

**“Maths is challenge, struggle and mistakes will grow
our brain”:**

**The impact on student attitude, confidence and achievement of
the introduction of inquiry-based learning into the mathematics
programme of a New Zealand primary school**

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A Proverb

Tell me and I will surely forget.

Show me and I might remember.

But make me do it, and I will certainly understand.

Abstract

Underachievement in mathematics in Aotearoa/New Zealand continues to be an issue for some students. Inquiry-Based Learning (IBL) has been described by research as one way of addressing these underachievement issues. Ongoing underachievement impacts on students' confidence which may exacerbate underachievement in a downward spiral. Research has shown that both confidence and achievement can be positively influenced by IBL, therefore IBL was trialled here at All Saints School. This thesis describes a research project which sought to determine the impact of an IBL teaching intervention with the aim of improving outcomes for students underachieving in mathematics. It examines the impact on students' attitude, confidence and achievement that resulted from the introduction of IBL into the mathematics teaching and learning programme of three classes, Years 3, 4 and 6, in a high socio-economic status (SES), high achieving, urban Catholic full primary school. The intervention drew on a professional learning community where the participant teachers explored literature on IBL and worked together to assist each other to add IBL to the teaching and learning programme for mathematics.

The study design was a mixed methods case study. Qualitative data were gathered through student interviews and surveys. The intervention was undertaken over a full school year, so quantitative achievement data were gathered from the school's usual assessment methods without the introduction of further external testing or assessment.

Student surveys and interviews from three classes totalling 51 students informed the research questions on student attitude and confidence. Over-all Teacher Judgement (OTJ) and Progressive Achievement Tests (PAT) provided quantitative data which informed the research questions on the impact IBL had on student achievement and the achievement gap between the highest and lowest achievers.

In this school setting students began the intervention with a very positive attitude to mathematics and only minor variations to this were observed. Students also began with a high level of confidence in their overall mathematical ability, but very low confidence in their problem-solving ability specifically. By the end of the intervention, their high level of confidence had extended to their problem-solving confidence also.

PAT achievement data revealed the Year 3 class and the Year 4 underachieving students both made mean achievement gains of a statistically significant level. The Year 4 class only just reached national averages, but the Year 3 and 6 classes exceeding national average results for their year level. A deeper exploration of the data revealed that the low achieving students made major achievement gains for the intervention year. The low achieving Year 4 and 6 students made gains that exceeded both national averages and their high achieving classmates by large margins. Taken together these results further add to the body of evidence that argues for the inclusion of IBL in schools' mathematics programmes.

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Chapter 1

Introduction

1.1 Context

Recent international studies such as the Trends in International Mathematics and Science Study (TIMSS) (Mullis, Martin, Foy, & Arora, 2012) and the Programme for International Student Assessment (PISA) (Organisation for Economic Co-operation and Development, 2014) showed a large percentage of students worldwide underachieving in mathematics. Calder and Brough (2013) state “Mathematics and statistics are inextricably linked with everyday life – they are key elements of being an informed participant within a diverse range of culture and social groupings and hence central to existing and contributing effectively in society” (p. 1). Therefore, mathematics needs to be made more accessible for those students who find success in mathematics a challenge.

1.2 Background to the Study

The 2012 PISA results placed Aotearoa/New Zealand, with a mean score of 500, above the OECD average of 494. This places Aotearoa/New Zealand above the United States of America (USA) (481) and the United Kingdom (UK) (494), but below Australia (504) and Ireland (501). PISA level two is considered the level of proficiency required to actively participate in mathematics related life situations (May, Cowles, & Lamy, 2013). Therefore, the percentage of students not reaching level two is one measure of a country’s lack of mathematics education success. On this measure, Aotearoa/New Zealand did not achieve well with 22.6 percent of our students not reaching PISA level two. This places Aotearoa/New Zealand slightly ahead of the USA (25.8), but behind Australia (19.7), the UK (21.8), and Ireland (16.9). At the other end of the achievement scale are the top performing students reaching level five or six (OECD, 2014). Internationally, Aotearoa/New Zealand (15%) fares better than Australia (14.8%), the UK (11.8%), Ireland (10.7%) and the USA (8.8%). These results identify Aotearoa/New Zealand as having one of the largest gaps in the OECD between high and low achieving students.

1.3 Aotearoa/New Zealand Context

Since 2003, the average scores of New Zealand students' mathematical ability, as measured by PISA, have been in decline, a decline that has been particularly marked since 2009 as shown in the graph below.

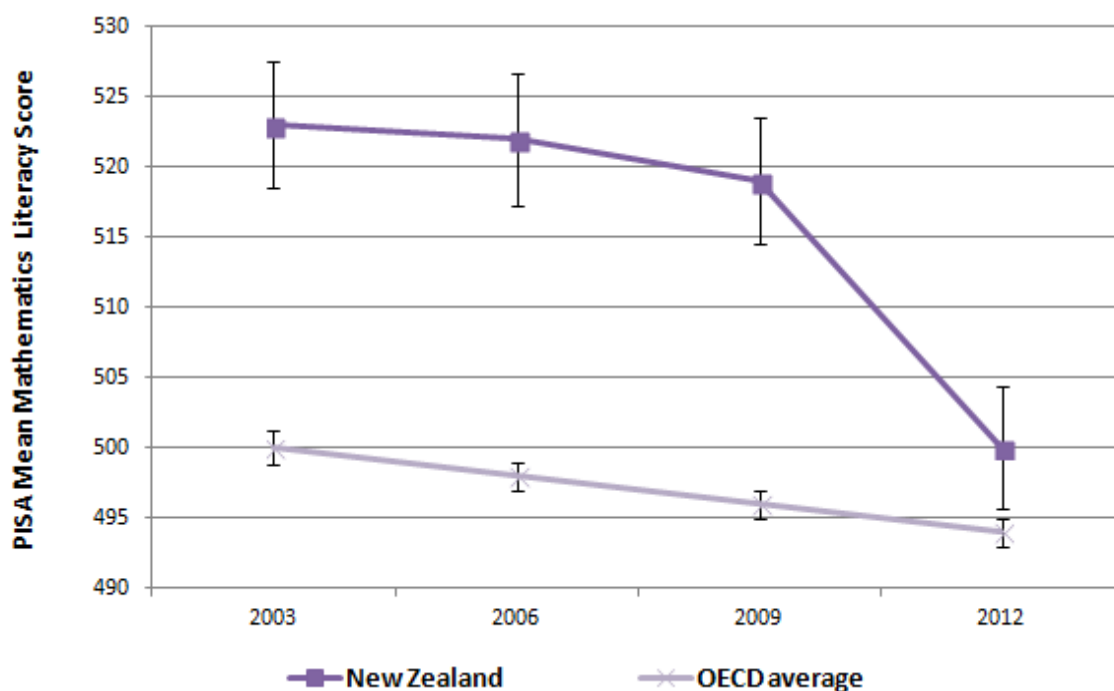


Figure 1.1 PISA mathematics literacy mean scores for New Zealand and OECD, 2003, 2006, 2009 and 2012¹ (Ministry of Education, n.d.a)

This decline has come despite the Ministry of Education's (MOE) investment in professional development for teachers through initiatives such as the Numeracy Development Projects. By 2012, the average PISA score for Aotearoa/New Zealand students had decreased from 523 to 500 (May et al., 2013). In 2003 Australia, Ireland, Poland, and Germany were all ranked below Aotearoa/New Zealand but by 2012 these same countries were all ranked above Aotearoa/New Zealand, due to both their increase in score and Aotearoa/New Zealand's decrease. The almost 23 percent of students not reaching PISA level 2 in 2012, while the same as the OECD average, still leaves nearly one quarter of Aotearoa/New Zealand students not achieving the mathematical skills they require for everyday life, with Bicknell and Young-Loveridge (2015) noting that "Māori, Pasifika, and students from lower decile schools are particularly disadvantaged" (p.2). As an initiative to increase achievement the MOE

¹ 2012 was the most recent data available at the time of writing.

identified groups of students for whom mathematics success appeared to be a greater challenge. Three groups of students were identified as having lower overall achievement statistics than the Aotearoa/New Zealand average and these three groups became MOE priority learners. The three sets of priority learners are Māori, Pasifika, and students from low socio-economic status (SES) areas. During the period 2003-2012 the average score for priority learners also declined.

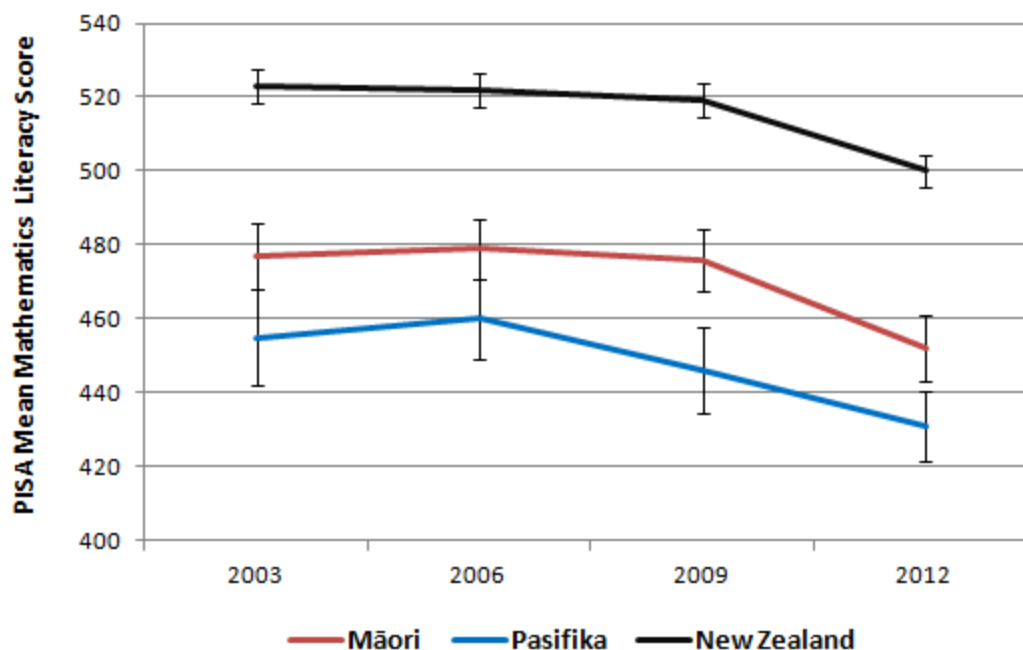


Figure 1.2 New Zealand PISA mathematics literacy mean scores by ethnic group 2003, 2006, 2009 and 2012 (MOE, n.d.a)

The average scores for these groups were below those of the Aotearoa/New Zealand average, with Māori (38%), Pasifika (46%), and 41 percent of low socio-economic students not reaching PISA level 2. Aotearoa/New Zealand has one of the widest gaps in achievement between highest and lowest scoring students of all countries in the OECD. This comes from a growing number of students with low performance in mathematics (May et al., 2013).

The 2010/2011 TIMSS study showed Aotearoa/New Zealand has a higher number of students in the very low achieving band in comparison with other countries with similar or higher overall average achievement (Caygill, Kirkham, & Marshall, 2013). The MOE National Standards data paint a similar picture with the 2015 data showing just 75.5 percent of students 'At' or 'Above' the National

Standard. Again the figures for Māori (65.4%) and Pasifika (63.3%) are lower than overall figures (MOE, n.d.b).

1.4 A Possible Solution

The international and Aotearoa/New Zealand situations outlined above suggest that change is needed in the way mathematics is taught if all students are to be successful. In 1938, John Dewey proposed a radical change to teaching and learning when he advocated inquiry as the vehicle for teaching science (Artigue & Blomhoj, 2013). Inquiry moved the focus from a teacher-centred approach to teaching and learning to a student-centred one. Alongside this, underpinned by the work of Vygotsky and Piaget (2.3.1), has been the development of sociocultural theory (2.3.2), which provides a theoretical foundation for inquiry as an approach to teaching and learning. Today, inquiry as a pedagogy is being advocated across a range of subjects, including mathematics where it has become known, amongst other titles, as Inquiry-Based-Learning (IBL). This change in approach to mathematics teaching and learning is being advocated by many researchers (e.g., Aulls & Shore, 2008; Engeln, Euler, & Maass, 2013; National Council of Teachers of Mathematics (NCTM), 2000; Silver & Stein, 1996) as a means of improving student attitude to, and raising student achievement within, mathematics.

In Aotearoa/New Zealand, with 24.5 percent of students failing to reach the National Standard in mathematics, change needs to happen to enable more students to achieve success. In All Saints School², which is in a high SES area, achievement levels were higher than the national average with 86.8 percent of students reaching the National Standard in mathematics at the end of 2014; however, these results still leave 13.2 percent of students not achieving success in mathematics. In 2010 I, as the mathematics lead teacher, was involved in the Accelerate Learning in Mathematics (ALiM) pilot programme (NZMaths, n.d.). This programme assisted All Saints School in raising levels of student achievement within mathematics. However, despite the success for some students involved in ALiM, there were still students for whom success in mathematics was challenging. My personal involvement in ALiM raised my interest in undertaking postgraduate study to improve my teaching skills with a view to further improving student achievement in mathematics. During this

² Pseudonyms are used for the school, teachers and all students.

postgraduate study I read about IBL, which appeared to offer a teaching and learning approach that may enable a greater number of students to achieve mathematical success. My reading about IBL and its effects led to the design and implementation of the intervention outlined in this study. The study aims to determine the impact of the introduction of IBL in mathematics on students in a New Zealand high SES full primary (Years 0-8) school. The outcomes of this study add to the international and Aotearoa/New Zealand research outlining the impact of IBL on students' mathematics outcomes, which to date has largely focused on schools in low SES areas.

1.5 Research Objectives

The major research question guiding the study was:

- Will a programme of reform based on IBL, implemented by the mathematics lead teacher in a high achieving, high SES Aotearoa/New Zealand primary school, have a positive effect on student attitude, confidence and achievement?

Specific Research Questions:

1. How does the introduction of IBL impact on student attitudes to mathematics?
2. How does the introduction of IBL impact on students' confidence in their mathematical ability?
3. How does the introduction of IBL impact on the mathematics achievement outcomes for all participating students?³
4. How does the introduction of IBL impact on the overall achievement gap between the highest and lowest achievers?⁴

1.6 Overview

Chapter 2 provides a definition of IBL, outlining its history and the foundational place of sociocultural theory in IBL. The Aotearoa/New Zealand mathematics education landscape is briefly reviewed prior to an overview of the pedagogical practice of IBL, student confidence is explored, and a review of the literature regarding the results that the introduction of IBL has on students' achievement.

Chapter 3 outlines the research setting, sample and schedule. Then the

³ Levels of achievement are determined by PAT scale score (3.5.1.1) and teacher OTJ (3.5.1.2).

⁴ Research questions are in the order they will be addressed in the thesis, not necessarily the order of priority.

methodology for the research project, including the choice of a Professional Learning Community (PLC) as the vehicle for working with the teachers to introduce IBL into their classrooms. Data collection and analysis are discussed along with the ethical considerations.

Chapter 4 presents the research results under the four headings of: How children see Mathematics; Attitude to Mathematics; Confidence in Mathematics; and Achievement in Mathematics. Further discussion of the research findings occurs in Chapter 5. The summary in Chapter 6 draws conclusions, outlines implications, and offers suggestions for further possible research.

Chapter Two

Literature Review

2.1 Introduction

Chapter One highlighted the persistent issue of student underachievement in mathematics which has impacted both the international and Aotearoa/New Zealand educational communities. This chapter addresses IBL, the teaching and learning approach being promoted by many researchers as an alternative to the traditional approach to mathematics instruction and which offers one potential way of addressing underachievement, both for All Saints School and the wider mathematics education community. Also considered is the theoretical approach to teaching and learning known as sociocultural theory, which is gaining growing recognition for its explanation of the student learning process, and which forms the theoretical foundation for the use of IBL in this thesis.

The history of IBL is discussed (2.2.2), before the definition of IBL as used in this thesis is given (2.2.3). Section 2.3 introduces sociocultural theory, from the foundational thinking of Lev Vygotsky and Jean Piaget (2.3.1), through its subsequent development (2.3.2), the nature of communities of practice is outlined (2.3.3), before the impact of sociocultural theory on pedagogy is considered (2.3.4). An examination of the current Aotearoa/New Zealand mathematics educational landscape in light of sociocultural theory follows in section 2.4. The following sections consider the specific implementation of IBL in classroom teaching and learning including: the role of the teacher and students (2.5.1); the development of norms (2.5.2); and challenges to implementation (2.5.3). Section 2.6 discusses student confidence before 2.7 reviews what research has to say about the results of implementing IBL, followed by a summary in section 2.8.

2.2 Inquiry-Based Learning

2.2.1 Introduction

Current mathematics education research literature promotes IBL as a potential solution to ongoing, world-wide issues of underachievement within mathematics (Engeln et al., 2013; Silver & Stein, 1996). Historical achievement patterns suggest conventional mathematics instruction holds “little hope for a further

closing of the gap (for underachieving students) in the near future” (Silver & Stein, 1996, p. 480). As outlined below (section 2.4), in Aotearoa/New Zealand, following initial gains made through the Numeracy Development Project, there is an emerging downward trajectory in overall student achievement data (Ministry of Education, 2013). There is, therefore, the need for a fresh approach if New Zealand schools are to meet the objective of successful educational outcomes for all students. This study is intended to determine whether that fresh approach might be in the form of IBL in this specific high SES, high achievement school setting.

2.2.2 History of Inquiry

Inquiry teaching and learning can trace its origins to John Dewey who in 1938 recommended the use of inquiry in the teaching of science (Artigue & Blomhøj, 2013). Dewey proposed a model in which students were actively involved and the teacher took on the role of facilitator and guide (Barrow, 2006). The shift was towards a student-centred approach rather than a teacher-centred learning environment. Dewey advocated for a problem-solving process in which the move is away from a separation between education and the real world in which “Inquiry.....combines mental reasoning and action in the world” (Schon, 1992, p. 121). The types of problems Dewey proposed students undertake were those in their areas of interest and their intellectual ability range. The perspective changed from seeing students as passive receivers of knowledge, to students being “guided to learn by doing” (Aulls & Shore, 2008, p. 155).

Curriculum reform in the USA during the 1960s had a focus on inquiry. However, there was some criticism of the inquiry approach in the 1970s, due to programmes not being within the student’s developmental range and the lack of emphasis on the social context of the work. Curriculum reform in the 1980s and 1990s again focused on the use of inquiry (Aulls & Shore, 2008), with documents like the *National Science Education Standards* (National Research Council, 1996) considering inquiry “as the overarching goal of scientific literacy” (Barrow, 2006, p. 268). This repeated and ongoing emphasis on inquiry is not limited to science with Aulls and Shore (2008) noting inquiry has been the focus of calls for reform in the teaching of “all basic subjects” (p. 2) for decades. In New Zealand there is an emphasis on problem solving in the 2007 revision of *The New Zealand Curriculum* (NZC), such as this from the section on Mathematics and Statistics: “These two

disciplines are related but different ways of thinking and solving problems” (MOE, 2007, p. 26). One vehicle for enacting that problem-solving approach in mathematics is the use of IBL. The introduction of an intervention based on IBL shifted problem solving to a more central focus of teaching and learning in mathematics at All Saints School.

2.2.3 Definition

The term IBL has been linked with a range of titles for what are essentially similar, or overlapping, approaches to teaching and learning. These include: Reform (Cheeseman, 2008; Manswell Butty, 2001), Inquiry (Artigue & Blomhoj, 2013; MaaB & Artigue, 2013), Discovery (Aulls & Shore, 2008), Realistic Mathematics Education (Treffers & Beishuizen, 1999), Cognitively Guided Instruction (Anthony, Bicknell, & Savell, 2001; Carpenter, Fennema, & Franke, 1996), Problem Solving (Coti & Zuljan, 2009), Communities of Mathematical Inquiry (Alton-Lee, Hunter, Sinnema, & Pulegatoa-Diggins, 2012; J. Hunter 2006; R. Hunter, 2008), and Communities of Mathematical Discourse (Ball, 1993; J. Hunter, 2009).

