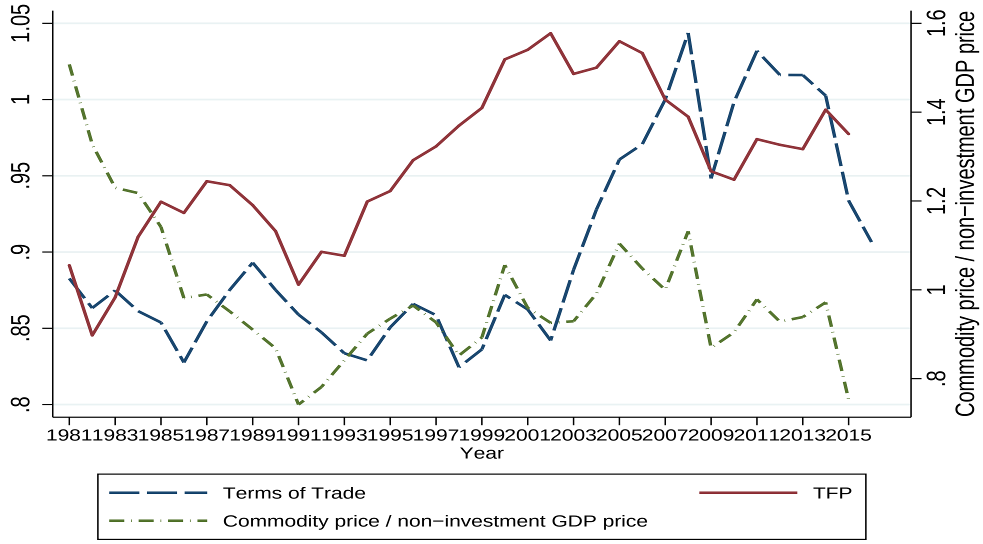


Figure B.2: Relative prices of Machinery and Equipment

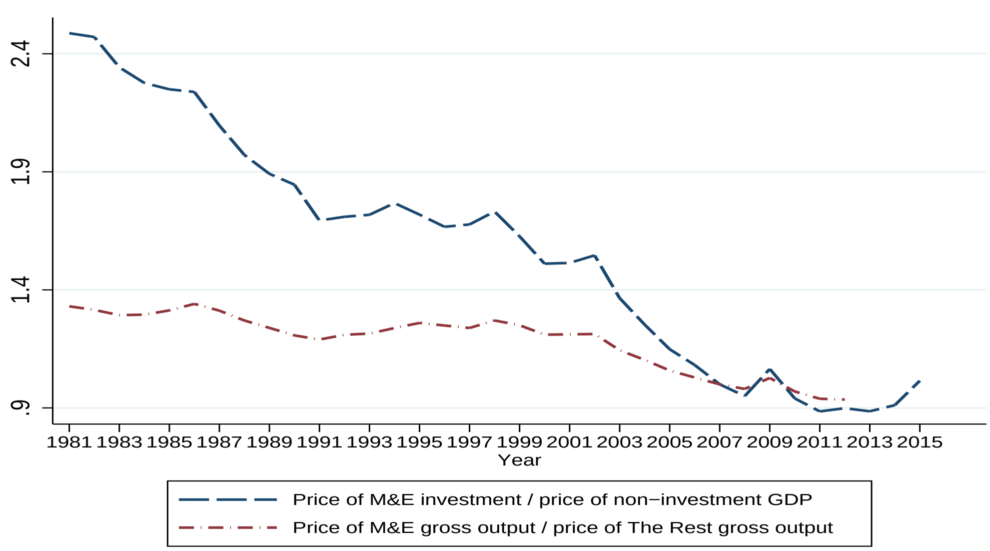


Figure B.3: Aggregate total factor productivity

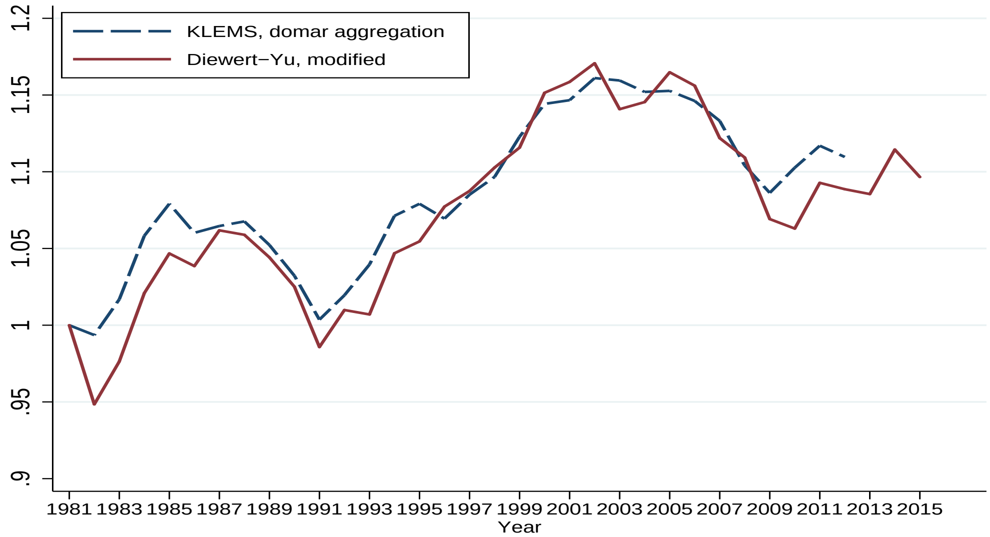