MaaB and Artigue (2013) state: “The term inquiry-based learning generally refers to student-centred ways of teaching in which students raise questions, explore situations, and develop their own ways towards solutions” (p. 780). This study uses the term IBL to refer to a student-centred approach (Aulls & Shore, 2008; Dorier & Garcia, 2013; MaaB & Artigue, 2013), in which the focus is on: first, mathematical understanding (Skemp, 1976), thinking and reasoning (Silver & Stein, 1996; Steinberg, Empson, & Carpenter, 2004); second, students working in mixed ability groups and as a whole class (Alton-Lee et al., 2012); third, problem-solving using non-routine or open-ended problems (R. Hunter, 2008; Steinberg et al., 2004); fourth, students presenting their work during whole class discussion in which they explain, justify, and argue mathematically (Ball, 1993; Steinberg et al., 2004); Fifth, using student errors as a resource for learning (Hunter & Anthony, 2011); Sixth, a press for sense making by all students (Skemp, 1976; Stein, 2007), leading to negotiated whole class conclusions (Ball, 1993).

Literature contains references to a range of potential benefits from using IBL, including that IBL fosters deeper understanding, and connections between classroom learning and students’ lives (Makar, 2007). Students create their own solution strategies which means that their work is always based on their existing knowledge,

which is then extended as they work to solve non-routine problems (White, 2003). Students, because they are creating the strategies, are actively involved learners not passive receivers of knowledge (Aulls & Shore, 2008; Yackel, 1995). Further, IBL allows students to build their own learning, or essentially discover for themselves through guided experiences what others have already discovered (Aulls & Shore, 2008). Finally, it has the potential to allow students' learning to move at a pace that best suits them.

As mentioned above IBL often focuses on non-routine problems. Kolovou, van den Heuvel-Panhuizen, and Bakker (2009) state:

...that genuine problem solving refers to a higher cognitive ability in which a straightforward solution is not available and that mostly requires analysing and modelling the problem situation. In order to be a true problem for students, it should not be a routine problem. (p. 37)

This study adopted this definition; non-routine problems, along with longer open-ended inquiry-based units, were the focus of the work with students. These longer IBL units were more project based where students were given a task and needed to problem solve the solution, for example the Olympic Event unit in Appendix C. IBL is an interactive and social, rather than individual approach to learning. Schön (1992) states: "Inquiry as Dewey conceived it is ...inherently social" as the individuals became "members of communities of inquiry" (p. 122). The following section examines IBL's theoretical underpinnings in the form of sociocultural theory.

2.3 Sociocultural Theory

2.3.1 Foundations

Sociocultural theory draws on, and links, the learning theories of Lev Vygotsky and Jean Piaget. Vygotsky defined learning as happening in a social context (Fosnot, 1996). Piaget emphasised the individual as constructing their own learning in a social environment (von Glasersfeld, 1990). The work of John Dewey, who saw communication, social life, and education as being interrelated (Dewey, 1944), complements the work of Piaget and Vygotsky.

The Piagetian constructivist approach emphasises learning as an individual process in which the learner constructs their own understanding in a social environment (von Glasersfeld, 1990). The focal point is the individual and the social aspects are the environment which enables the individual to construct their learning.

The social process leads to disequilibrium between what a learner already knows and what they are experiencing in the social interaction. This disequilibrium then leads to new learning (Fosnot, 1996; Lerman, 2000; Palincsar, 2005).

Vygotsky saw the development of human personality as a social construct: “The concept ‘personality’ is, thus, a social, reflective concept...” (Vygotsky, 1983, p. 324, cited in Valsiner & van der Veer, 2005). Learning, as viewed by Vygotsky, is said to happen twice; first, the learning is created within the social interaction, then secondly, in the mind of the learner (Lerman, 2000). This distinguishes him from Piaget for whom the emphasis was on the individual. However, in a similar fashion to Piaget, Vygotsky also viewed learning as a developmental or constructive process (Fosnot, 1996). This learning occurs, socially, through the use of “mediational means such as tools and language” (Lerman, 2000, p. 34).

Central to Vygotsky’s approach to teaching and learning is the Zone of Proximal Development (ZPD). The ZPD is the area that bridges the learner’s existing knowledge and understanding with the new learning that comes from their interaction within the social group. For the learner to be developing new knowledge and understandings the social groups’ interaction must be connected to the learner’s existing level of development (Steele, 2001). When the learner is working in the ZPD they are capable of achieving and assimilating new learning which is beyond what they can master individually (Lerman, 2001; Palincsar, 2005). Despite their different emphases, both these theorists inform our understanding of sociocultural theory due to their shared understanding of learning as a constructive process which is related in some fashion to the social setting in which it occurs.

2.3.2 The Development of Sociocultural Theory

The developing theory of Socioculturalism combined the ideas of Piaget and Vygotsky to create a unique theory of learning. In this theory learning is seen as being constructed (Noddings, 1990), with the construction occurring within the combined activity of the group, not the mind of the individual (Palincsar, 2005; Salomon & Perkins, 1998). This shifts sociocultural theory away from early constructivism which focused on the construction of learning within the mind of the individual in a social environment. Within this sociocultural view learning is socially constructed by all of the participants and the learning is shared across the minds of

all the individuals who make up that specific community (Brown, Collins, & Duguid, 1989; Rogoff, 2003; Salomon & Perkins, 1998).

In sociocultural theory learning is one aspect of active participation in situated social communities of practice (Palincsar, 2005; Rogoff, 2003). Lave and Wenger state: “learning is an integral and inseparable aspect of social practice” (p. 31). Learning as participation was developed by Lave and Wenger (1996) in their concept of “legitimate peripheral participation” (p. 35). This concept, which developed out of the apprenticeship model, sees learning taking place within a community, or social group of practice where all members are seen as active participants. The legitimate peripheral participation theory sees the “teacher/learner dyad” (Lave & Wenger, 1996, p. 56) replaced by a model in which participants are either newcomers or old-timers. As newcomers grow in knowledge, skill and discourse, they develop towards the role of old-timers, who are described as “full participants” (Lave & Wenger, 1996, p. 37). Other newcomers join the community of practice at a later date, participate, and begin their own journey towards becoming old-timers (Brown et al., 1989; Lave & Wenger, 1996). Rogoff (2003) defines this process as the transformation of the individual’s participation within the social context.

For the intervention at All Saints School the combined focus of small group and whole class discussion, alongside both group and personal responsibility to ensure everyone understands the mathematics being discussed, ties directly into the social nature of learning as outlined by Piaget and Vygotsky. The selection of problems for use in IBL also allows students to begin working on a problem with the prior knowledge they have, thus placing the learning directly in their ZPD.

2.3.3 Communities of Practice

Communities of practice are situated in specific locations of time and space and the knowledge that grows out of that community of practice is unique to them (Brown et al., 1989; Rogoff, 2003; Salomon & Perkins, 1998). This is because the knowledge grows out of the participation of the members of the community as they use the tools specific to that community (Brown et al., 1989; Rogoff, 2003; Salmon & Perkins, 1998). Further, as members function within their ZPD within these communities of practice “children learn to use the intellectual tools of their community, including literacy, number systems, language, and tools for remembering and planning.”

(Rogoff, 2003, p. 282). A different community of practice will contain different participants and therefore will form their own unique journey in development of knowledge and skills (Salomon & Perkins, 1998). Within each community the members create a socially mediated learning environment (Rogoff, 2003). Each member grows in their ability to participate, through their participation (Lave & Wenger, 1996; Salomon & Perkins, 1998). Within this socially mediated environment the action of the members collectively enables the learning of each community member. This is best summed up in this quote from Lave and Wenger (1996): “All of this takes place in a social world, dialectically constituted in social practices that are in the process of reproduction, transformation and change” (p. 123).

We see then that a focus on the apprenticeship model, or as Aulls and Shore (2008) state, “being guided to learn by participation” fits well with Lave and Wenger’s (1996) concept of legitimate peripheral participation which dovetails with the concept of communities of practice. When IBL in mathematics is considered in light of the sociocultural theory just outlined, we can see the links between sociocultural theory and the definition of IBL in mathematics as outlined above, and the practice of IBL as outlined below. What was undertaken in the intervention at All Saints School was the creating of a community of practice within each participating classroom where students were encouraged to view themselves as mathematicians.

2.3.4 Sociocultural Theory applied to Pedagogy

Accepting socioculturalism as a theory explaining the way in which cognitive development takes place requires one to develop a sociocultural approach to pedagogy. In this view the community of learners is seen as the key to teaching and learning (Cobb, Wood, & Yackel, 1993; Lerman, 2001) with participants discussing and solving problems together with an emphasis on small group work and whole class discussion. As Brown et al. (1989) state: “Collaboration also leads to articulation of strategies, which can then be discussed and reflected on. This in turn, fosters generalizing, grounded in the students ‘situated understanding’” (p. 39). Sharing and reflecting is key to assisting students to develop multiple strategies for solving each problem which is seen as an essential aspect of their developing participation in the community of practice. Discussion and cooperative work is where Vygotsky and his followers see learning as occurring. The community, rather

than the individual, becomes the place where new learning is constructed through the active participation of each of the individuals (Cobb, 1994, 2000; Cobb, et al., 1993; Rogoff, 1994). Rogoff (1994) states: “The community-of-learners model based on theoretical notions of learning as a process of transformation of participation in which responsibility and autonomy are both desired...” (p. 210). Incidents of students’ reasoning are seen as acts of participation in the communal practices. These practices are enacted by the teacher and students themselves (Cobb, 2000).

This sociocultural approach to pedagogy differs from the “one-sided” models of either adult-centred, where the focus is on the teacher as the active participant and the students as passive receivers of information which is ‘dispensed’ transmission style by the teacher, or child-centred, where the focus is on the students as the active participants and the adults are simply guides (Rogoff, 1994). In a sociocultural pedagogy the students and teacher are both seen as active and both actively position themselves as learners (Lerman, 2001). The classroom becomes a learning place for the teacher and the students (Cobb et al., 1993). Within the classroom community they create their own normative set of practices for doing mathematics (Seah, Atweh, Clarkson, & Ellerton, 2008). These practices become the taken-as-shared understanding of what it means to participate in mathematical activity. The practices they create are unique and do not exist apart from the community that creates them (Cobb, 2000).

In a sociocultural approach, students are seen as learning as they work together with other students and adults in solving problems and carrying out activities connected with the practices of the discipline in which they are working. So when carrying out activities and solving problems within mathematics the students are learning to participate in the community of mathematicians and engage in the general use of mathematics by the wider society (Cobb, 1994; Lerman, 2001; Yackel & Cobb, 1996). In a sociocultural model mathematics is seen as a “complex human activity rather than as disembodied subject matter” (Cobb, 2000, p. 65).

The shift to the sociocultural model requires, in the words of Rogoff (1994), “a paradigm shift like that of learning to live in another culture” (p. 215). When a sociocultural understanding of teaching and learning is fully adopted, it is no longer appropriate for teachers to view student learning as building up “a good (if not yet complete) replica of the ideas in the teacher’s mind” (Maher & Alston, 1990, p. 147).

Rather, it requires teachers to radically revise their ideas to viewing themselves and the students collectively as a community, who together construct their own learning. Teaching in this way requires a shift in teacher beliefs and understandings as well as a shift in their classroom practice. These shifts are not easily made and teachers will need a sound reason to undertake the shifts required.

A note of caution needs to be raised here. While one can see the links that can be made between sociocultural theory, the sociocultural pedagogy outlined here, and the teaching and learning approach of IBL within mathematics, it does not lead automatically to the belief that sociocultural theory stipulates the IBL approach to pedagogy. In the words of Simon (1995): “it [sociocultural theory] does not tell us how to teach mathematics; that is, it does not stipulate a particular model.” (p. 115). Sociocultural theory is a theory of how people learn, not a practical ‘how to’ guide for teaching (Fosnot, 1996; Lave & Wenger, 1996).

The shift to a sociocultural approach to teaching and learning required a shift in thinking for both participating teachers and students at All Saints School. The teachers needed to see themselves as learners within the classroom and develop new ways of teaching, while the students need to begin taking more responsibility for their own learning and learn a new way of learning. All of these shifts were new for all of the participating teachers and students.

As outlined in the previous chapter, student underachievement in mathematics is a world-wide problem, an issue in Aotearoa/New Zealand, and at All Saints School. This does not mean, however, that the causes and reasons behind that underachievement are the same in every context around the world, nor necessarily that the same solution will work everywhere world-wide. So, before going on to outline in detail what IBL looks like in practice (2.5), it is appropriate to briefly consider the current New Zealand mathematics education context.

2.4 The Aotearoa/New Zealand Context

In New Zealand the aims and objectives of mathematics education are outlined in the NZC (MOE, 2007) and then further defined in *The New Zealand Curriculum: Mathematics Standards for Years 1-8* (MOE, 2009). In the New Zealand Curriculum, it states:

By studying mathematics and statistics, students develop the ability to think creatively, critically, strategically and logically. They learn to structure and to organise, to carry out procedures flexibly and accurately, to process and communicate information, and to enjoy intellectual challenge. (p. 26)

The NZC includes achievement objectives for Number and Algebra, Geometry and Measurement, and Statistics. These achievement objectives are arranged in a series of ascending levels from one to eight designed to encompass a student's entire schooling from Year 0 to Year 13. These achievement objectives are further refined within the *Mathematics Standards* (MOE, 2009) which indicate specifically what children are expected to achieve at each year level of their schooling until they reach secondary school.

A large proportion of the achievement objectives in the NZC, particularly those in the Number Strand, relate historically to *The Numeracy Projects*. *The Numeracy Projects* were developed following the *Count Me In Too* (Years 0-3), and *Numeracy Exploratory Study* (Years 4-6) pilot studies undertaken in 2000 (Young-Loveridge, 2004). The Numeracy Projects were developed as part of a government initiative targeted at building the mathematics teaching capability of primary school teachers as a means of raising student achievement (Young-Loveridge, Bicknell, & Lelieveld, 2013).

Five projects were developed: *The Early Numeracy Project (Years 1-3)*; *The Advanced Numeracy Project (Years 4-6)*; *The Intermediate Numeracy Project (Years 7-8)*; *The Secondary Numeracy Project (Years 9-10)*; and a version for use in Māori medium schools called *Te Poutama Tau* (Young-Loveridge, 2004). As part of this programme an overarching framework called *The Number Framework* was developed describing a series of research-based developmental progressions (Young-Loveridge, Bicknell, & Lelieveld, 2013). This further refines the Curriculum achievement objectives and The National Standards specific learning objectives to describe the knowledge and strategies students should have if they are to make the required progress through their schooling journey. The eight stages of the Numeracy Project include a student's mathematics schooling from Year 0 to Year 10, whereas the eight Levels of the Curriculum cover the entire education of students across all subjects from Year 0 to Year 13. These projects were introduced across the country through an extensive professional development programme which ran from 2001 until 2009 when the initial phase was completed (Young-Loveridge, 2009).

The Numeracy Projects added a greater range of skills to the primary school teacher's toolkit, while alongside the addition of those skills came a strong focus on ability grouping of students. However, the MOE states that following initial gains made from the Numeracy Development Project there is now "an emerging downward trajectory" in mathematics achievement (MOE, 2013, p. 1). In the All Saints School context a similar pattern was seen with initial gains and then a stagnation in mathematics achievement levels. Therefore, a new approach was required to continue closing the achievement gap between highest and lowest achieving students. This led to the current IBL intervention. We now move on to what the sociocultural shift means for classroom practice in what Lerman (2000) has called "the recontextualisation of ideas into pedagogy (p. 20). The following section will give an overview to what the literature tells us about the nature of IBL and what it looks like within the classroom context.

2.5 IBL in Practice

2.5.1 The Role of the Teacher and Students

In IBL the teacher takes the stance of a facilitator, rather than the keeper of all knowledge and sole arbitrator of mathematical correctness (Ball, 1993; Grant, Hiebert, & Wearne, 1998; R. Hunter, 2006; Stein, 2007; White, 2003; Yackel & Cobb, 1996). The place of authority for determining mathematical correctness shifts from being solely the role of the teacher, to the shared responsibility of both students and teacher (Ball, 1993; Grant et al., 1998; R. Hunter, 2006; Stein, 2007; White, 2003; Yackel & Cobb, 1996). The teacher moves from a show and tell model to "responsive guidance in developing pupils' own thinking" (Anghileri, 2006, p. 33). This makes "guided participation...central to the role of teachers in the classroom" (Anghileri, 2006, p. 35). Guided participation is a significant shift and requires a fundamental change in teacher beliefs about mathematics, and mathematics pedagogy (Anthony et al., 2001). Research suggests that in a teacher-centred classroom up to 80 percent of talk is teacher talk (Kotsopoulos, 2007), whereas IBL requires the teacher to talk less, allowing the students to talk more. IBL also requires that the teacher undertakes a variety of roles such as facilitator of the discussion, participant in the discussion, and commentator on the discussion (Alton-Lee et al., 2012; Aulls & Shore, 2008; Manswell Butty, 2001; R. Hunter, 2006). It becomes the teacher's responsibility to promote the students' mathematical understanding by

focusing on, and developing, the discussion that takes place in both whole class and small group settings (Anghileri, 2006).

Teachers need to have high expectations for the achievement of all students (Fravillig, Murphy, & Fuson, 1999). In IBL this means they must continually press to ensure all students make sense of the mathematical ideas and concepts that emerge during the small group and whole class phases. This requires them to support the strugglers and at the same time challenge those who are succeeding (MaaB & Doorman, 2013). Teachers use a variety of methods for achieving this support/challenge balance depending on their class and situation, but one important aspect of achieving this balance, within IBL in general, is the use of flexible mixed ability grouping of students. This flexibility and changing of grouping assists with students' learning through peer collaboration rather than teacher instruction (Anghileri, 2006).

In IBL the entire lesson cannot be scripted beforehand (Jacobs & Ambrose, 2008) but evolves as students' responses are articulated. The teacher needs to adapt and refocus the discussion as it happens to ensure the mathematical goals for the lesson are achieved. These goals are developed from the teacher's growing knowledge of the students' thinking and their knowledge of problem solving during the planning of each lesson (Steinberg et al., 2004).

This introduction of IBL is a significant shift for students, as they have to transition to a new way of doing and learning mathematics. To enable this shift the teacher must facilitate the establishment of a new set of social norms for the way mathematics is done, and a new set of sociomathematical norms for what is and is not acceptable mathematically (2.5.2). The teacher is responsible for initiating and guiding the development of these norms (Yackel, 1995), although their exact nature emerges out of a collaboration between the teacher and the students.

For most teachers these requirements represent a significant expansion of their pedagogical content knowledge (Makar, 2007; Shulman, 1986). Many writers highlight the challenging nature of this transition for teachers (e.g., Artigue & Blomhoj, 2013; Ball, 1993; R. Hunter, 2008; Manouchehri, 2007; Staples & Colonis, 2007). Despite the acknowledged difficulty, Stillman (2013) adds a positive note: "Teachers' overall teaching practices improved within one year of beginning to learn about and use mathematical inquiry units in their classroom" (p. 914).

The intervention at All Saints School involved participating teachers adding IBL to their range of teaching strategies and gaining in their confidence and ability over the period of the study to draw out the main learning goals in a lesson.

2.5.2 Norms

Introduction

IBL requires both teacher and students to work in new ways which are expressed in terms of classroom norms, and which in this instance come in two distinct types, sociocultural norms, and sociomathematical norms. A norm is a way of doing things, a taken-as-shared understanding between teacher and students as to what behaviours, actions, words, ways of working, discussing, relating and interacting are acceptable in a particular setting (Yackel & Cobb, 1996). Negotiating these norms is about developing an understanding of what, when, and how we work (Bauersfeld, 1993, as cited by Yackel & Cobb, 1996). In an IBL classroom, neither sociocultural nor sociomathematical norms is a set of predetermined criteria the teacher introduces, rather they are negotiated between teacher and students in the process of doing mathematics and continue to evolve and be modified throughout the year (Yackel & Cobb, 1996). This means the norms may be different in different classes. Therefore, no definitive list of norms, or specific definition of each norm can be stated. However, there are some norms generally common to most IBL mathematics classrooms which will now be elaborated.

Sociocultural Norms

Sociocultural norms include the ways in which a classroom is organised and lessons are conducted. In IBL mathematics those norms are quite different from non-IBL mathematics and include ways of responding to the teacher and fellow students, expectations in terms of work output, and the roles of both teacher and students.

A distinctive feature of IBL is the emphasis on discussion in both whole class and small group settings. In IBL the focus shifts from traditional teacher-centred discussion patterned on question-response-evaluation, to discussion focused on student to student interactions. This shift promotes greater valuing of students' ideas as their ideas are the focus of the discussion (White, 2003). This focus on student ideas and student to student interaction has several benefits: first, it involves greater participation in terms of numbers of students who can contribute; secondly, it increases the depth of student thinking as they are required to explain, challenge,

justify, and debate both their own and other students' mathematical ideas (Fraivillig et al., 1999); thirdly, it builds the learning on the students' prior knowledge as this becomes the starting place for the ideas discussed (White, 2003).