Figure B.4: Sector-specific total factor productivity in gross output

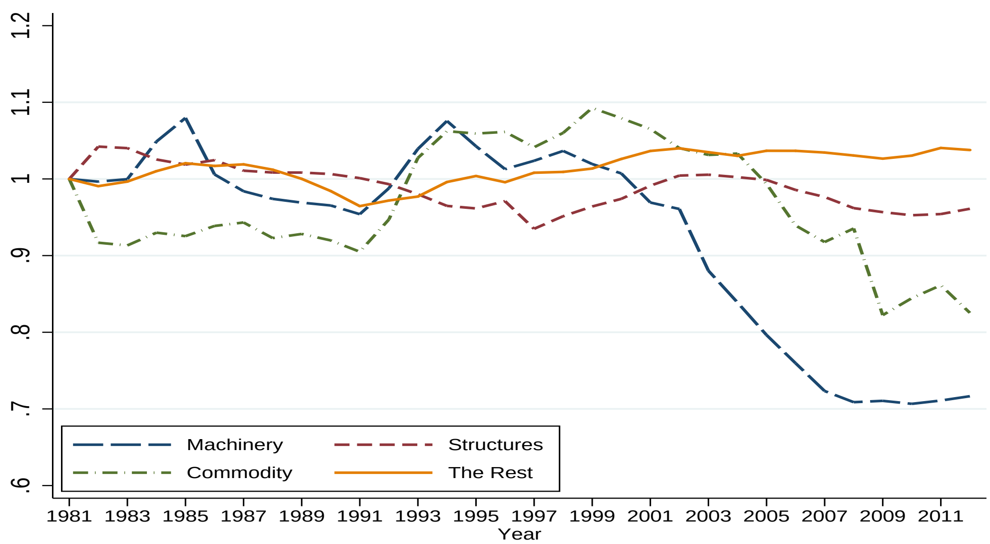


Figure B.5: Price of domestic sales relative to price of export

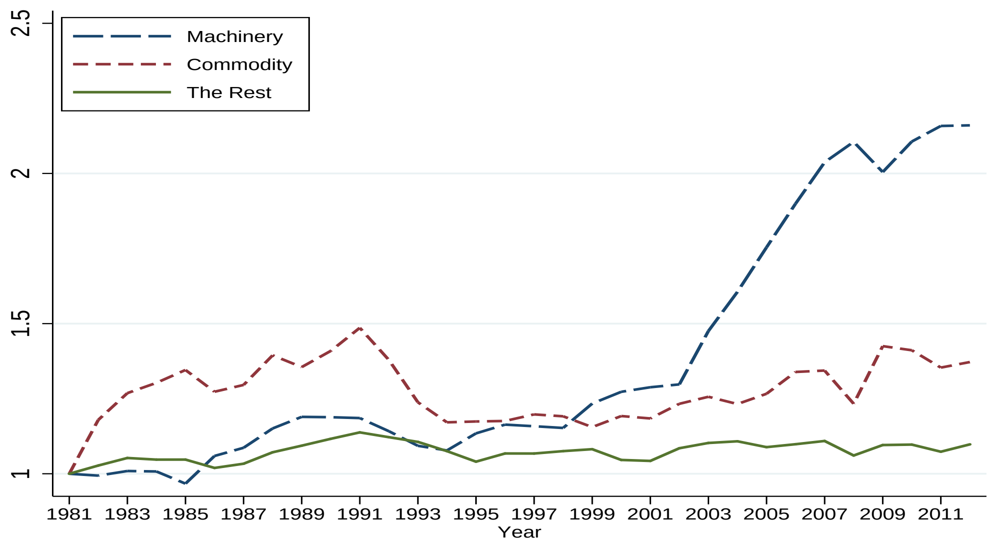