Mathematical discourse has the potential to facilitate the development of students' mathematical thinking (Anghileri, 2006; Aulls & Shore, 2008; White, 2003). It becomes the teacher's responsibility to promote the students' mathematical understanding by helping to develop the discussion so that key mathematical ideas are explored (Anghileri, 2006). Care needs to be taken to ensure all students are included, and it is the teacher's responsibility to decide when and how to draw the quieter students into the discussion (White 2003). The skillful use of probing questions is central to the teacher shaping the discussion to explore the key mathematical concepts they wish to bring out during the lesson. This discussion is marked by students explaining the methods of other students, questioning each other, building on the thinking of their classmates, and evaluating the mathematical merit of each other's work. With classroom discourse playing a key part in IBL, the teacher is required to facilitate, participate in, and comment on classroom discussion (Alton-Lee et al., 2012; Aulls & Shore, 2008; R. Hunter, 2006; Manswell Butty , 2001). The balancing of these three roles is a key skill that teachers who wish to successfully implement IBL need to develop.

In an IBL classroom there is the expectation that everyone can be successful in mathematics. The teacher holds high expectations for all students and expects all students to tackle all problems. Likewise, the students are encouraged to see themselves as mathematicians who are able to solve all given problems.

Student grouping in an IBL classroom is heterogeneous rather than homogeneous. Students work in mixed ability groups with all students expected to undertake the problem being worked on. In these deliberately mixed ability groups students interact and learn from each other and all students are expected to be able to both solve the problem and explain and justify the group's strategy. Students are expected to work together discussing solutions to problems, to share their solutions with others, both in small groups and in whole class discussion (White, 2003; Yackel, 1995), rather than working individually as is the case in many non-IBL mathematics classrooms.

In IBL classrooms students are expected to explain their solution strategies not just their answers (White, 2003; Yackel & Cobb, 1996). Kazemi and Stipek

(2001) maintain “an explanation consists of a mathematical argument, not simply a procedural description” (p. 59). So it is expected that as students share they do more than just say “This is what I did” rather, they explain why they did what they did and what makes that an acceptable method. When students do this the teacher focuses on the students’ conceptual understanding by asking, or expecting students to justify their strategies mathematically (Kazemi & Stipek, 2001). The expectation is that each new student who shares will present a mathematically different solution strategy.

A single solution strategy is not sufficient in an IBL mathematics lesson. Students are encouraged to create more than one strategy within their group and as a whole class (Fraivillig et al., 1999; McClain & Cobb, 2001; Yackel, 1995). No single solution strategy is *the* correct one; rather, all strategies are accepted, compared, contrasted and evaluated for efficiency. There is no one right way to solve a problem but a number of ways and students can either keep their own strategy or select someone else’s that they see as being easier or more efficient.

Persistence is an important sociocultural norm in inquiry mathematics (Kazemi & Stipek, 2001). Students are expected to continue to wrestle with a problem until they have created a solution of their own. This is an important aspect of students’ learning, as they are more likely to remember a solution strategy which they have had to work hard to achieve than they are a strategy the teacher explains, which students may have forgotten by the next day or the next week.

A key norm in an inquiry classroom is the expectation students actively participate at all times including listening attentively to their classmates when they are speaking (Fraivillig et al., 1999; McClain & Cobb, 2001; White, 2003). Listening is fundamental to each student understanding the mathematics being discussed (Yackel, 1995), and leads to the sociocultural norm of a continual press for all students to gain a full understanding of the mathematics involved. While there is a much greater emphasis on collaborative work in an IBL classroom, this collaborative work is balanced by a strong emphasis on individual accountability. Students are given the responsibility of ensuring they understand the mathematics that is being presented and if they do not it is their responsibility to ask and keep asking questions until they do.

An inquiry classroom is often a noisy place, characterised by challenge, discussion, and uncertainty. It requires patience, flexibility, a balance of cooperation

and independence, and a willingness to expect the unexpected (Makar, 2007). For the participating teachers and students at All Saints School these sociocultural norms represented a major shift in the way that they both undertook and thought about mathematics. It was not a shift that they were able to make overnight, rather, they developed their way into working in a classroom characterised by these norms. An especially challenging shift was accepting the differing roles teachers had in discussion as students still tended to see the teacher as having the answers and so initially teacher comments often brought an end to student discussion. Undertaking mathematics in mixed ability groups also took time to fully evolve as initially the low achieving students seemed less willing to contribute and teachers needed to learn to facilitate their participation effectively.

Sociomathematical Norms

Sociomathematical norms refer specifically to the mathematics itself, not to the way the mathematics is undertaken. Sociomathematical norms set the basis for the mathematical thinking within the classroom (R. Hunter, 2008; Kazemi & Stipek, 2001; McClain & Cobb, 2001; Yackel & Cobb, 1996). These norms “set forth a way of interpreting classroom life that aims to account for how students develop specific mathematical beliefs and values” (Yackel & Cobb, 1996, p. 458). Sociomathematical norms differ from sociocultural norms in their focus. Yackel and Cobb (1996) give the following example: “... the understanding that when discussing a problem students should offer solutions different from those already contributed is a social norm, whereas the understanding of what constitutes mathematical difference is a sociomathematical norm” (p. 461).

Sociomathematical norms cover such things as: what is a mathematically acceptable solution? what qualifies as a mathematical explanation? what is a mathematically acceptable justification? what constitutes a mathematically different, sophisticated, or efficient solution? (McClain & Cobb, 2001; Yackel, 1995). An important feature of sociomathematical norms is that the process of creating them causes students to recognise and think specifically about their mathematical thinking and beliefs (McClain & Cobb, 2001). This process of metacognition assists the students in the learning process. These norms are negotiated between students and teacher during the course of mathematical discussion throughout the year. The negotiation process is itself a learning opportunity for students as they consider

mathematical concepts, and for the teacher as they have the opportunity to hear students thinking about important mathematical concepts.

The sociomathematical norm for what constitutes an acceptable explanation and justification requires being able to demonstrate and understand them through actions on mathematical objects the students can manipulate (McClain & Cobb, 2001). Therefore, a solution is only acceptable when it is both mathematically correct and the other students are able to understand it.

Comparing and contrasting the range of solution strategies shared by the class is an important sociomathematical norm in an IBL classroom and a single solution strategy is not sufficient (Fraivillig et al., 1999; McClain & Cobb, 2001). Individuals, groups, and the whole class are expected to produce multiple strategies for each problem. Students are expected to examine similarities and differences between the various strategies (Kazemi & Stipek, 2001), and to search for the easiest, most efficient strategies (Fraivillig et al., 1999). An important sociomathematical norm is that of mathematical difference. An alternative solution strategy is only acceptable if the mathematical process is different, or as McClain and Cobb (2001) observe, if it can be symbolised in a different way. This need for alternative solution strategies causes challenges to students thinking that would not occur if alternative strategies had not been required (McClain & Cobb, 2001). There are no pre-set criteria for what counts as mathematically different, this emerges through negotiation between students and teacher in the process of exploring differences between solutions (Yackel & Cobb, 1996).

Students are expected to agree or disagree with a classmate's solution and to be able to give mathematical reasons for their answers (Kazemi & Stipek, 2001; White, 2003). Students accept or reject a classmate's solution based not on who they believe to be the most mathematically able, but rather a serious exploration of mathematical thinking where all students are expected to be able to justify their answers by giving mathematical reasons for them (Yackel & Cobb, 1996).

Another sociomathematical norm in an IBL classroom is the handling of student errors which become a resource for learning (Hunter & Anthony, 2011). Initially the focus is on student ideas, on their solution strategies and on the justification of their answers, not the "correctness of their answers" (Chapin, O'Connor, & Anderson, 2009, p. 18). Rather than rejecting student errors teachers appropriate them and they become the material for guiding student learning (Boaler,

2015; Chapin, et al., 2009; Fraivillig et al., 1999). As the steps in the reasoning are explored and the source of the error uncovered, students are often led to correct their own mistakes (Anghileri, 2006; White, 2003). Participation in this process helps students learn how to review and examine their own and others' thinking to determine the mathematical correctness of the solution (Chapin et al., 2009). This acceptance of errors has the effect of making it safe for students to take a risk and share mathematical ideas knowing that they will not be ridiculed or laughed at if they get it wrong, which leads to a greater willingness to attempt more complex problems. "They gradually lose some of the anxiety and avoidance behaviours that many students display when confronted with complex mathematical ideas" (Chapin et al., 2009, p. 17).

Boaler (2016) suggests that brain research shows making errors actually has a positive effect on students' brain activity and therefore on their learning. "What this means is that we want students to be making mistakes, and we should not be giving students work that they get mainly correct" (Boaler, 2015, p. 4). She further suggests that teachers should be giving students open tasks, not work that is quickly answered right or wrong. In an IBL lesson errors are seen as an opportunity "to reconceptualise a problem, explore contradictions in solutions, and pursue alternative strategies" (Kazemi & Stipek, 2001, p. 59). Examining errors is a part of the process of building conceptual understanding, an important goal of IBL (Kazemi & Stipek, 2001; McClain & Cobb, 2001).

These sociomathematical norms represented a major shift in approach to teaching and learning for the participating teachers and students at All Saints School. As with the sociocultural norms, adopting and working with the sociomathematical norms was a process rather than an event. The shift led to a deeper exploration of the mathematics than previously, requiring deeper mathematical thinking from both the teachers and the students. The change in approach to student errors represented perhaps the biggest difference for the participating students at All Saints School and it took most of the year before a significant percentage of students felt comfortable discussing the mistakes they had made.

2.5.3 Challenges to Implementation

Some of the challenges to implementing IBL that are highlighted in the literature include: that it is time consuming; there can be issues of student participation; it

requires new skills; it requires student higher order thinking; there can be challenges in keeping track of student learning; lack of teacher first-hand experience; low teacher content knowledge; and teacher challenge and frustration.

Introducing IBL in mathematics can be a time consuming process in two distinct ways. Firstly, it is not a change that teachers can decide to make and have up and running the next week. Developing the classroom norms necessary for eliciting and supporting student thinking is not a simple process and takes time and skill (Anghileri, 2006; Fraivillig et al., 1999). Secondly, the process of teaching through inquiry takes longer than more traditional forms of instruction as noted by Aulls and Shore (2008): “When enacting the curriculum, learning content through investigation, projects, or research demands more time than do traditional approaches to instruction” (p. 20). This time-consuming nature of IBL can mean that there is less curriculum coverage than with traditional instruction methods with the trade-off being that the mathematics that is undertaken is covered in more depth. The issue of curriculum coverage can be a problem for teachers who have external high stakes testing to prepare their students for, making them “unwilling to risk changes and omissions in covering the required curriculum material” (Aulls & Shore, 2008, p. 143).

There are also challenges with regard to student participation. As in any group work situation it is possible for some students to let the others do most of the work, and learning, while they contribute little and learn little as a result. This is especially so in heterogeneous groups such as those advocated for use in IBL. These mixed ability groups can leave those who have lower self-confidence sitting back and letting those they see as more able do the work (Kazemi & Stipek, 2001). Boaler (2008) sees this as a common problem teachers need to overcome to ensure all students are achieving to their full potential.

IBL pedagogy in mathematics also requires teachers to develop a new, wider range of skills (Fraivillig et al., 1999). “That means quite a radical change in teachers’ practices.” (Dorier & Garcia, 2013, p. 838). They need to be able to facilitate student learning while allowing students a lot more control over what happens in the classroom (Aulls & Shore, 2008; McClain & Cobb, 2001). Teachers are required to draw out the meanings behind student responses rather than simply accepting their answers and evaluating them as correct or incorrect (Anghileri, 2006). Teachers must be able to think on their feet, recognising and choosing between a wide range of student responses, making decisions about which avenues to

pursue at any given moment (Ball, 2000). Makar (2007) sees the teacher as needing to have a high level of innovation and states in the shift to IBL “the learning curve for this approach is steep” (p. 50).

IBL requires higher orders of student thinking than more traditional mathematics teaching. Higher order thinking is not something students pick up for themselves; like the skills for co-operative group work, it needs to be developed. If students’ primary concern is with gaining the highest possible grades, there can be little motivation for engaging in co-operative work or higher order thinking skills (Aulls & Shore, 2008). “Some research studies clearly demonstrate that dialogue helps mediate students’ higher-order thinking” (Aulls & Shore, 2008, p. 20).

A student-centred pedagogy that encourages a diverse range of solution strategies makes the ongoing assessment and recording of learning required for reporting to parents and school boards more challenging. Teachers and schools need to identify ways to assess problem-solving skills and the content that is being learnt through IBL. “Teachers require new and adapted tools to assess changes in inquiry strategies and the content that is learned through participation in inquiry instruction” (Aulls & Shore, 2008, p. 22). This need for new and innovative ways of assessment can provide a further barrier for teachers already having to learn new innovative ways of teaching.

Many current primary and secondary teachers have no experience of IBL either as a teacher, or previously as a student (Dorier & Garcia, 2013). Therefore, many have difficulty understanding what inquiry-based teaching and learning actually looks like in practice (Fosnot, 1996). Lack of first-hand experience and a heavy teacher dependence on text books (Aulls & Shore, 2008) can make implementing an IBL pedagogy a significant challenge.

Issues concerning teacher content knowledge may also potentially present a challenge in the introduction of IBL. There has been much discussion about the impact of low teacher content knowledge in mathematics, as noted by Makar (2007): “Low content knowledge has also been repeatedly named as a barrier to improved practice” (p. 51). Makar does note that some researchers (e.g., Kennedy, 2005) have questioned this problem, claiming the “category of ‘content knowledge’ is overly broad” (Makar, 2007, p. 51). While some questions exist, Makar (2007) notes there is research evidence to support the claim of low teacher content knowledge in primary school teachers, which could be “a barrier to learning” (Makar, 2007, p. 51).

Developing new IBL teaching skills for an experienced classroom teacher can also be a frustrating process. The skill development can be made even more frustrating and challenging because not only is the teacher learning how to teach in this fashion but students are also learning to learn in a new way. This adjustment can be just as challenging for students who also have expectations for how teaching and learning happen based on a transmission approach (Makar, 2007). For the teacher a further source of frustration can be that “participation in such reform degrades competence and confidence, at least initially” (Sykes, 1996, p. 464).

These challenges raise the question as to whether it is worth all this effort to introduce IBL into mathematics teaching and learning. This question will be addressed below (section 2.7) where we consider what research has to say about the effectiveness of introducing IBL in real classrooms, to real students, with real learning needs. However, before doing so we will consider the issue of student confidence in mathematics because student confidence has a significant impact on their achievement (Mullis et al., 2012).

The participating teachers at All Saints School encountered all of the challenges to implementation discussed above. The high levels of commitment meant participating teachers responded to issues of low teacher content knowledge and other challenges with a high work rate. Ensuring full student participation required on-going monitoring and input as did the need to push students to higher order thinking. Initial challenges with assessment were mostly overcome through the use of the Assessment Resource Bank (Assessment Resource Bank, n.d.). The high levels of commitment from both teachers and students meant that considerable progress was made in overcoming these challenges during the intervention period. However, as teachers and students continue to work with IBL it is expected that both their skills and the resulting student outcomes will continue to rise.

2.6 Student Confidence

Student confidence has a significant impact on their achievement (Mullis et al., 2012). Successive TIMSS assessments have shown strong positive relationships between student confidence and their achievement in mathematics: “The relationship is bidirectional, with confidence and achievement mutually influencing each other” (Mullis et al., 2012, p. 19). Internationally, just one-third of fourth grade students expressed confidence in their mathematics ability but their mathematics achievement

was higher than for the ‘Somewhat Confident students’. The students lacking confidence (21%) had the lowest achievement (Mullis et al., 2012). Students’ confidence affects their beliefs about their academic ability, their academic aspirations, the level of motivation, the effort they expend, and their overall academic accomplishments (Bandura, 1997).

Student confidence impacts on their interest in, and attitude to a subject. Those with high mathematics confidence will have a greater level of interest in, and a more positive attitude to mathematics, while those with less mathematics confidence are more likely to have lower levels of interest and a more negative attitude to mathematics (Bandura, 1997).

Student confidence can be tied closely to how easily they undertake and solve problems. The easier the task, the greater the students’ confidence. However, there is an emerging trend towards valuing challenge and struggle in student learning. Dweck (2006) identifies two alternative mindsets: either a fixed mindset where people believe intelligence is something we are born with and therefore, people are either smart or they are not; or a growth mindset in which people believe that intelligence is an open quality that can be increased with hard work. Boaler (2016) highlights the growth mindset as an important aspect of student learning in mathematics: “Students with a growth mindset take on hard work, and view mistakes as a challenge and motivation to do more” (p. 7). Boaler further asserts that we need to be giving students work that will “prompt disequilibrium” (p. 17). Acceptance of these views requires a shift in thinking about challenge, struggle and mistakes for both teachers and students. Positive messages are required from teachers if students are to make this transition (Boaler, 2016). Therefore, this research will consider the students’ mathematics confidence generally, and their problem solving confidence specifically.

2.7 The Impact on Student Learning Outcomes of Implementing IBL

In order to review the impact of IBL on student achievement a systematic search of the research literature was completed. The list of studies used in this section (Appendix A, first column) was derived from a search of the available literature on inquiry-based learning using the following data bases: A+ Education, Eric, Proquest, and JStor. These data bases were searched using either ‘All Dates’ or with the dates

unspecified, depending on the way the data base functioned. Using the word 'mathematics' as a consistent starting place, these data bases were searched using the following terms: inquiry-based learning, IBL, realistic mathematics education, problem-based learning, discovery learning and cognitively guided instruction. Each of these terms was used in conjunction with each of the following terms for student achievement: student achievement, student outcomes, benefits, goals, and results. These searches produced a large number of journal articles, books and book chapters. As a further resource the reference lists of a range of articles on IBL were searched in pursuit of further information on the impact of IBL on student learning. From all this information only those sources that included the use of quantitative data to substantiate the results were included in the list. This process culled the large range of books, chapters, and articles down to just the 19 included in appendix A.

These articles include a range of student ages and numbers, and a number of different strategies for reporting the outcomes of the interventions, all of which means it is not possible to combine the information from the various studies and give a single overall result on student achievement when IBL is introduced into the classroom. However, an examination of each of the interventions does allow some trends to be seen and conclusions to be drawn.

The collected data show the results of: 11 studies on primary aged students with a student sample size of 16,458, and a further 642 primary aged students acting as representatives of larger groups, giving a total sample size of 17,100; four studies representing 23,866 secondary aged students; one crossover study giving the results from a mixed group of primary and secondary students with a sample size of 2128; and one study with a student sample size of 3212 showing the impact of IBL on tertiary students. There is also one study in which the age of the students is unspecified, and in which 12 students are reported on as representatives of a larger but very mobile student group. Overall this gives a total sample size of 46,318 students (Appendix A).

The outcomes of these studies show a significantly positive impact on student achievement as a result of the introduction of IBL. Eight studies (Boaler, 1998, 2008; Erbas & Yenmez, 2011; Higgins, 1997; Kogan & Laursen, 2014; Mistretta, 2005; Newmann, Marks, & Gamoran, 1996; Shayer & Adhami, 2010) report an overall statistically significant difference in student achievement as a result of the introduction of IBL into a traditional mathematics programme. Boaler, in her 2008

study, also noted IBL assisting in closing the achievement gap between different ethnic groups: “Railside teachers [the IBL teachers] were extremely successful at reducing the achievement differences between groups of students belonging to different ethnic groups” (p. 177).

Five studies (Cotic & Zuljan, 2009; Manswell Butty, 2001; Rakes, Valentine, McGatha, & Ronau, 2010; Schorr, 2000; Taylor & Bilbrey, 2012) show a mixture of positive results, and neutral results for IBL for various sub-groups. One of those studies (Taylor & Bilbrey, 2012) showed that the introduction of IBL, as well as raising achievement, compressed the achievement gap between the lowest and highest performing students. Three studies (Fennema et al., 1996; Hiebert & Wearne, 1993; Villasenor & Kepner, 1993) showed overall gains in standard deviations for the IBL-taught students. One study (Morgan, Farkas, & Maczuga, 2014) showed a neutral result for most students, but a positive result for teacher directed learning over IBL for students with mathematical difficulties. One study (Olander & Robertson, 1973) showed the IBL students gaining a statistically significant benefit for retention and application of mathematical knowledge, but the expository taught students gaining a statistically significant benefit on computation. Overall, this study showed benefits for IBL students with those students starting off higher than the expository-taught students and continuing to improve at a greater rate. One study (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) showed a neutral result overall on the ITBS (Iowa Test of Basic Skills) computation test; however, those classes that were lower overall on the pre-test showed greater gains for classes taught with IBL, but for classes that were judged “very high” on the pre-test the control group classes outperformed the experimental group classes.

These results indicate there is a very good chance that the introduction of IBL will result in positive gains for students. Therefore, the introduction of IBL was used in this study with the intention of improving students’ attitude, confidence and achievement. The IBL approach is further supported by this by John Hattie (2009) in his syntheses of meta analyses:

Overall, inquiry-based instruction was shown to produce transferable critical thinking skills as well as significant benefits, improved achievement, and improved attitude towards the subject. (p. 21)

2.8 Summary

Socioculturalism is a theory of learning which views learning as being socially constructed within the combined activity of the group rather than in individual minds as early constructivism proposed. This social construction was further refined by Lave and Wenger (1996) in their concept of legitimate peripheral participation which frames the teacher and students as together forming a socially mediated learning environment in which all are participants in actively creating the learning, defined as participation in the community of practice. One of the approaches to teaching and learning that fits well with this sociocultural theory of learning is IBL where students work together to solve real world problems and share their solutions with the rest of the class for discussion and debate. The research literature contains a range of examples of how IBL has impacted positively on student attitude and achievement. Therefore, IBL was used in this study with the intention of improving attitudes, confidence, and achievement, particularly for lower achievers. This thesis examines the introduction of an IBL approach to teaching and learning of the mathematics programme of three teachers and their classes in a high SES school and the impact this had on student attitude, confidence, and achievement.

Chapter Three

Research Design and Methodology

3.1 Introduction

This chapter describes the design and methods used in this study along with the setting and professional development process. Section 3.2 outlines the setting, sample and schedule for the research. Research design and justification for the selection of a mixed-methods approach is provided in section 3.3; section 3.4 details the role of the researcher; the nature and structure of the intervention are outlined in section 3.5; section 3.6 details data collection methods, including Progressive Achievement Tests (PAT), Overall Teacher Judgements (OTJs), student surveys and student interviews; section 3.7 considers aspects of data analysis; validity and reliability are covered in section 3.8 and ; section 3.9 addresses potential ethical considerations, before a summary in section 3.10.

3.2 The Research Study: Setting Sample and Schedule

This section provides the setting of the study, participant details, and the schedule for data collection and the teaching experiment. All Saints School⁵ was selected as the site for this study as the researcher was employed as a teacher at this school at the time of the research. The participant teachers were approached for involvement because of their interest in mathematics teaching and learning and their availability; they did not have other curriculum responsibilities at the time of the research. The principal of the school was included in the PLC because of her interest in the research and desire to be involved.

Table 3.1 The Research Setting and Teacher Participants

Name	Details
All Saints School	Suburban Catholic full primary school (Year 1-8) with a role of approximately 320 students in a high SES setting.
Teacher-researcher	Primary school teacher with: <ul style="list-style-type: none"> • 12 years' experience and • School Mathematics Lead Teacher Post Graduate Diploma in Education and Professional Development.
School Principal (Mrs Taylor)	School Principal with: <ul style="list-style-type: none"> • 12 years' experience as a principal and • 16 years' experience as a teacher Master of Education.
Teacher 2 (Mrs Preston)	Primary school teacher and syndicate leader with: <ul style="list-style-type: none"> • 16 years' experience Bachelor of Arts and Graduate Diploma of Teaching.
Teacher 3 (Mrs Vance)	Primary school teacher: <ul style="list-style-type: none"> • in her second year of teaching (second career) Bachelor of Arts and Graduate Diploma of Teaching.

The students who were invited to participate in this research were selected because they formed the classes of the teacher volunteers. The Year 3 students were in a combined Year 2/3 class, but only the Year 3 students were invited to participate as Year 2 students do not undertake the PAT used as a measure of achievement.

Table 3.2 *Student Participants*

Class	Details	Ethnic mix of Participating Students
Year 3	<ul style="list-style-type: none"> • A class of 25 students • 17 Year 3 students and • 8 Year 2 students Of the 17 Year 3 students 10 participated in the study.	4 New Zealand European/Pākehā 4 Filipino 1 Cambodian 1 Māori
Year 4	<ul style="list-style-type: none"> • A class of 29 students of whom 19 participated in the study • Broad range of historical achievement in mathematics 2 participants left during the year.	8 New Zealand European /Pākehā 3 Māori 2 Pasifika 3 Filipino 1 Chinese 1 Indian 1 Middle Eastern
Year 6	<ul style="list-style-type: none"> • A class of 30 students of whom 22 participated in the study Broad range of historical achievement in mathematics	10 New Zealand European/Pākehā 5 Filipino 5 Indian 1 Middle Eastern/Latin American 1 Chinese

The schedule for the study was chosen with two benefits in mind: firstly, using a full school year allowed the students to have the maximum time to experience IBL; secondly, the spacing of the before and after testing over a full calendar year allowed for the use of the school's existing assessment timetable, eliminating the need for additional testing.

Table 3.3 *Research Schedule*

Date/Timing	Item
January 2015	First meeting of PLC, which then met regularly throughout the year.
Mid February 2015	Information and consent forms sent home to all parents.
26/2/15	Information evening for parents.
First week in March 2015	Final parent consent forms returned; student information and consent forms given out, discussed and completed.
March 2015	Teachers began using IBL with classes.
March 2015	PAT, student survey and interviews time 1.
November 2015	Student survey and interviews time 2.
December 2015	2015 student OTJs.
March 2016	PAT time 2 conducted.

3.3 Research Design and Justification

This study was conducted using a pragmatic theoretical lens adopting the view that gathering data from multiple sources, using both qualitative and quantitative approaches, would allow for a richer, fuller understanding of the impact that introducing IBL had on students (Creswell, 2009). The research questions determined the use of a mixed methods approach (Tashakkori & Teddlie, 2003), thereby allowing the collection of qualitative data to determine the impact on students' attitudes and confidence and quantitative data to determine the impact on student achievement (Creswell, 2009).

The aim of this study was to investigate the impact of IBL on student confidence, attitude, and achievement within mathematics, in a New Zealand primary school. A triangulated mixed-methods approach (Tashakkori & Teddlie, 2003) was adopted as the study aimed to describe and quantify the impacts of the introduction of IBL on student outcomes. A qualitative approach best suited gaining an understanding of student confidence; shifts in achievement results were best determined through quantitative methods; and a mix of both was used to examine IBL's impact on student attitudes. An advantage of qualitative data is the depth and richness of student thinking that can be obtained, giving the research deeper insight into the thoughts and actions of the students. The disadvantage of qualitative data is that it is subjective and open to interpretation which can lead to the possibility that

different conclusions could be drawn from the same data by different researchers (Cohen, Manion, & Morrison, 2007), a disadvantage overcome by also gathering quantitative data. Further, a mixed approach helps to strengthen a study and overcome the weaknesses in each individual approach (Bonne, 2012). The simultaneous collection of qualitative and quantitative data gives this study a parallel design, the merging of the various data sources in the findings making it a convergent parallel design (Creswell, 2014).

A key advantage of quantitative data is that it gives information that can be statistically analysed, and it involves the collection of data which other teachers and researchers are able to see and analyse for themselves. It has the disadvantage of only giving a number or word as a result which gives little insight into how the students are thinking and why they are doing and saying the things they are (Cohen et al., 2005), a disadvantage overcome by also gathering qualitative data.

A case study was conducted as “A case study examines a *bounded system*” (McMillan & Schumacher, 2010, p. 24), in this case three classes within the same school, through the collection of multiple data sources (Stake, 2005; Teddlie & Tashakkori, 2006). Case study (Stake, 2005) allows the collection of both qualitative and quantitative data: “case study does not claim any particular methods for data collection or data analysis” (Merriam, 2001, p. 28). Rather, a case study can “obtain information from a wide variety of sources” (Merriam, 2001, p. 31). A case study is the product of an intensive study of a phenomenon in a specific situation, and is particularly useful in education as it “can affect and perhaps even improve practice” (Merriam, 2001, p. 41).

Quantitative data were collected in a quasi-experimental fashion as the school having insufficient student numbers to allow for both experimental and control groups. However, the comparison of student achievement data against national average achievement data for the intervention period and against previous achievement data with the same data from the intervention period allowed for a quasi-experimental approach. A quasi-experimental method examines differences between pre-existing groups, without random assignment, and is “often used to address casual questions” (Gravetter & Wallnau, 1985, p. 13).

A teaching experiment involves the researcher gaining first-hand experience of “students’ mathematical learning and reasoning” (Steffe & Thompson, 2005, p. 267). It enables the researcher to create and maintain an innovative teaching

initiative and monitor the results throughout the intervention (Steffe & Thompson, 2005). A teaching experiment was selected as the research design for the qualitative aspects of the study for four reasons. First, this approach most accurately reflected the dual roles of teacher and researcher undertaken by myself (Kelly & Lesh, 2000). Secondly, the study explored the impact of introducing IBL within the environment of a working classroom, a key component of a teacher experiment (Steffe & Thompson, 2000). Thirdly, a teaching experiment fits with the broad theoretical perspective of socioculturalism which underpins this study (2.4). Socioculturalism's emphasis on students' construction of learning in a social environment fits with a teaching experiment where the focus is on enabling researchers, through conceptual analysis, to understand the learning which students construct (Steffe & Thompson, 2005). Fourthly, teaching experiments allow teacher-researchers to gain "understanding [of] the progress students make over extended periods" (Steffe & Thompson, 2000, p. 274), in this case one full academic year.

3.4 The Role of the Researcher

The study design meant my role as researcher was multifaceted. Along with the common tasks of the researcher, in this study I was also a teacher participant introducing my own class to IBL in mathematics, and a lead teacher responsible for introducing IBL into the mathematics pedagogy of the participant teachers. As well as sourcing and creating the quantitative data instruments, gathering the qualitative data meant being the instrument for gathering and analysis (Merriam, 2001).

There are advantages to "Working on the Inside" (Ball, 2000, title). First, as a colleague of the teacher volunteers I was readily accessible to informally discuss the students and how they were responding to the IBL lessons and to visit classes and observe students at work, which also improves reliability of data through further direct observations. These informal conversations gave insight into what was happening for the participant students which an external researcher may not have gained. This perspective gives a further insight into the research findings by entering "a teacher's voice and perspective into the discourse of scholarship" (Ball, 2000, p. 375).

Being an insider with already established relationships with the staff meant time spent building such relationships was instead able to be focused on students and

their response to IBL. This meant more available time for fine tuning classroom practice to best meet the needs of these specific students.

An identified limitation to being an insider is failing to see important aspects due to being too familiar with the situation, staff and students. This was mitigated by having the school principal as a member of the PLC, giving added authority, and having lecturers from Victoria University of Wellington oversee the research project. Having the researcher's supervisors separated from the school and the PLC was an additional way of mitigating potential familiarity blindness.

3.5 The Intervention

Professional Learning Communities (PLC)

The intervention was based on a professional learning community where the principal and teacher participants worked together to assist each other with the introduction of IBL into each classroom. Literature contains many references to the benefits of teachers working collaboratively to improve pedagogy and raise student achievement, which was the driving focus of the intervention (e.g., Cobb & Jackson, 2011; DuFour, 2004; DuFour, Eaker, & DuFour, 2005; Franke, Carpenter, Levi, & Fennema, 2001; Makar, 2007). PLCs focus on learning for *all* students and how this can be influenced by teachers working collaboratively; the culture of PLCs includes the expectation that through working together teachers can enable students to make achievement gains (Schmoker, 2005a); PLCs include an open and honest focus on teacher's pedagogy and how this can be improved to aid in raising student achievement; teachers hold themselves (not students) accountable for teaching and learning results; members commit to a shared problem solving approach focused on overcoming barriers to higher student achievement. The motivation for using a PLC in this research is perhaps best summed up by Mike Schmoker (2005b): "The use of PLCs is the best, least expensive, most professionally rewarding way to improve schools" (p. 137).

Teacher Professional Learning and Development Programme

An important precursor to this research was the PLD undertaken with the teacher volunteers. The PLD design took into account a range of theoretical factors. It is important to note that I was very aware of the pressures and demands of the everyday workloads experienced by busy classroom teachers. Therefore, the PLD programme was designed around a balance of four competing demands: firstly, the need for an

understanding of the theory and research behind IBL and its impact on student achievement; secondly, guidelines research literature has for its practical implementation; thirdly, challenges involved in introducing a significant new pedagogical practice to both the teacher and the students: and fourthly, the need not to overload already busy teacher volunteers. The professional development had six main features, derived from the research literature:

- 1) The need for an extended time period if it was to produce lasting change (Cobb & Jackson, 2011; Higgins & Parsons, 2011; Makar, 2007; Timperly, 2008).
- 2) Participation in a professional learning community (DuFour et al., 2005).
- 3) A clear focus on student learning outcomes (Makar, 2007; Timperly, 2008).
- 4) An introduction to the theory of IBL, including both theoretically based and practically based readings (Higgins & Parsons, 2011; Timperly, 2008).
- 5) Classroom observations both of and by the participating teachers (Grant et al., 1998).
- 6) Regular opportunities for reflection (Makar, 2007; Sykes, 1996).

Ball (2000), in discussing first person research, observed: “The design work does not proceed linearly; instead, the design is iteratively adjusted in the course of the research” (p. 387). The PLD was designed for the possible dissemination of IBL across the wider school in the following year. A final note at the outset of the PLD programme comes from Heaton, cited in Ball (2000) who argued that: “learning to change one’s teaching is not helpfully constructed as complete abandonment of past practices.” Instead it requires “a skillful merge of old and new practices” (p. 392). The thinking behind this study was that IBL would complement teachers’ existing pedagogical skills, not replace them.

IBL

IBL was new to both the teachers and the students; therefore, a staged introduction to this new way of working was undertaken. Despite the range of benefits that IBL offers, Aulls and Shore (2008) warn that IBL cannot totally dominate a student’s learning experiences. Therefore, it was never intended that IBL

completely replace the existing programme; rather, a blend of both IBL and the existing mathematics programme was undertaken.

Following an introduction to the theory and an opportunity to observe the researcher teaching using IBL, teachers planned and undertook the teaching of two IBL lessons during Term 1. The teaching of the second IBL lesson was planned so that the participating teachers and principal had the opportunity to both teach an IBL lesson and to observe another IBL lesson being taught. Term 2 saw the participating teachers undertaking an IBL lesson once a week and the principal teaching using IBL once in each of the three classes. Observations between classes for both teachers and the principal were undertaken twice during the term. During term three the weekly IBL lessons continued and a week-long inquiry unit was undertaken by each of the teachers. This pattern of a week-long IBL unit and weekly IBL sessions continued during term four.

IBL lessons focused on non-routine problems as defined in 2.3 above, an example of which is “Three Dice” in Appendix C. An example of a week-long IBL unit is “An Olympic Event” also included in Appendix C. The IBL sessions were planned to ensure that they covered the full range of mathematics strands, for example the measurement in “An Olympic Event” alongside the number strand work.

3.6 Data Collection

As would be expected in both a mixed methods and teacher experiment study, data collection used multiple instruments. Data were collected in line with the dates listed above (3.2).

3.6.1 Qualitative Data

Student Surveys

Surveys have the advantage of allowing a glimpse into the thinking of those who undertake them. They provide a view into the respondent’s world and have the advantage of being able to capture perspectives of more respondents than allowed by interviews (Cohen et al., 2007). The student surveys contained researcher-devised open questions based on those used by Tait-McCutcheon (2008) and Tanner and Jones (2003). “In qualitative research you ask open-ended questions so that the participants can best voice their experiences unconstrained by any perspective of the

researcher or past research findings” (Creswell, 2014, p. 240). The survey also contained Likert type scale questions as they “build in a degree of sensitivity and differentiation of response while still generating numbers” (Cohen et al., 2007, p. 325). This allowed some qualitative data to be analysed in a quantitative fashion.

The survey was trialed on a group of non-participating Year 3 students and subsequently refined before being used with participant students. The main refinement was to question nine, which initially asked students to circle any words they thought applied to mathematics out of a set of 15 (Year 3) and 19 (Years 4 & 6); this question was changed as a result of the trial to placing a cross on a continuum, marked 1 to 4 between paired sets of words, with six pairs for the Year 3 survey and nine pairs for the Year 4 and 6 survey (Appendix D). This refinement added “Semantic Differential Scale” questions to the survey (Cohen et al., 2007). The reason for the change was to gain greater clarity of student thought as a number of students in the trial group circled opposing words, such as easy and hard. “Rating scales are widely used in research ... for they combine the opportunity for a flexible response with the ability to determine frequencies, correlations and other forms of quantitative analysis” (Cohen et al., 2007). As a result of data received from time 1 surveys, two questions (12 & 13, which appear in italics in the survey in Appendix D) were added to time 2 surveys.

The responses to open questions are subjective, and open the possibility of respondents simply telling the surveyor what they think he or she wants to hear, or alternatively being contrary for the sake of it. The use of more than one question and the consideration of the entire data set obtained in this fashion helped to mitigate the potential for possible problems caused by these issues (Cohen et al., 2007).

Student Interviews

Interviews were used to allow further exploration of student beliefs and motivation and to seek reasons or explanations for student responses (Silverman, 2006). Interviews were semi-structured allowing for reordering and further examination of responses as appropriate (Cohen et al., 2007). Interviews included a problem-solving task with responses recorded on an interview instrument (Appendix E), and an audio recorder enabling full transcription for coding and analysis. The problem they were shown was chosen for being suitable to their mathematics level; therefore, three problems were used (Appendix G). The problems were not completely open-ended as I was concerned

that a totally open-ended style of problem would prove too confusing for the interviewed students at the beginning of the intervention. The problems were sourced from the problem-solving section of the NZMaths website. The students were not asked to solve the problem, rather they were asked if they thought they would be able to. The focus was on their problem-solving confidence, not their problem-solving ability. They were shown the same problem at time 1 and time 2 and these problems were not used in any classwork between the time 1 and 2 interviews.

Students were interviewed in pairs of matching ability. The reasons for the use of ability pairs were: it enabled students to spark ideas off each other; having like ability prevented the possibility of a less able student simply deferring to a more able student; having students interviewed in pairs provided an additional personal safety measure for both students and interviewer.

The selection of students to be interviewed used the following process: students were separated into three achievement bands (stanines 1-4, 5 & 6, 7-9) based on the results of the PAT test they had just undertaken; students were further separated by gender; from each gender specific, achievement band group a single student was randomly selected by having their name drawn out of a container. This gave a sample size of six students per class, three each male and female. The make-up of the students interviewed was as follows:

Table 3.4 *Student Interview Sample*

Year group	Achievement band	Ethnicity
Year 6	High <ul style="list-style-type: none"> • Female • Male Mid <ul style="list-style-type: none"> • Female • Male Low <ul style="list-style-type: none"> • Female Male	<ul style="list-style-type: none"> • Chinese • Pākehā • Pākehā • Indian • Middle Eastern Filipino
Year 4	High <ul style="list-style-type: none"> • Female • Male Mid <ul style="list-style-type: none"> • Female • Male Low <ul style="list-style-type: none"> • Female Male	<ul style="list-style-type: none"> • Samoan • Indian • Pākehā • Pākehā • Asian Filipino
Year 3	High <ul style="list-style-type: none"> • Female • Male Mid <ul style="list-style-type: none"> • Female • Male Low <ul style="list-style-type: none"> • Female Male	<ul style="list-style-type: none"> • Pākehā • Māori • Pākehā • Filipino • Cambodian Filipino

The selection process makes this group completely representative in terms of achievement levels and gender, and reasonably representative of the ethnic make-up of All Saints School.

3.6.2 Quantitative Measures

Qualitative measures were used to gain understanding of the students' attitudes and confidence as these measures best suited the gathering of a richer deeper data set. Changes in achievement were most accurately measured using data that could be statistically analysed and therefore quantitative data were chosen for this.