Figure B.6: Contribution to annual aggregate TFP growth (percentage points)

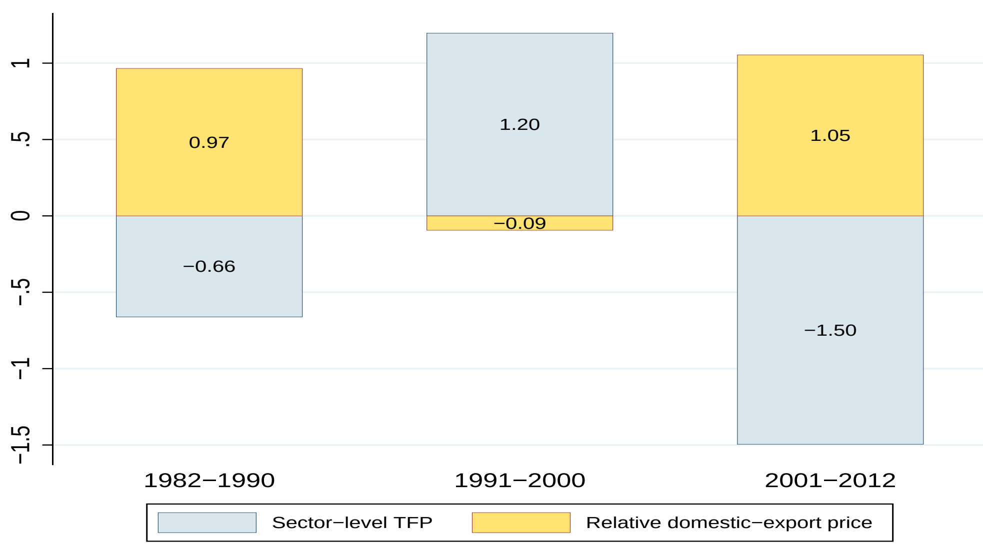


Figure B.7: Contribution to annual aggregate TFP growth (percentage points)