Progressive Achievement Test [PAT]

The New Zealand Council for Educational Research (NZCER) created PAT which are used for testing students' abilities in relation to the educational achievement of all

students across the country in mathematics and other subjects. Once students complete the test they are given a raw score. This raw score is converted into a scale score on the Mathematics Achievement Scale which covers students from Year 3 to Year 10. Scale scores allow a student to be compared to nationally representative groups on whom the test had been trialled. Scale scores are also converted into stanine scores which run from 1 lowest, to 9 highest, with each stanine covering a range of scale scores (NZCER, n.d.). Scale scores were compared for each participating student in Years 4 and 6 from 2014, 2015 and 2016, and from 2015 and 2016 for Year 3 students (as they do not complete PAT before Year 3), during data analysis. Scale score progress was also compared with the national average progress for each year group (Appendix B).

There are several advantages to the use of PAT for this research: the tests were already created and used nationally giving a recognisable and reliable measure of achievement; PATs are created and trialled for each specific age group, therefore, it was possible to guarantee the tests as being age group appropriate; PAT formed part of the existing assessment regime at All Saints School preventing the need for further external testing. There are, however, some potential limitations to use of this type of testing: this is a one-off assessment and therefore may not give a full account of a student's overall abilities; All Saints School has a relatively high percentage of English Language Learners and the nature of PAT, with their high reading content, may hinder some students for whom English is not their first language; PAT are multi-choice tests and do not specifically involve problem-solving questions. As far as possible these potential disadvantages were allowed for in the study design by the inclusion of OTJ as a second measure of student achievement.

Overall Teacher Judgement (OTJ)

OTJs are an assessment measure based on a student's entire year's work, made against the National Standards in Mathematics Years 1-8 (MOE, 2009). The teacher considers a range of evidence from each student, both formative and summative, and compares that with the National Standard. Selections of OTJs from every class are moderated by a group of teachers within the school. OTJs from 2014 were compared with those from 2015 for all participating students. There are a number of advantages to the use of OTJ for longitudinal research such as this: it allows for the inclusion of a student's entire year's mathematics work rather than just a snapshot; it enables a judgement about a student's assessment to be made on the basis of their mathematics work with less

potential for hindrance from any reading or language issues; it bases the assessment with the teacher who works with the student on a daily basis; and assessment is made against a set of nationally used criteria. As with any assessment tool teacher OTJs also have potential limitations. The main disadvantage is that the assessment is subjective as it is not based on repeatable tests but a teacher's considered professional opinion. This raises the possibility that different teachers may make different judgements on the same student's work. At All Saints School this issue is dealt with openly and honestly through the school's moderation processes. However, concerns about the reliability of OTJs persist (Ward & Thomas, 2015).

Likert Type Scale Scores

Student surveys (Appendix D) included a section which focused primarily on qualitative data but also included Likert type scale questions to provide insights into students' perceptions of attitudes toward mathematics. While Likert type questions give data that can be treated in a quantitative fashion, the focus of the student surveys was qualitative in nature examining students' perceptions of their mathematics experiences. These questions were based on the attitude questions that accompany e-asTTle mathematics tests (e-asTTle, n.d.). Likert type scale scores have the advantage of giving data that can be treated in a quantitative fashion and subjected to statistical analysis. However, care needs to be taken as responses are not more straight-forward yes, no, right, wrong type answers but more subjective personal feelings or impressions given a numerical value and therefore cannot be treated as numerical data. Students' responses were compared between the time one surveys and the time two surveys.

3.7 Data Analysis

Education is a complex undertaking and the classroom is a dynamic place with the teacher-student and student-student interactions making each classroom a unique environment of its own. Data analysis was conducted with this complexity firmly in mind and conclusions only drawn when clearly warranted by the available data. Data came from four different sources: student surveys; student interviews; teacher end of year OTJ; and norm referenced PAT. Each source was considered individually and then areas of similarity and cross-over were compared between data sources to gain a clearer picture and to enable opportunities for triangulation.

Student surveys were administered at the beginning and end of the intervention and compared to determine change between time 1 and time 2. Comparison was made at an individual and year group level. Students' responses to each Likert scale question at time 1 and time 2 were compared and changes noted, with those who changed their rating for any question by more than one place (e.g., 'Not at all' at time 1 to 'Some' at time 2 constitutes a two place move) being examined more closely. At the year group level, the collected number of each response to each question at time 1 was compared to that at time 2 using graphs. Students' responses to the multi-response questions were examined at the collected year group level and compared between time 1 and 2, again using graphs.

Student interviews were transcribed and included with the written responses to open-ended questions from student surveys to give a database which was read through several times to gain a preliminary exploratory analysis (Creswell, 2014). Data from the combined set of survey open questions were then hand coded (Creswell, 2014) to identify emerging ideas, before these were collapsed into major themes. Themes were then compared to the hand-coded interview data to determine a clear picture as to the possible impact of the introduction of IBL on student confidence and attitude in response to the research questions (1.5).

PAT scale scores were used to ascertain student achievement and learning progress using a standardised tool that enabled comparison: within a student's individual learning progress; between students; and between students and national achievement averages as outlined in the table below.

Table 3.5 *PAT Data Analysis*

Achievement Band	Comparison Group	Analysis
Year 3		
All	National Average	Individual scale score progress against national average. (Figure 4.12)
All	National Average	Mean scale score progress 2015-2016 against national average using <i>t</i> -test (section 4.5.2)
Low	Rest of cohort	Mean scale score progress 2015-2016 against rest of cohort using <i>t</i> -test (section 4.5.4)

Mid	Rest of cohort	Mean scale score progress 2015-2016 against rest of cohort using <i>t</i> -test (section 4.5.4)
High	Rest of cohort	Mean scale score progress 2015-2016 against rest of cohort using <i>t</i> -test (section 4.5.4)
Year 4		
All	National Average	Individual scale score progress 2015-2016 against national average (Figure 4.13)
All	National Average	Individual scale score progress 2014-2015 against national average and 2015-2016 against national average (Figure 4.15)
Low	Rest of cohort	Mean scale score progress 2015-2016 compared to national average using <i>t</i> -test (section 4.5.4)
Mid	Rest of cohort	Mean scale score progress 2015-2016 against national average using <i>t</i> -test (section 4.5.4)
High	Year 4 Students	Mean scale score progress 2015-2016 against national average using <i>t</i> -test (section 4.5.4)
Year 6		
All	National Average	Mean scale score progress direct 2015-2016 compared to national average (figure 4.14)
All	National Average	Individual scale score progress 2014-2015 and 2015-2016 compared to national average (Figure 4. 16)
Low	Rest of cohort	Mean scale score progress 2014-2015 compared to 2015-2016 using <i>t</i> -test Section 4.5.4)
Mid	Rest of cohort	Mean scale score progress 2014-2015 compared to 2015-2016 using <i>t</i> -test (section 4.5.4)
High	Rest of Cohort	Mean scale score progress 2014-2015 compared to 2015-2016 using <i>t</i> -test (section 4.5.4)

Quantitative data were analysed in two different ways. The first of these was a direct comparison; this was done at an individual level, and compared scale score progress for the intervention year with the previous year. This method was used with the Year 4 and 6 students who had PAT scores from 2014 to 2015 to compare their progress with 2015 to 2016. The Year 3 students did not have 2014 PAT scores so this method could not be used for them. All three year groups were compared with national averages using *t*-test and the Year 4 and 6 classes' individual 2014-2015 scale score progress was compared to their 2015-2016. Then the Year 4 and 6 classes' mean scale score progress was compared to the national average using *t*-tests. Finally, each class was divided into low (under stanine 5), mid (stanines 5 & 6) and high (stanines 7, 8, & 9) and the mean scale score for each group was compared to the rest of their cohort again using *t*-tests⁶. *T*-tests were used because "This method allows researchers to compare the means of two variables measured on the same individuals or sample elements to evaluate whether or not the means for those variables are statistically significantly different from each other" (Sage Research Methods, 2005). A statistical significance level of $p=0.05$ was set as this is the generally accepted level giving 95 percent confidence that the result is due to the intervention rather than another cause (Sage Research Methods, 2005).

Each student's OTJ from the end of 2014 was compared with their OTJ from the end of 2015. The OTJ data were then examined alongside the PAT data to give a fuller picture of each student's achievement, and the overall achievement of each year group as a cohort.

Student movement for all data, except PAT, between time 1 and time 2 was tracked through the use of 'Student Shift Data Charts' (Ward & Thomas, 2015), an example of which is available in Appendix H. These charts show the student movement between time 1 and time 2. Bold data show the percentage of students who selected the same answer at time 2. The other data in each column show the percentage of students who changed their response and what they changed it to. The bottom row shows the number of students who selected that response at time 1. This

⁶ De Winter (2013) examined the use of *t*-tests with small groups and concluded "there is no principal objection to using *t*-test with *N*s as small as two" (p. 1). This can be seen as employed by Teahen (2015) who used *t*-tests with groups of five.

allowed an overview of how stable the data were and in which ways the data had changed between time 1 and time 2.

Quantitative data from the student survey were compared with qualitative data from the surveys and student interviews to gain an overview of possible shifts in student confidence and attitude. The qualitative and quantitative data results were then examined together giving an overview of the likely overall impact on the students of the introduction of IBL. The data were then gathered under four headings: how children see mathematics; attitude to mathematics; confidence in mathematics; and achievement in mathematics.

3.8 Validity and Reliability

In a mixed-methods study validity and reliability for both quantitative and qualitative approaches must be addressed. Cohen et al. (2007) state: “It is unwise to think that threats to validity and reliability can ever be erased completely” (p. 105). Rather, by attention to design and conduct, possible threats can be mitigated. Validity refers to the correctness of the conclusions drawn based on the data, while reliability refers to the consistency, stability and repeatability of results (Johnson & Christensen, 2010). The use of a mixed-methods approach allows for the minimisation of potential weaknesses in validity by using a range of data sources allowing for data triangulation (Johnson & Christensen, 2010).

Internal validity refers to the trustworthiness of findings on the basis of the data collected (Johnson & Christensen, 2010). The use of students’ regular assessment regime for quantitative data, using a nationally normed test (PAT) and then triangulating that with moderated OTJ, allows for strong internal validity for the quantitative assessment data. Internal validity for the qualitative data was strengthened through use of multiple data sources (interviews and surveys) and triangulation of these with the Likert type quantitative data also gathered through the survey. The scope of the data collected and the use of student voice through low inference descriptors in the results section further strengthen reliability and validity of the qualitative data (Cohen et al., 2007).

The following aspects were designed to further strengthen reliability and validity for quantitative and qualitative data: the design allows for temporal ordering validity; controls for possible confounding extraneous variables through the use of three separate class groups; having three groups to overcome one group pretest-

posttest threats; use of previous data to help prevent history effects; a range of age groups used to help mitigate maturation effect; using the same instruments to help prevent an instrumentation effect; and a stable student group helps mitigate an attrition effect (Johnson & Christensen, 2010).

As much as possible reliability and validity concerns have been mitigated in the deliberate design of the study to give the most valid and reliable results possible. External validity will always be open to reliability concerns when it comes to the transferability of the results. However, Cohen et al., (2007) state: “it is possible to assess the typicality of a situation - the participants and settings - to identify possible comparison groups, and to indicate how data might translate into different settings” (p. 109), indicating that the results obtained in this research may be generalisable to similar settings, hence a detailed description of the setting was provided (3.2). A final comment comes from Bonne (2012); “Blending quantitative and qualitative methods was intended to take advantage of the inherent strengths of both methodologies, and at the same time minimise their weaknesses, thereby contributing to the validity of the findings” (p. 112).

3.9 Ethical Considerations

This study met the ethical guidelines of Victoria University of Wellington for research involving human participants (Victoria University of Wellington, n.d.). Key components of this policy include: cultural respect, informed and voluntary consent, respect for all participants, truthfulness, and confidentiality. Ethical approval was sought and granted (approval number 21444). All participants were given all relevant information and informed consent was obtained before data gathering commenced (Appendix F). Given the participants were under 15 years of age, all parents were provided with relevant information and their informed consent obtained.

As a teacher-researcher and a member of the New Zealand Educational Institute, the Institute’s code of ethics (NZEI, n.d.) was upheld at all times. This included conducting the research in an ethical way, and maintaining a focus on best education and welfare outcomes for all student participants.

Ethical considerations specific to this research include the setting which required me to be working with existing colleagues. The potential for ethical issues was mitigated in three ways: the principal was a full member of the PLC; the researcher’s supervisors, who were based off-site, were not members of the PLC, but

had full and free access to all PLC documentation including meeting agendas and minutes; and the assessment data gathered were on the students and their achievement, not on the teachers and their pedagogy.

Potential harm to students was minimised by using the regular assessment practices of the school as the basis for data gathering, having all students complete the survey, and always interviewing the students in pairs. While the teachers and students were all known to each other, meaning complete anonymity was impossible, all reporting and discussion involved the use of pseudonyms to prevent identification by anyone not involved.

3.10 Summary

A mixed-methods design was used for this study so that student achievement, attitude and confidence could be considered. A teaching experiment was undertaken as this best reflected my role as teacher-researcher. Quantitative aspects drew data from PAT and teacher OTJ, while qualitative data were drawn from a student survey (attitude) and interviews (confidence). Strict ethical guidelines were adhered to, while at all times student welfare and education were upheld. Having outlined the design and methodology, it is now time to turn to the results.

Chapter Four

Results

4.1 Introduction

In this study, data are examined under four headings: How children see Mathematics; Attitude to Mathematics; Confidence in Mathematics; and Achievement in Mathematics. Sections 4.2 - 4.4 present findings from the qualitative analysis of the survey and interview data; section 4.2 outlines the way students have defined mathematics; section 4.3 considers students' attitude to mathematics; and section 4.4 examines students' confidence in their mathematical ability. Section 4.5 presents quantitative analysis of the students' achievement data from OTJs and PAT, before a brief summary in section 4.6 concludes this chapter.

4.2 How Children see Mathematics

Gaining the best possible understanding of students' attitudes to mathematics requires we have as clear a picture as possible of what students understand mathematics to be. Therefore, consideration of the research results begins with an exploration of student perceptions of what constitutes mathematics. Data are drawn from time 1 and time 2 student interviews and multi-response and open questions within the student surveys at time 1 and time 2.

Student surveys included questions on students' views of the usefulness of mathematics to them, both outside of school and when they had finished school. The results showed a high level of awareness for the usefulness of mathematics including nearly 90 percent of Year 4 and 6 students rating mathematics usefulness outside of school at 'Some' or 'A lot' at time 2. The highly positive attitude was also seen in the responses to the question on mathematics usefulness once school has finished. Responses of between 60 and 95 percent for 'A lot' at time 1 showed small declines at time 2. However, these small declines need to be seen in the context of responses to both questions showing All Saints School participating students have a high awareness of the importance of mathematics to them outside of, and after they finish school.

Interview and survey data also strongly suggested that the students view mathematics primarily as working at activities within the number strand, as defined by the NZC: “Number involves calculating and estimating, using appropriate mental, written, or machine calculation methods in flexible ways” (MOE, 2007, p. 26).

Mathematics as number can be seen during time 1 interviews where students’ responses to the two questions “What do you like about maths?” and “What do you not like about maths?” typically included responses such as:

- S3.1: I like fractions, subtraction and counting;
- S3.5: I like adding, subtracting and working on my own;
- S4.18: I like adding, subtracting and working from textbooks;
- S6.2: I don’t like doing division and decimals.

Eleven of the 18 students referred only to number activities when talking about the mathematics they liked and five referred only to number activities when answering the question “What do you not like about maths?” Remaining comments referred to a mix of number, geometry and basic facts. This response prompted the addition of the questions “What is maths?” and “Why do we do maths?” to time 2 student interviews and student surveys, and “What do you do if you get stuck in maths?” to the time 2 interviews.

The clearest theme to emerge from the time 2 surveys was also mathematics as number. Over 70% of Year 3 and 4 students and 50% of Year 6 students mentioned number strand activities either exclusively or in conjunction with problem solving. Responses from the time 2 surveys included comments such as:

- S3.8: It is divide, add, takeaway, times;
- S4.7: Plus, times, divide-by and worksheets;
- S6.23: Maths is adding, subtracting, multiplying and dividing, it is working with numbers and getting answers, you problem solve and learn basic facts.

The focus on number was reinforced in the time 2 interviews with 12 of the 18 students either exclusively referring to number, or to number and problem solving. The remaining comments mentioned volume and geometry twice each, and problem solving exclusively three times.

Other data also supported this finding; for example, several mentions of basic facts at both time 1 and time 2, such as this from S4.2 “Practice, repeat their times tables” in suggesting how students improve, is suggestive of an understanding of mathematics as number. The emphasis on number activities specifically was clearly

seen in the student interviews and supported by the responses to the survey questions “What is maths?” and “What do people who are good at maths know, or do, which makes them good?” Taken together the findings are suggestive of a perception of mathematics as being about number.

An examination of the MOE documents *The NZC* (MOE, 2007), *The National Standards* (MOE 2009), and *The Numeracy Development Project* materials suggests a possible reason for students’ thinking of mathematics as number. All these MOE documents heavily favour time spent on number over other strands of mathematics. A 2010 MOE document (poster) gave these recommendations for time spent on number:

- Beginning School – Year 4: “During these years, Number should be the focus of 60-80 percent of mathematics teaching time.”
- Year 5-6 “During these school years, Number should be the focus of 50-70 percent of mathematics teaching time.”
- Year 7-8 “During these school years, Number should be the focus of 40-60 percent of mathematics teaching time” (MOE, 2010).

It is therefore not surprising that the students in the Year 3, 4 and 6 classes involved in this research viewed mathematics predominantly as the number strand.

Further data were sought from the student survey question about students’ favourite mathematics activities, to allow some triangulation between open questions in the interviews and surveys, and to give a fuller picture of the students’ views of mathematics. Data from this revealed maths games featuring strongly across all year groups at both time 1 and time 2. Alongside maths games, across the Year 4 and 6 data the high number of students selecting ‘working in their maths books’, ‘text book work’ and ‘maths tests’, suggests students viewed mathematics as an individual written activity rather than a shared verbalised activity, indicating they saw mathematics as having a more traditional drill and practice approach to teaching and learning. With little change between time 1 and time 2 it would seem IBL had little influence on students’ favourite mathematics activities.

Understanding the perceptions children hold about mathematics is an important frame of reference for interpreting their responses to questions about their attitudes to mathematics which are examined next. A narrow definition of mathematics as number has the potential to influence their attitude to mathematics

and their response to the introduction of IBL. In the following section it is important to bear in mind what children are thinking of when talking about mathematics.

4.3 Attitude to Mathematics

This section examines students' attitude to mathematics with data sourced from student interviews, and Likert scale questions, multi-response questions and open questions in the student surveys. The picture that emerged was of a group of students who have a positive attitude to the subject. A trend to note in the following data is that there was often much more movement on an individual student basis between time 1 and time 2 data than is seen in cohort level change. This happened in a generally common pattern with the Year 3 students changing the most, the Year 4 less so, and the Year 6 students being the most stable. Evidence of this trend can be seen in 4.3.3 below which discusses students who have moved more than one place.

4.3.1 Enjoyment of Mathematics

This section examines students' enjoyment of mathematics. Data are sourced from two Likert questions in the student surveys and the rating for enjoyment of mathematics in the interviews. The data will show a high level of enjoyment of mathematics at both time 1 and time 2.

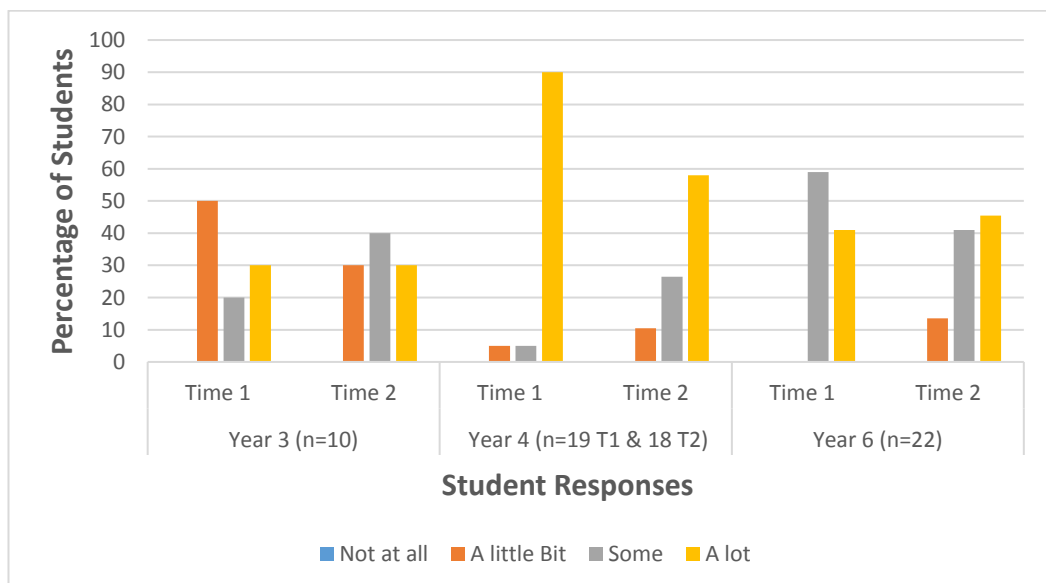


Figure 4.1 How much do you enjoy maths at school?

Figure 4.1 shows a group of students who are positive about the mathematics that they undertake at school. A feature is that the ‘Not at all’ response does not appear at either time 1 or time 2 for any of the year groups. The data give a positive picture of the students’ enjoyment of mathematics at school at both time 1 and time 2.

There has been a positive shift in the Year 3 enjoyment of mathematics at school. This positive shift is seen in the decrease in students selecting the rating of ‘A little bit’ in the time 2 data and an increase in the number of students selecting ‘Some’ between time 1 and time 2. There is a downward trend in the time 2 data for the Year 4 and 6 cohorts. The Year 4 data show a large decrease in the ‘A lot’ rating and an increase in the ‘A little bit’ and ‘Some’ ratings at time 2. The Year 6 data show a decrease in the ‘Some’ rating and an increase in the ‘A little bit’ and ‘A lot’ ratings at time 2. While this trend is important to take seriously, overall there are high levels of positive responses at both time 1 and time 2. The time 2 data reveals 70 percent, or more, of the students in each year group choosing either ‘Some’ or ‘A lot’ for their enjoyment of mathematics at school. These results suggest a possible decrease in enjoyment which, given the overall nature of the shifts may, or may not, be due to the introduction of IBL. These results will be further discussed in the section on students attitude to challenge and struggle (5.2.3).

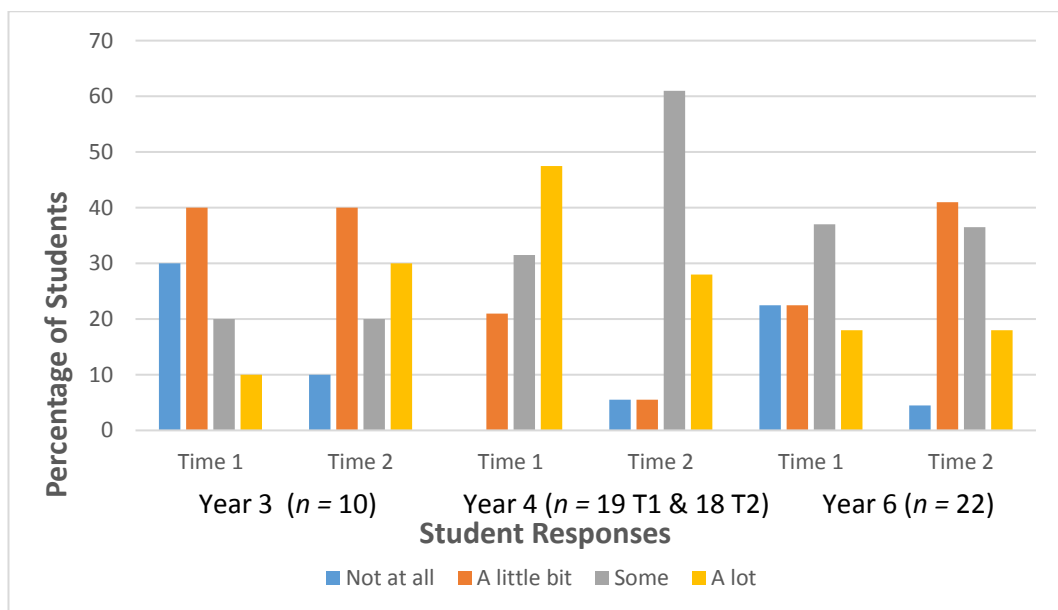


Figure 4.2 How much do you enjoy maths in your own time?

Figure 4.2 shows a wider spread of responses to this question than to the previous question from all three year groups. Each year group has the 'Not at all' rating in either time 2 or both data sets. Across the year groups the two lowest ratings of 'Not at all' and 'A little bit' are more prominent than was evident in the enjoyment of mathematics at school data. The Year 4 and 6 data are again more positive at time 1 than the Year 3 data where 70 percent of the students selected one of the two lowest ratings of 'Not at all' and 'A little bit'.

There has been a positive shift in the Year 3 students' enjoyment of mathematics in their own time. This positive shift is seen in the decrease in students selecting the rating of 'Not at all' in the time 2 data. Alongside this there is an increase in the number of students selecting 'A lot' between time 1 and time 2. This is not to suggest that students moved directly from 'Not at all' to 'A lot'; rather, there was a wider shift, as only just under half of the students moved upwards in their rating of enjoyment of mathematics in their own time (Appendix H).

The Year 4 data at time 2 reveal the appearance of the 'Not at all' rating and a decrease in the 'A little bit' rating. An increase in the rating of 'Some' between time 1 and time 2, occurred alongside a corresponding decrease for 'A lot', leaving the combined totals with a very small increase. While not especially large it does still represent a small positive shift in students' attitude to mathematics in their own time.

The Year 6 data reveal a decrease in the selection of 'Not at all' and an increase in 'A little bit' in time 2 data. Overall, the data reveal an increasingly positive attitude to doing mathematics in their own time. A possible reason for this may be the increased awareness of the mathematics in the world around them, due to the real life problem-solving activities undertaken during the year.

Overall, data from these two questions show a positive attitude to mathematics. Generally, the attitude to mathematics at school is higher than it is to mathematics in their own time. However, there is a slight decrease in enjoyment of mathematics at school and a slight increase in the enjoyment of mathematics in their own time, at time 2. Together these findings suggest a relatively neutral impact on student attitude for the introduction of IBL.

The interview data further reinforce the high level of mathematics enjoyment indicated by the survey findings, with 16 of the 18 students rating at seven or above (1 lowest to 10 highest) at time 1 and 17 rating at seven or above (with 9 at 9 or above) at time 2. Within this highly positive attitude to mathematics is a small

anomaly revealed during the time 1 interviews, that is, a quite strong negative reaction to struggle and challenge, with nine of the 18 students making comments to this effect, such as:

S3.4 I don't like to do the harder ones;

S3.3 I like doing the easier ones and less hard (ones); and

S4.3 I don't like doing the hard sheets.

While these nine students expressed a negative reaction to having to work hard and struggle with problems, they all still maintained a positive attitude to mathematics overall at both time 1 and time 2. A feature to emerge from time 2 interviews was the commitment to struggle to solve problems. This came through with five of the 18 students who, when asked what they would do if they got stuck, referred firstly to the things they could do themselves to try and solve the problem before asking for help. Other data, such as this from a time 2 survey: S4.8: "Maths is challenge, struggle and mistakes which will grow our brain", also supported this finding with a greater incidence of students acknowledging that focus, concentration and persistent effort was a part of what made students good at mathematics. While the overall numbers making such comments for this are small, at this stage, it is an important movement as this emerging theme is accompanied by a reduction at time 2 in the negative reaction to challenging tasks and struggle evident at time 1.

Along similar lines is the emergence of 'practice' as the clearest response to the question "What do people who are good at maths know, or do, which makes them good?" Fifty-eight percent of the Year 4 responses at time 1 and 2 included 'practice', while the Year 6 response of practice increased from 47 to 56 percent between time 1 and time 2. Students appear to be aware that focus and persistent effort are a part of what is required to be successful at mathematics, and that if they apply themselves they too can be successful. This positive shift in attitude, at least for Year 6, suggests a possible impact of the introduction of IBL on ideas about perseverance and practice leading to success.

4.3.2 Time spent on Mathematics

These data are included to triangulate with the data on enjoyment of mathematics assuming that generally children will choose to do more of things that they enjoy and less of things they do not enjoy. Data come from student survey

responses to the question ‘Would you like to do less, the same amount, or more maths at school?’ The data continue to show a positive attitude to mathematics.

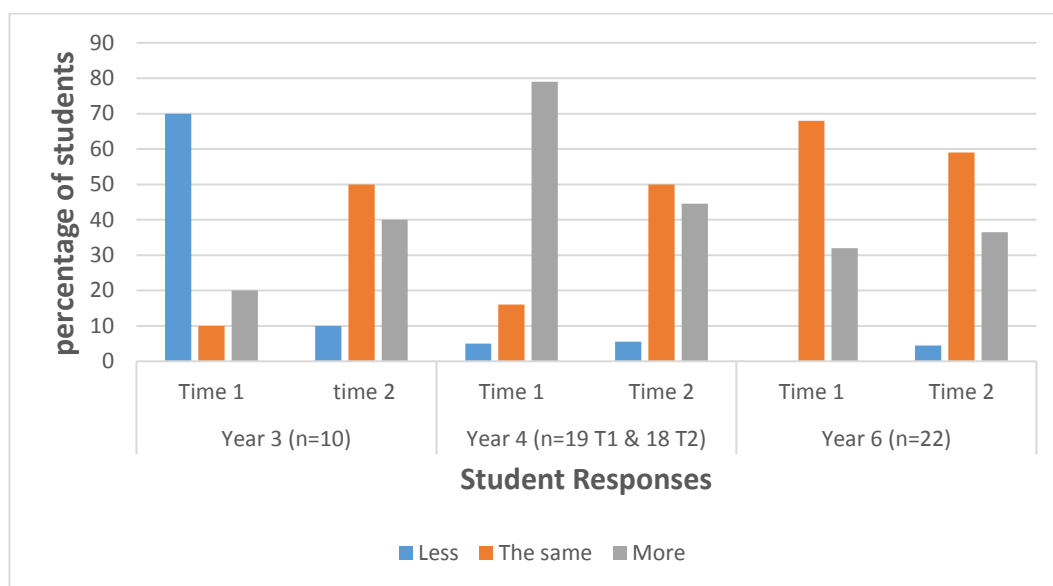


Figure 4.3 More, the same, or less time on mathematics?

Analysis shows the Year 3 class raising their desired time, while the Year 4 data showed a shift from ‘More’ to ‘The same amount’ and the Year 6 had two students move from ‘The same amount, one to ‘Less’ the other to ‘More’. These data need to be viewed in the context of their showing a very positive attitude to mathematics. The students who chose less mathematics decreased in Year 3, remained static in Year 4, and increased by one in Year 6. There were just eight students who opted for less mathematics at time 1 and at time 2 the number was down to three, suggesting a small positive move which, given the small nature of the shifts, may, or may not be the result of the introduction of IBL.

4.3.3 Students who moved more than one place with regard to Attitude

As mentioned above the small movements in overall data for each of the year groups masks a larger number of individual shifts. The following students were considered more closely in an attempt to determine whether these larger shifts indicated a pattern which might further inform the research. The chart below identifies those students who moved more than one place, for example from ‘Not at all’ to ‘Some’ would constitute a two place shift as would ‘A lot’ to ‘A little bit’.

Table 4.1 *Students who have moved by more than one place.*

Student Identification	How much do you enjoy maths at school?	How much do you enjoy maths in your own time?
S6.6		Up 2
S6.2		Down 2
S6.1		Up 2
S4.18	Down 2	
S4.15		Down 2
S4.13	Up 2	
S4.1	Down 2	
S3.10		Down 2
S3.9		Up 2
S3.8		Up 2
S3.7		Up 2
S3.5	Up 2	

There are 12 students who have made a move of two places or more, five Year 3, four Year 4, and three Year 6, with 8 girls and 4 boys. All 12 of these students appear just once. Movements of two places or more occurred in both of the Likert type questions discussed in this section, and included both increases and decreases. All are shifts of two places. The responses of these 12 students were examined to determine whether they may indicate emerging trends or patterns that would further inform the research questions. However, with all students only appearing once and the 12 students representing five decreases and seven increases, no pattern or trend emerged. This mixed result for the larger shifts in student attitude again leaves a relatively neutral implication for the introduction of IBL. With little pattern to the shifts it would appear that the changes were relatively random and so little can be attributed to them in terms of their impact on the intervention results.

4.3.4 Summary

The participating students at All Saints School have a very positive attitude to mathematics. An interesting feature to emerge during the year was an increased awareness of the value of struggle, effort, and mistakes to their mathematics learning. While the overall movements between the time 1 and 2 data in this section were relatively minor, they do mask a much larger individual movement within each year group. The Year 3 were the least stable of the three groups with the Year 6

being the most stable. This is a trend that will continue to emerge from the remaining data. Students who made larger shifts in response to any of the questions were considered more closely to determine if these shifts indicated a possible trend or pattern. However, no pattern or trend emerged. When taken together the results from this section show only small shifts in students' attitude to mathematics leaving a generally neutral outcome for the first research question. Section 4.4 now considers students' confidence in their mathematical ability.

4.4 Confidence in Mathematics

This section examines students' confidence in their ability in mathematics generally (4.4.1) and in problem-solving specifically (4.4.2). Data for confidence in general were sourced from student surveys and for problem-solving confidence from student interviews revealing a large increase in problem solving confidence at time 2 and a high but similar level of confidence in mathematics generally at both time 1 and time 2. Again, there is a larger movement on an individual student basis than appears in the cohort level data.

4.4.1 Confidence in Mathematics Generally

Data for this analysis came from two Likert type questions in the student survey: "How good do you think you are at maths?"; and "How good do you think your teacher thinks you are at maths?" The survey data show a high level of confidence in their ability in mathematics at both time 1 and time 2.

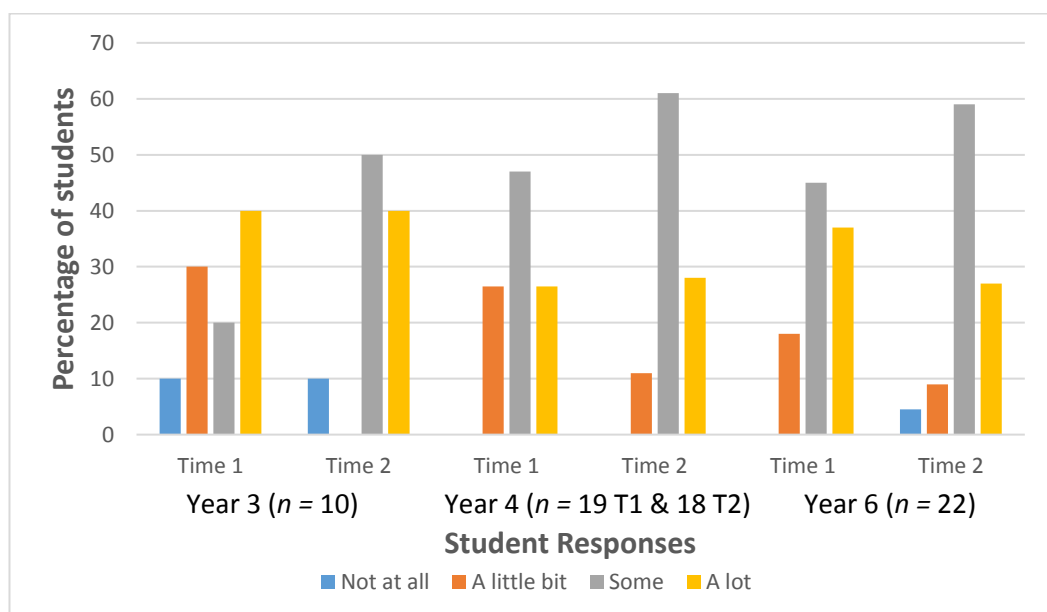


Figure 4.4 How good do you think you are at maths?

Figure 4.4 reveals a cohort of students most of whom are positive about their ability in mathematics. The trend is again towards a small increase in the selection of ‘Some’ and a small decrease in the selection of ‘A lot’ by the Year 4 and 6 students within an overall positive data set.

There has been a positive shift in the Year 3 students’ data. This can be seen in the fact they moved completely away from selecting ‘A little bit’ and have increasingly selected ‘Some’, representing a positive shift.

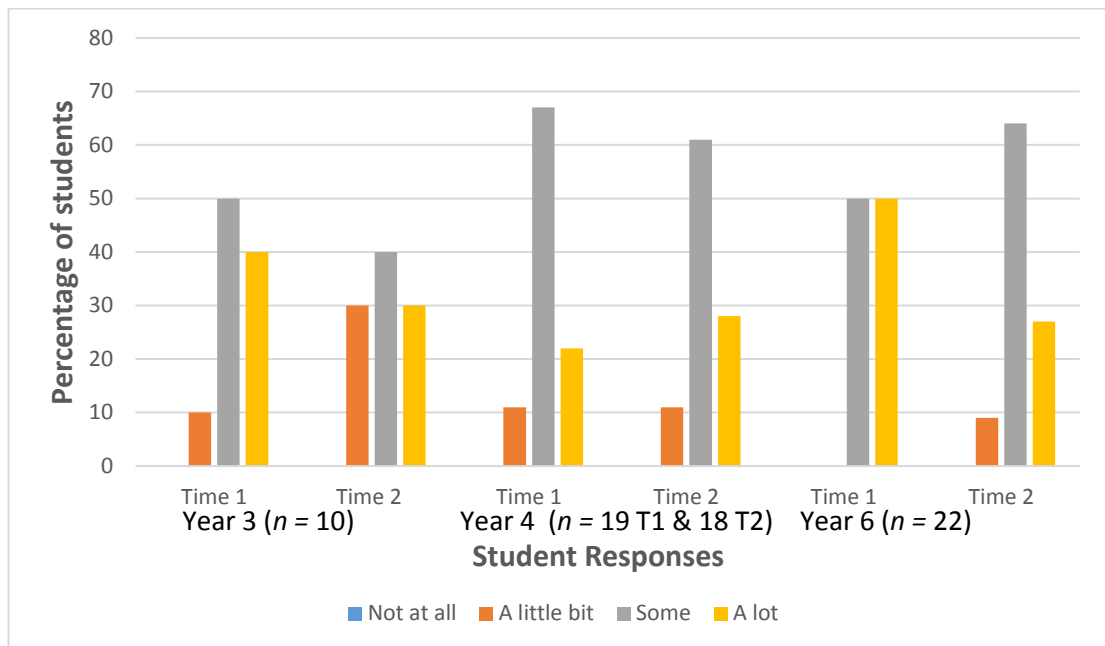


Figure 4.5 How good do you think your teacher thinks you are at maths?

Figure 4.5 reveals a data set in which the lowest rating of ‘Not at all’ does not appear for any year group at either time 1 or 2 revealing a group of students who have a high opinion of the ability their teacher thinks they have in mathematics. The Year 3 data show a decrease in the ‘A lot’ and ‘Some’ ratings and an increase in the rating for ‘A little bit’ at time 2 meaning a slight decrease overall. The Year 4 data show a small increase in the rating for ‘A lot’ and a decrease in the ‘Some’ rating. The Year 6 rating for ‘A lot’ decreases, while the ‘Some’ rating increases and the rating ‘A little bit’ which was not used in time 1 appears at time 2.

Overall, this data set reveals small downward movements representing a decrease in confidence which was greatest in the Year 3 data. Despite this decrease, the data represent a highly positive view of the ability students believe their teacher thinks they have. The Year 4 and 6 data show nearly 90 percent of students think their teacher thinks highly of their ability in mathematics at both time 1 and time 2.

This section also contains data from the students' response to the question 'How much do you like helping other people with their maths?' The data is included here as the students' willingness to help others can be seen as a reflection of their belief in their ability to actually supply that help, therefore making it a reflection of their confidence in their mathematical ability. What we see here is again a strongly positive view of their mathematical ability.