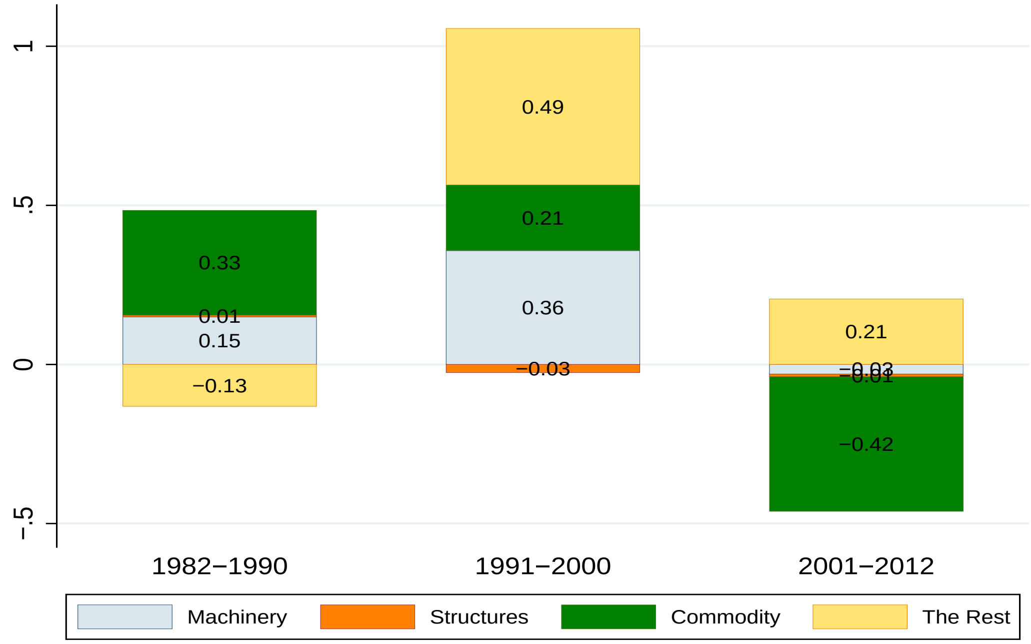
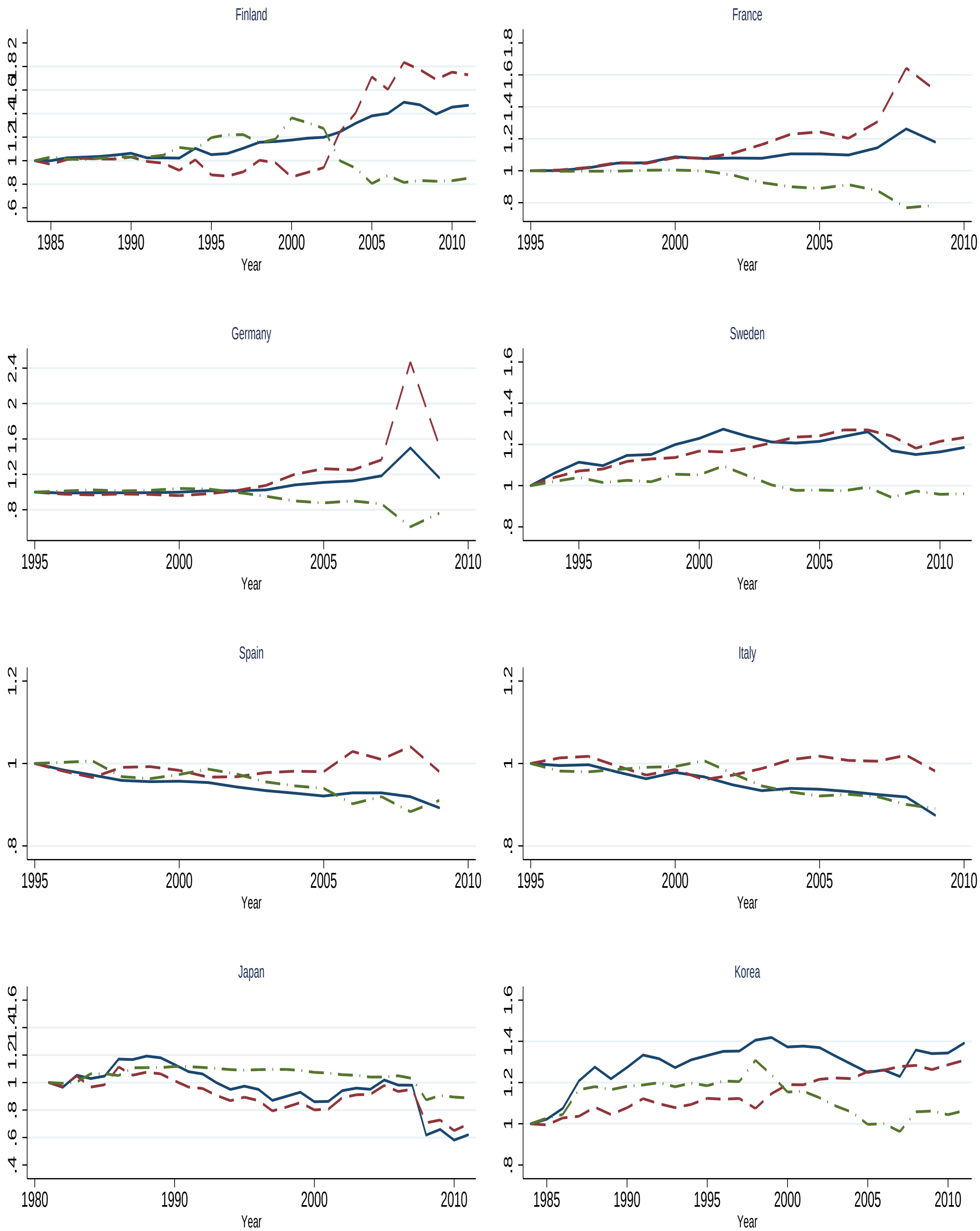
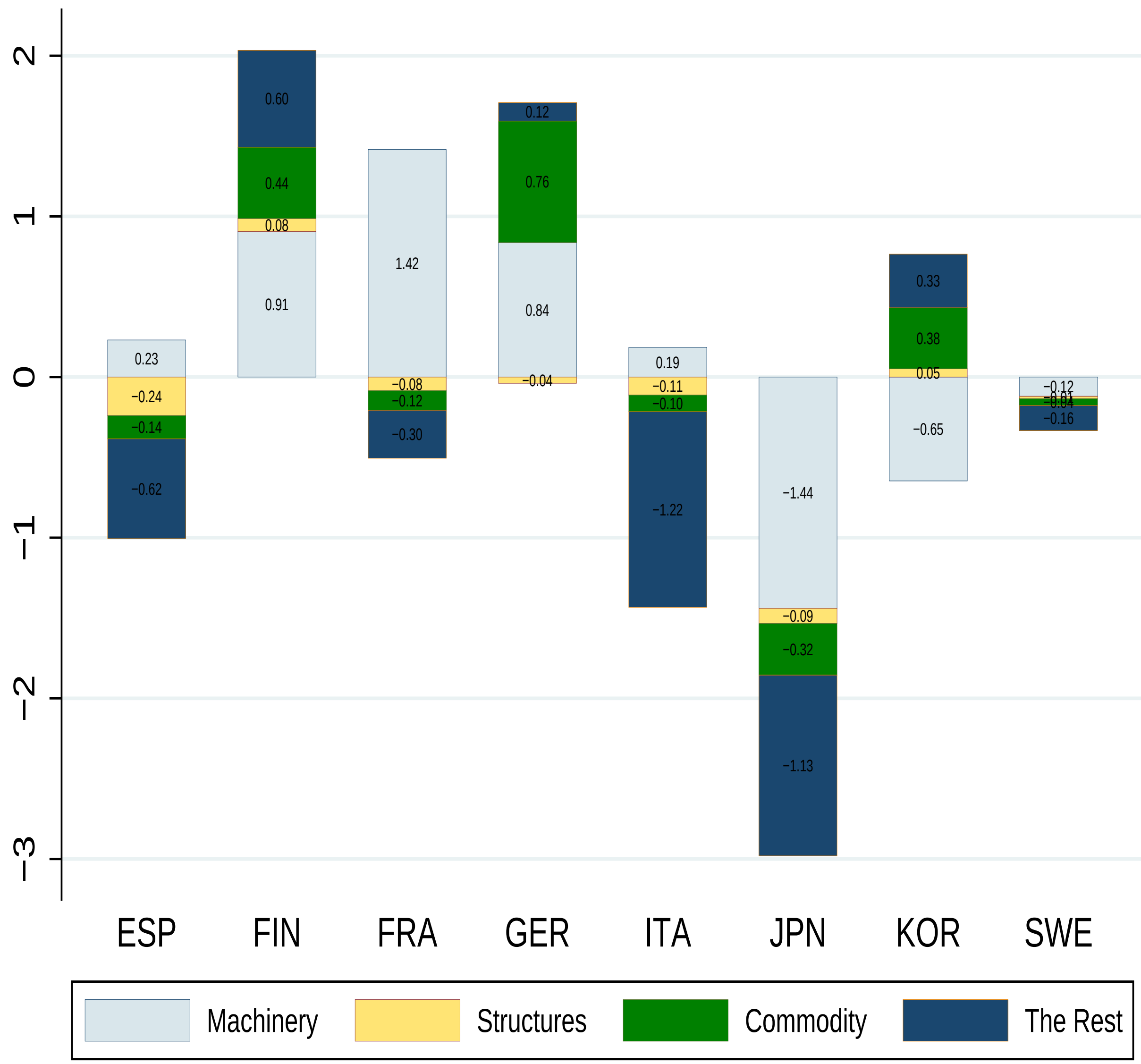


Figure B.8: Aggregate TFP and relative price



Note: Data source are WIOD and various sources of KLEMS. Solid line represents the aggregate TFP, dashed line represents the contribution of sector-level TFP to aggregate TFP, and dash-dotted line represents the contribution of relative domestic-export price to aggregate TFP.

Figure B.9: Contribution to aggregate TFP growth 2001 to 2011, by sector (percentage points)



Source: WIOD and EU KLEMS

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