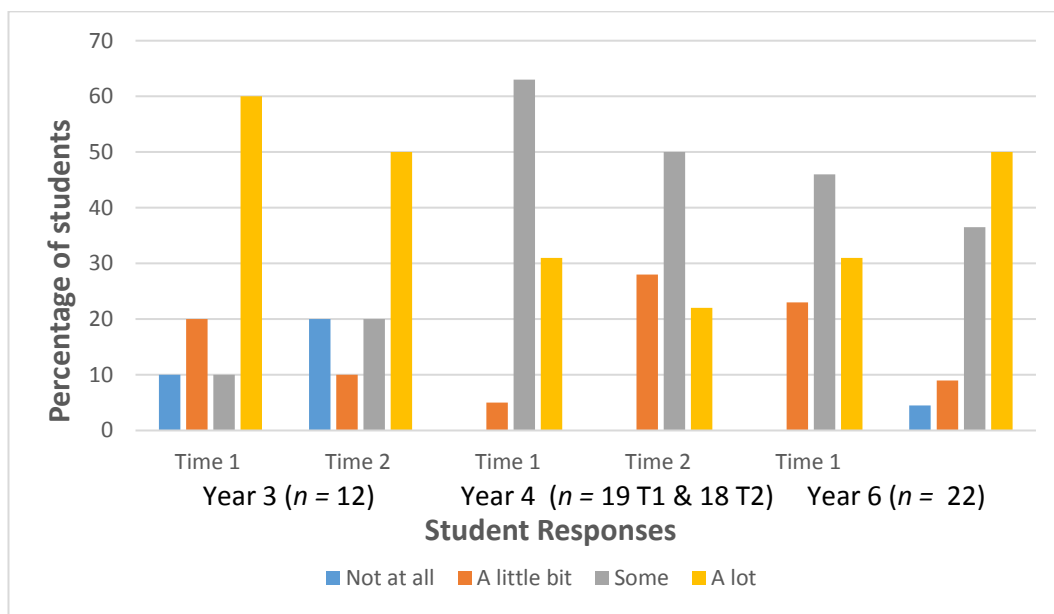


Figure 4.6 How much do you like helping other people with their maths?

Figure 4.6 reveals a mixed set of shifts within the data. There has been an increase in the rating of 'Not at all' in both the Year 3 and 6 data, but not the Year 4 data where the 'Not at all' rating does not appear; however, there is an increase in the Year 4 selection of 'A little bit'. Both the Year 3 and 4 data show a small decrease in the selection of 'A lot', while the Year 6 data increase for this rating. Overall there is a slight positive increase in the Year 6 students' willingness to assist their classmates but a slight decrease in Year 3 and 4 willingness. IBL involves a high level of group work and co-operation and these small shifts in willingness to assist others may be a reflection of a small number of students' response to their levels of enjoyment of that group work, or to those with whom they were working. These results suggest a small negative impact on confidence which, given the size of the movement, may or may not be due to the introduction of IBL.

Responses to the question "Who helps when you are stuck?" are included in the confidence section as they are an indication of the confidence students have in

their classmates' ability in mathematics. It is an indication not of their classmates' actual ability but of their confidence in each other's ability.

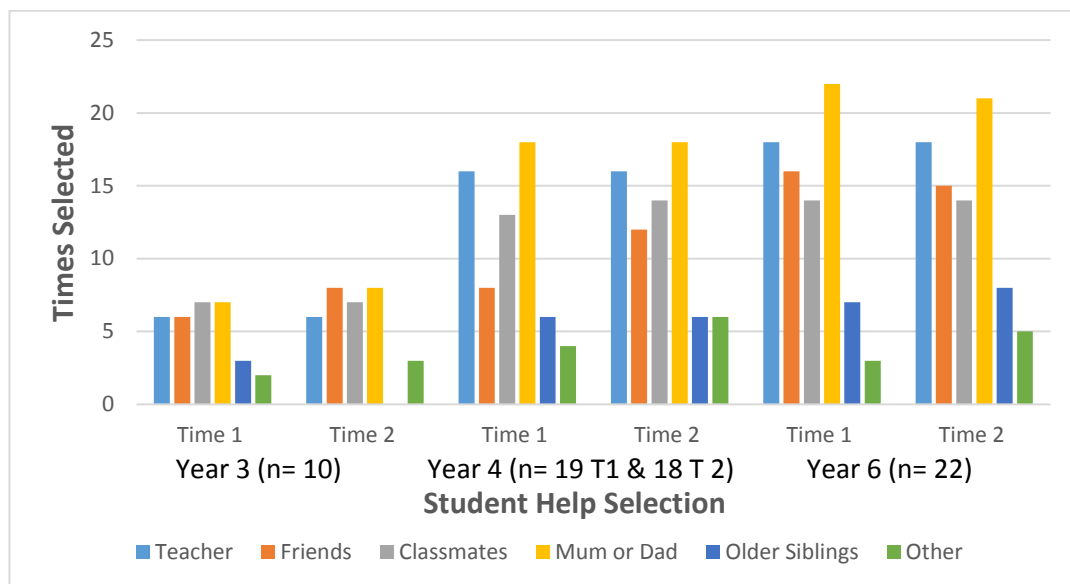


Figure 4.7 Who helps when you are stuck?

Analysis of these data shows some consistent trends across all three year groups in both time 1 and time 2 data. When asked who they got to help them with their maths when they got stuck, the strongest response at time 1 and 2 was parents, followed by their teacher. The third most common place students went to for help was a mixture of friends and classmates. The Year 3 and 4 preferred classmates, while the Year 6 preferred friends. The rating 'other' appeared quite consistently in the data with a total of 11 selections across the year groups at time 1 and 18 selections at time 2. Students who chose this category most commonly defined this as cousins, grandparents, uncles and aunts.

The data reveal a slightly different picture of the confidence students have in each other's ability with mathematics. Here they rate each other below parents and teachers suggesting that they may not have as much confidence in each other's mathematical ability as they have in their own. Students also rate parents slightly higher than teachers as their place to go for help with mathematics. A possible reason for this may be ease of access and availability rather than having to compete with classmates for the teacher's attention.

4.4.2 Problem-Solving Confidence

Data for this section come from the student interviews and reveals a picture of growing confidence across the intervention period. Interviews were conducted in accordance with the protocols outlined in 3.5.1.

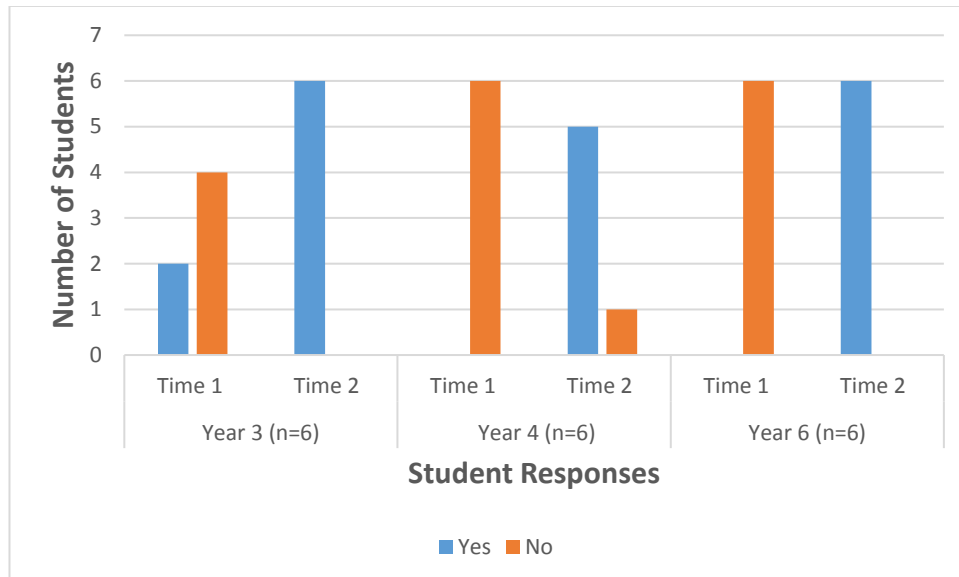


Figure 4.8 Do you think you can solve this problem?

Figure 4.8 reveals a large increase in students' problem-solving confidence. These figures reveal an 83 percent increase between time 1 and time 2 in the number of students who are confident in their problem-solving ability. At the beginning of the intervention just two students (S3.7 and S3.12) said that they would be able to solve the problem. When questioned a little further both students began to explain strategies that strongly suggested they indeed would be able to solve it. The remaining 16 students said they would not be able to solve the problem with a number saying things such as "Too hard". By the time 2 interviews 17 of the 18 students confidently said they could solve the problem with comments such as this: S4.14: "Yes, I think I got better at problem solving. We can use the maths you already know." Student (S4.7) was not sure; his response was "Maybe, but it's a bit hard". How this overall confidence relates to their actual problem-solving ability would be a topic for further research. These data represent a strongly positive impact as a result of the introduction of IBL.

4.4.3 Students who moved more than one place with regard to Confidence

As noted in section 4.4, the small movements in overall data for each of the year groups mask a larger number of individual shifts. The chart below identifies those students who moved more than one place.

Table 4.2 *Students who have moved more than one place*

Student Identification	How good do you think you are at maths?	How much do you like helping other people with their maths?
S6.10		Up 2
S6.8		Up 2
S6.7	Down 2	
S4.19	Down 2	
S4.18		Down 2
S4.7	Down 2	
S4.1		Down 2
S3.8		Down 2
S3.1	Up 3	

The first feature of note is that there are fewer students on this list than there were on the previous list (4.3.4). There are 9 students, 2 Year 3, 4 Year 4 and 3 Year 6, with 5 boys, and 4 girls. Also there are no shifts of two or more places in response to the question ‘How good does your teacher think you are at maths?’ whereas in the attitude to mathematics section both questions produced such shifts. This raises the possibility that students were more settled in their confidence in mathematics than in their attitude to mathematics; however, the truth or otherwise of that claim would require further research.

There are three students who appear in both this list and the previous list of students who moved more than one place (4.3.4) and these students were each considered to determine if there were any overall trends appearing.

Student S3.8 has decreased her rating for how much she enjoys helping others with their mathematics from ‘A lot’ to ‘A little bit’, which is suggestive of decreasing confidence in her mathematical ability. However, this needs to be balanced against her two place upward shift in her enjoyment of mathematics in her own time, suggesting a more mixed picture in her mathematical confidence and attitude.

Student S4.1 appeared in the previous list with a shift down in his enjoyment of mathematics at school and shows a two place downward shift here in his rating for how much he enjoys helping others. Student S4.18's previous appearance was a two place downward shift in her enjoyment of mathematics at school and this is now matched by a two place downward shift in her enjoyment of helping others with their mathematics. These changes suggest these students have a decreasing confidence in their ability at mathematics as students generally enjoy what they think they are good at, and like to share this with others, but are less keen on things they find more challenging.

These patterns seem to reflect the changing perceptions and confidence of these students individually rather than any overall trend.

4.4.4 Summary

The picture revealed is that of a group of participating students at All Saints School who began the IBL intervention highly confident in their own ability and their teacher's sense of their ability at mathematics. This response was in marked contrast to their problem-solving confidence which, at the beginning of the intervention, was very low. However, by the end of the intervention they were as confident in their problem-solving ability as they were in their mathematics in general, most clearly demonstrated in 17 of the 18 (94%) interviewed students' saying they would be able to solve the problem they were shown. The large growth in problem-solving confidence suggests a positive impact on student confidence due to the introduction of IBL.

4.5 Achievement in Mathematics

This section examines the achievement data for participating students. Data sources are student OTJ from 2014 and 2015, and PAT scale scores from 2014, 2015, and 2016, with the exception of the Year 3 students who completed PAT for the first time in 2015. The OTJs for each student in 2014 have been collected and compared to their OTJ for 2015 as outlined in the graphs below.

4.5.1 OTJ Data

OTJ is made by the classroom teacher based on a student's entire year's work. The data were collected for the end of 2014 and at the end of the intervention in 2015.

The results are shown in the graphs below.

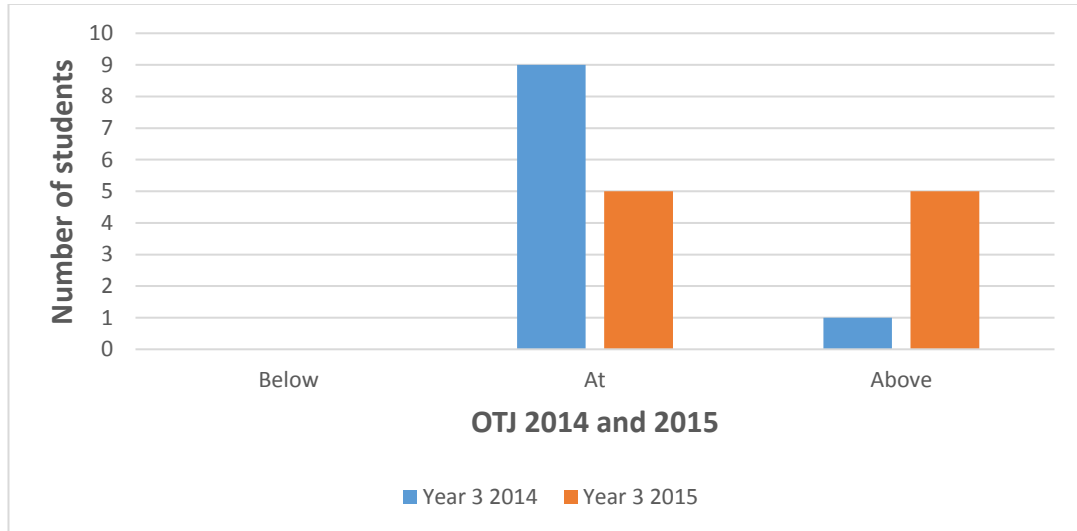


Figure 4.9 Year 3 OTJ data

Figure 4.9 reveals a positive shift in overall achievement for participating Year 3 students as measured by their OTJ. The absence of students assessed as *'Below'* and *'Well Below'* the standard suggests that perhaps the Year 3 students' confidence in their ability in mathematics is justified. Despite a high level of achievement at the beginning of the intervention, there is still a positive rise in achievement by the end of the intervention with 40 percent of students progressing from *'At'* the standard to *'Above'* the standard.

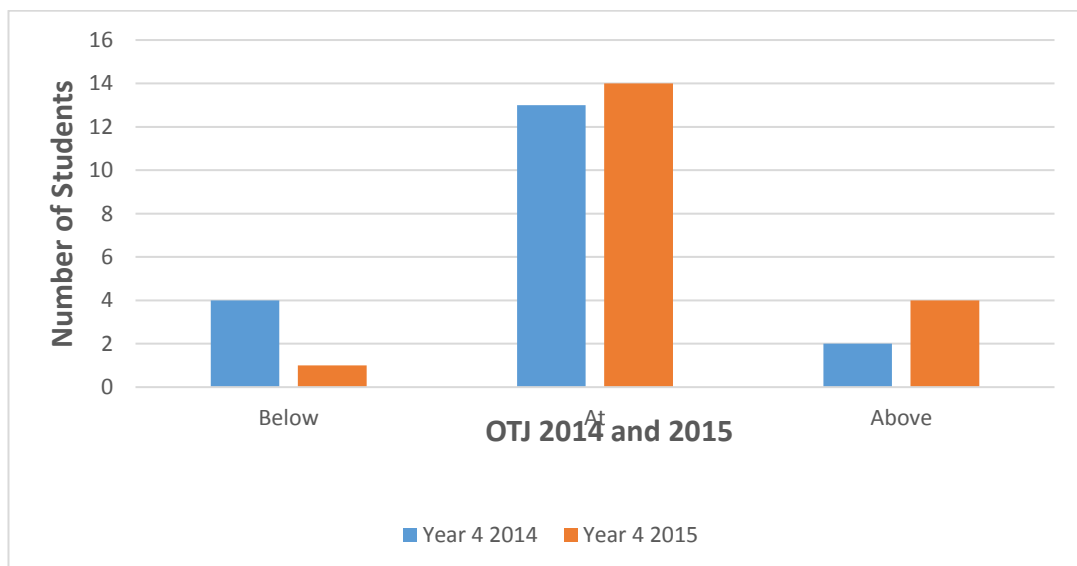


Figure 4.10 Year 4 OTJ Data

Figure 4.10 reveals a small positive rise in achievement for the participating Year 4 students as measured by the end of intervention OTJ. There are only a small number of students who began the intervention *Below the standard* and there are no students *Well Below the standard*. There is a 75 percent decrease in students *Below the standard* and a 50 percent increase in the number of students assessed as *Above the standard*.

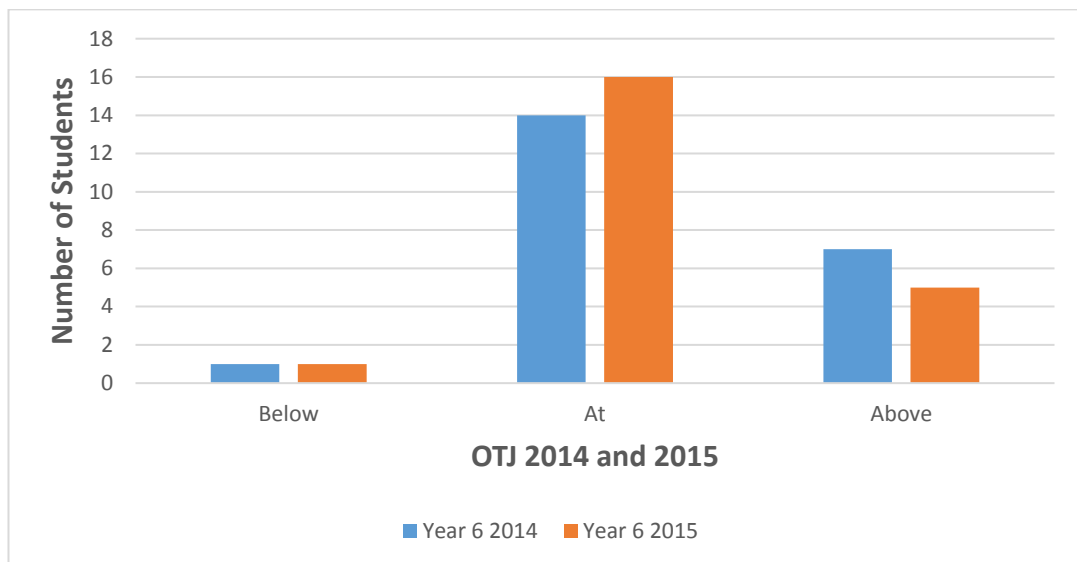


Figure 4.11 Year 6 OTJ Data

Figure 4.11 reveals a small achievement decrease for participating Year 6 students. At the beginning of the intervention just one student was *Below the standard* and no students *Well Below the standard*, and this situation remains the same at the end of the intervention. For this year group there has been a slight decrease in achievement with slightly fewer students assessed *Above the standard* at the end of the intervention than there were at the beginning. Discussions with the classroom teacher revealed that the two students who decreased from *Above* to *At* were both very close to being *Above*, suggesting only a very small decrease in achievement levels.

Summary

The OTJ data show a small decline in achievement for the Year 6 students; balanced against this is the rise in achievement for the Year 3 and 4 students suggestive of an overall positive impact on OTJ for the introduction of IBL. We now move to triangulate the OTJ data with the PAT data.

4.5.2 PAT Data

PAT scale score was analysed in a range of ways (3.7). The first of these was comparing student scale score progress against National Average⁷ scale score progress, which is the only method that could be applied to every year group.

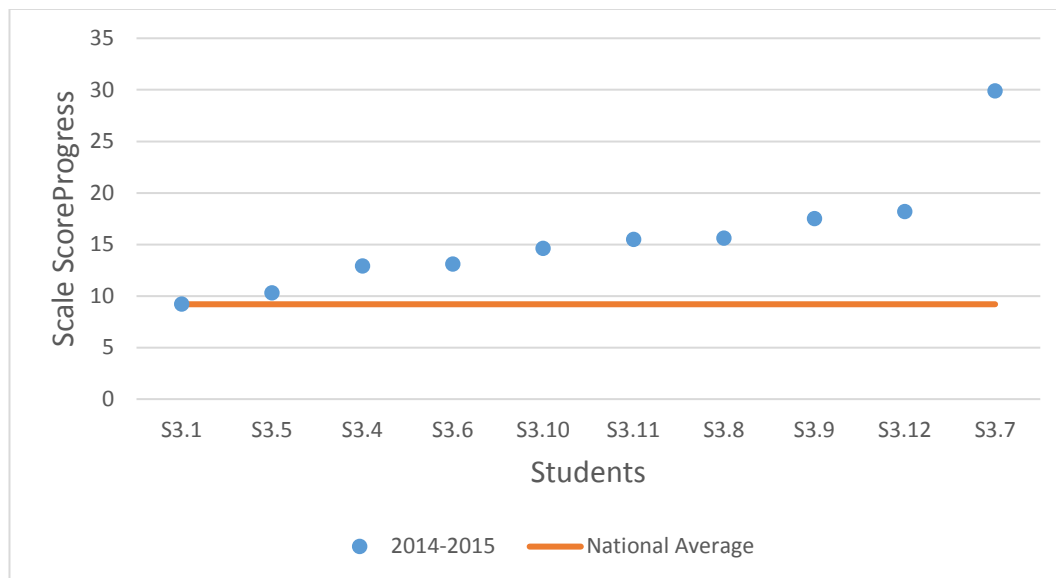


Figure 4.12 Year 3 PAT Scale Score 2016 against National Average

Figure 4.12 shows a group of students who have made excellent progress, with most students making more progress than the national average, and a small number making well above the national average. The average scale score progress for the students as a cohort was 14.68 or 160 percent of the 9.2 national average. When the student's scores are compared with the national average in a *t*-Test ($t(9)=2.69$, $p=0.02$), a statistically significant result against the national average is confirmed. Given the size of the progress, this suggests a positive impact for the introduction of IBL. When the progress of the students who began 2015 *below stanine 5* is considered, we see they have scale score progress between 9.2 and 15.6 with mean progress of 12.63. When compared with the national average of 9.2 in a *t*-test we get ($t(2)=1.5$, $p=0.42$), not a statistically significant result.

⁷ It should be noted that achievement above national average could normally be expected for a high decile school (Ward & Thomas, 2015). National average achievement data in PAT for decile bands was not available at the time of writing.

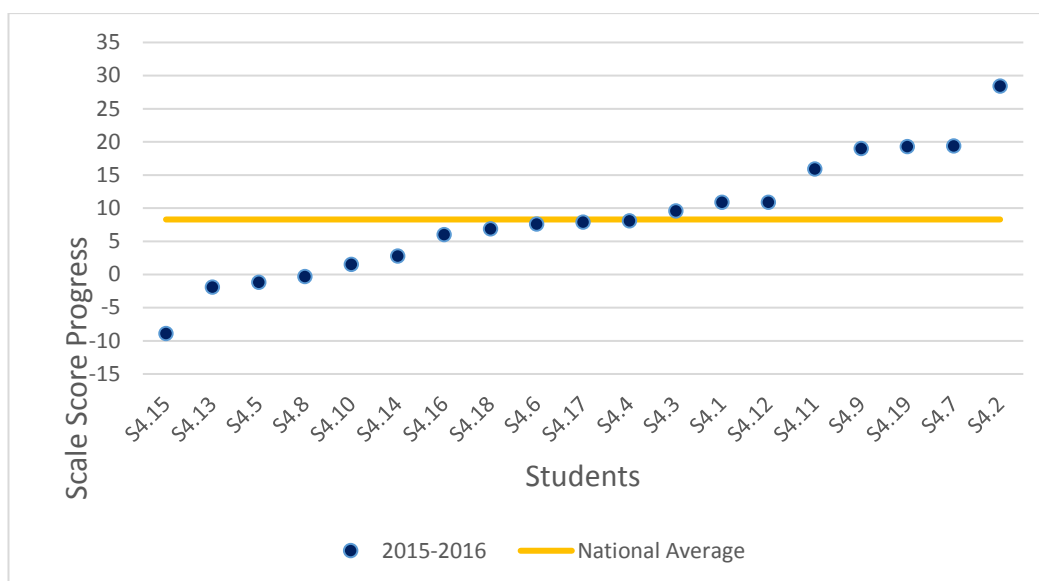


Figure 4.13 Year 4 Scale Score progress 2015-2016 against National Average

Figure 4.13 shows a broad range of results against the national average. Four students have negative scale score progress and several others have very little progress; these students will need close monitoring going forward. At the opposite end of the spectrum some students have made gains between 2 and 3 times the national average. The cohort average is 8.5, just above the national average of 8.3. When examined using a t -test ($t(18)=0.093$, $p=0.9$), this does not reach the level of statistical significance. However, of particular interest in the data is the progress of the lowest achieving students in comparison to the national average and the achievement of the rest of the cohort. There were three students who started 2015 below stanine 5 in their PAT, these three students made scale score progress between 15.9 and 19.4 scale points compared to the national average of 8.3 and the average for the rest of the cohort at 6.725. This gives these three students an average of 18.1 which is 218 percent of the national average and 269 percent of the rest of the cohort average. These figures give ($t(2)=7.232$, $p=0.012$) against the national average, and ($t(2)=8.395$, $p=0.0002$) against the rest of the cohort, both of which are statistically significant results. It should be noted that these results come from a very small sample size and so need to be treated with some care. However, the progress achieved is an important result for these three previously low achieving students. While the class mean is average, the result for the underachieving students suggest a positive impact for the introduction of IBL.

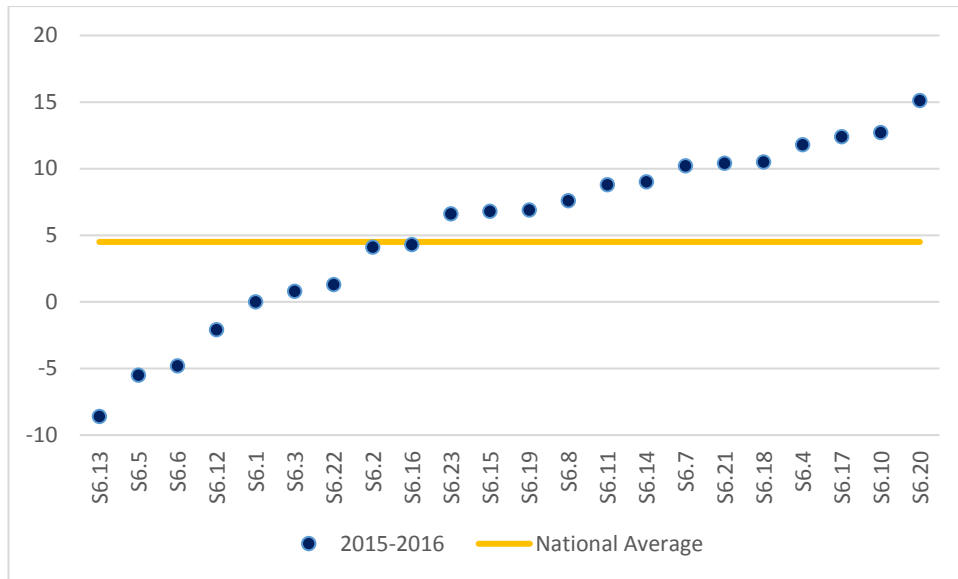


Figure 4. 14 Year 6 Scale Score against National Average

Figure 4.14 shows a diverse set of results for this cohort. There are students who have negative scale score progress; again they will need careful monitoring going forward. Others have made more than double the national average progress. This cohort has an average scale score achievement of 5.38 compared to a national average of 4.5, giving 120 percent of the national average. A t -test of ($t(21)=0.62, p=0.53$) fails to reach the level of statistical significance. In this class there were two students who began the year *below stanine 5 in their PAT*. These two students had an average scale score progress of 8.3 which is 184 percent of the national average and is 165 percent of the average for the rest of the cohort at 5.045. Examination using t -test produces ($t(1)=3.83, p=0.116$) against the national average and ($t(1)=3.25, p=0.073$) against the rest of the cohort, neither of which reach the level of statistical significance. Again these are based on a very small sample size so need to be treated with care. However, these previously low achieving students are out-performing the rest of the cohort suggesting an important result for these students, and a positive impact for the introduction of IBL.

4.5.3 PAT Against Previous Year

To triangulate these data a comparison was undertaken against the previous year's scale score progress. The analysis compared 2014-2015 scale score progress against 2015-2016 scale score progress.

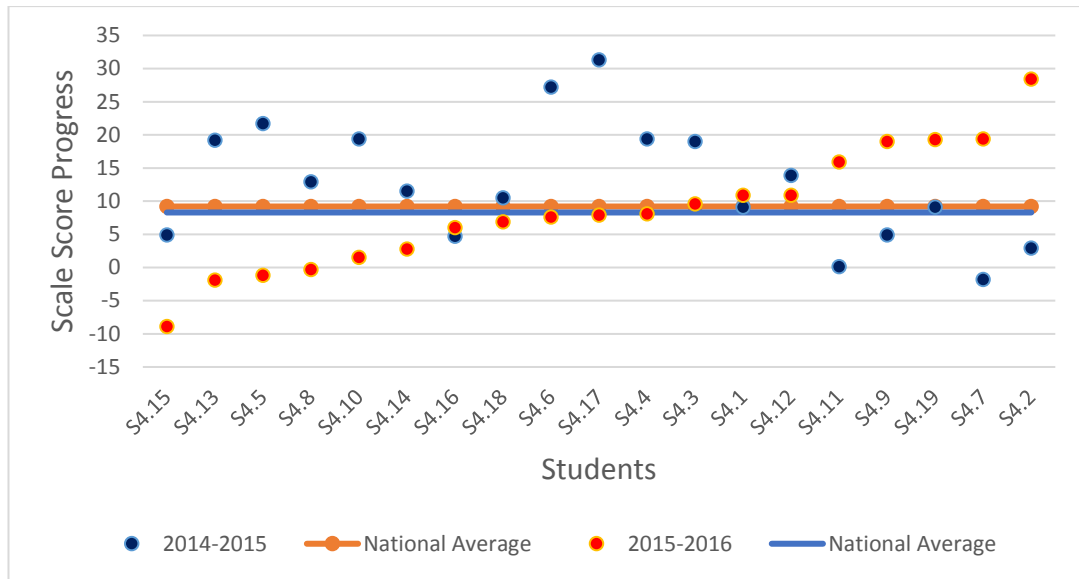


Figure 4.15 Year 4 compared to National Average 2014-2015 and 2015-2016

Figure 4.15 displays scale score progress against the national averages for 2014-2015 (blue line and markers), and 2015-2016 (red line and markers). This allows individual student progress to be seen between the two timeframes and against national averages. The data represent an average scale score progress between 2014 and 2015 of 12.63, or 137 percent of the national average. On the other hand, the 2015-2016 data has an average of 8.5 or 103 percent of the national average. This represents a decrease in average progress against the previous year for participating students, which is suggestive of a possible negative impact for the introduction of IBL; however, this needs to be balanced against the huge progress made by the underachieving students, which strongly suggests a positive impact for IBL.

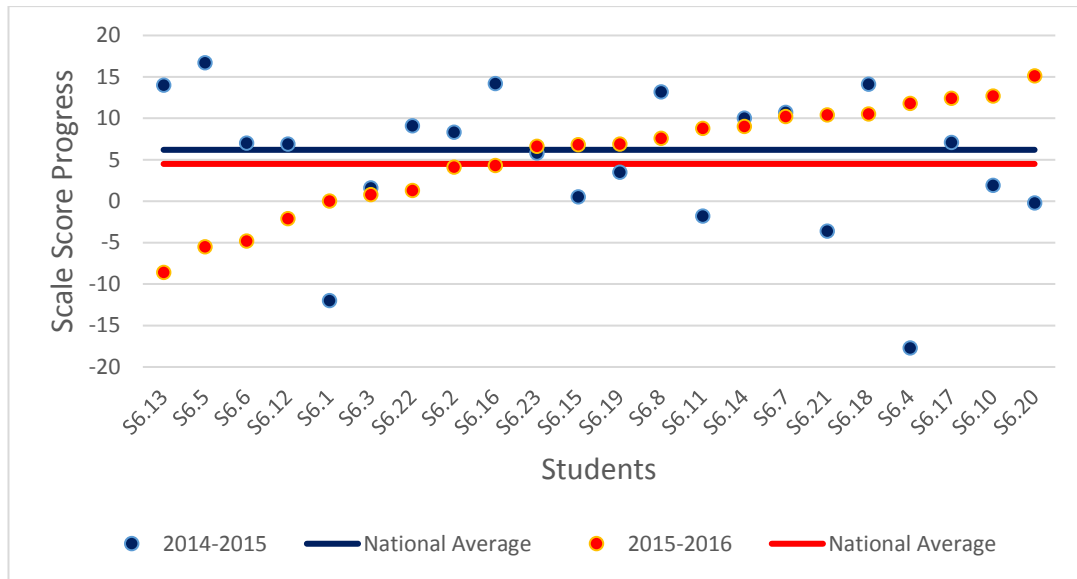


Figure 4.16 Year 6 compared to National Average 2014-2015 and 2015-2016

Figure 4.16 displays scale score progress against national averages for 2014-2015 and 2015-2016 allowing individual student progress to be seen between the two timeframes and against national averages. These data represent an average scale score progress between 2014 and 2015 of 4.97, or just 80 percent of the national average. On the other hand, the 2015-2016 average progress is 5.38, or 120 percent of the national average. This, along with the higher progress rate for the lowest achieving students suggests, a positive impact of IBL in this school situation.

4.5.4 Performance in Achievement Bands in Comparison to the rest of the Cohort

Given that the primary motivation for the introduction of IBL and the associated research was an attempt to raise the achievement levels of those students who were underachieving, a closer examination of student achievement in relation to previous achievement was undertaken. This section examines the students' achievement, as measured by PAT results at the beginning of 2015, in the bands of low, mid and high achievement against the rest of their cohort. The criteria for each band are: Low, below stanine 5; Mid, stanines 5 & 6; and High, stanines 7, 8, & 9. Each band's achievement is compared to the rest of their cohort for each year group in the following table.

Table 4.3 PAT Scale Score tests in Achievement Bands

Achievement Band	Year group	2015-2016 progress	Comparison progress	t-test	P value	Statistically Significant
Low	3	12.63	15.56	($t(2)=-1.28$, $p=0.421$)	0.421	No
Mid	3	15.56	12.63	($t(6)=0.936$, $p=0.421$)	0.421	No
High	3	No high achievers at beginning of 2015				
Year 4						
Low	4	18.1	6.725	($t(2)=8.395$, $p=0.0002$)	0.0002	Yes
Mid	4	8.313	8.678	($t(7)=-0.162$, $p=0.982$)	0.928	No
High	4	5.138	10.98	($t(7)=-1.389$, $p=0.215$)	0.215	No
Year 6						
Low	6	8.3	5.045	($t(1)=3.25$, $p=0.073$)	0.073	No
Mid	6	5.85	4.917	($t(9)=0.532$, $p=0.743$)	0.743	No
High	6	4.24	6.259	($t(9)=-0.791$, $p=0.487$)	0.487	No

The data show promising signs for the progress of lower achieving students in the Year 4 and 6 cohorts as they made up ground on the rest of their classmates. What the data reveal is that the low achieving students at the beginning of the intervention markedly out-performed the rest of their cohort during the intervention, and in the case of the Year 4 low achievers out-performed their cohort to a statistically significant level.

4.5.5 Summary

The results outlined above show statistically significant achievement gains for the Year 3 students' mean scale score progress when compared with the national average for that year group. The Year 4 low achievers also showed statistically significant achievement gains when compared to both the rest of their cohort and the national

average. These two results show a positive impact on student achievement for these groups following the introduction of IBL.

While the achievement gains for the low achieving Year 6 students were indicative of a potentially positive impact from the introduction of IBL, this was not to a statistically significant level. The PAT data show mean scale score positive results against national average achievement for the Year 3 and 6 students, but not to a statistically significant level, while the Year 4 result was very similar to the national average. When their previous year's progress against national averages is compared to the intervention year, a decrease is seen for the Year 4 cohort, but a large increase for the Year 6 cohort, but again not to a statistically significant level.

Five students across the three classes began this intervention *below the standard*; that number had dropped to just two at the end of the intervention. There was an increase in students assessed *above the standard* in the Year 3 and 4 cohorts, but a small decrease for students *above the standard* in the Year 6 cohort. Overall the OTJ results are suggestive of a small positive impact on student achievement which may be the result of the IBL intervention.

4.6 Summary

The main area of interest in the achievement data came in the Year 3 students' PAT scale score progress to a statistically significant level over the national average as outlined above. The PAT scale score progress of the underachieving Year 4 students also saw large achievement gains over national averages and the rest of their cohort to a statistically significant level, outlined above. This positive impact on the achievement of the previously underachieving students shows a significant impact from the introduction of IBL. The Year 6 low achieving students also saw major gains in their PAT scale score but not to a statistically significant level. Taken together these results suggest a possible positive impact for the introduction of IBL. The intervention saw a rise in achievement as measured by OTJ for the Year 3 and 4 students, but a small decline for the Year 6 students as outlined above.

The participating students at All Saints School began with, and maintained, an overall positive attitude to mathematics. There were some small shifts, with the Year 3 students tending to move upwards and the Year 4 and 6 students moving downwards from the highest enjoyment rating to the second highest rating for mathematics at school. For mathematics outside of school the Year 3 and 6 students

saw positive gains in enjoyment while the Year 4 data showed a small decline. The largest impact on attitude came in students' responses to challenge, struggle and mistakes which was quite negative at the beginning but students were showing a more positive response to these things at time 2. These data do not show any significant impact for the introduction of IBL on students' attitude to mathematics.

Students began and ended the intervention highly confident in their mathematical ability, with just some minor movement within this positive data set. The largest impact on students' confidence came within problem-solving. Students began the intervention with very low problem-solving confidence, but ended it with very high problem-solving confidence. This data set shows a positive impact on students' problem-solving confidence which is most likely due to the introduction of IBL.

Chapter Five

Discussion

5.1 Introduction

This chapter discusses the results reported in the previous chapter. Results will be considered in light of the themes identified, literature, and possible implications, underpinned by the research aim and questions.

An important aim of this study was to determine what impact the introduction of IBL would have on students' attitude to mathematics and their confidence in their mathematical ability. A further key aim was to consider the impact the introduction of IBL would have firstly on the overall achievement of all participating students, and secondly, on the mathematics achievement gap between highest and lowest achieving students.

The questions underpinning this research were:

1. How does the introduction of IBL impact on student attitudes to mathematics?
2. How does the introduction of IBL impact on students' confidence in their mathematical ability?
3. How does the introduction of IBL impact on the mathematics achievement outcomes for all participating students?
4. How does the introduction of IBL impact on the overall achievement gap between the highest and lowest achievers?⁸

The discussion examines these research questions in three sections: 5.2 Attitudes (RQ 1); 5.3 Confidence (RQ 2); and 5.4 Achievement (RQs 3 & 4). Finally, the overall impact of the introduction of IBL is considered.

This discussion is undertaken with an awareness that education is a complex undertaking and that in many, but not all cases, these findings are open to a range of possible interpretations as to the cause of the changes discussed. The classroom is a dynamic and complex place where the culture is created through an interaction of the teacher-student and student-student relationships. Good and Brophy (2002) note "In a single day, an elementary teacher may engage in more than a thousand interpersonal exchanges with students" (p. 23), meaning that no two classrooms are

⁸ Levels of achievement are determined by PAT scale score and teacher OTJ.

ever the same. Attributing change to a specific cause is, therefore, not a straightforward process. When this complexity is combined with the small number of participants involved in this study the conclusions and implications are, therefore, necessarily emerging and tentative in nature.

Given this complexity, it is important to consider other factors that were possible influences on the results outlined in chapter four. All participant students began the year with a new teacher, as would normally be the case at All Saints School. A new teacher means a different teaching style and inevitably a new teacher-student relationship which again may have had positive effects on some and not so positive effects on others. Teachers at All Saints School also discussed fixed and growth mindsets with students as part of their approach to encouraging students to see themselves as able to grow their intelligence as they learn (Dweck, 2006). This approach may have influenced some of the students in this study. However, this mindset work only began later in the second half of the year and given the shorter timeframe is, therefore, less likely to be an influence on the results reported here. The most significant change made to the teaching and learning programme for all participant students was the introduction of IBL into the mathematics programme. It is, therefore, more likely that the changes reported here result largely from the introduction of IBL.

5.2 Attitudes

This section examines the themes of students' understandings of, and attitudes to mathematics (Muis, 2004). The views students hold about mathematics have the potential to impact on their achievement in mathematics and are, therefore, an important focus for research. If we are to develop clear insight into the views students hold about mathematics, it is important that we try to ascertain their understanding of what constitutes mathematics and this was addressed in the present study in both student surveys and interviews.

5.2.1 Perceptions of Mathematics

Data from the qualitative sources strongly suggested that the students viewed mathematics primarily as number (Section 4.2). The same theme emerged from student surveys and student interviews with data from both sources giving a strong sense of students' perceptions of mathematics primarily as what the NZC defines as

the number strand: “Number involves calculating and estimating, using appropriate mental, written, or machine calculation methods in flexible ways” (MOE, 2007, p. 26). The IBL programme used a range of problem-solving activities from all strands of the mathematics curriculum, yet the time 2 surveys and interviews continued to show that students turned first to the number strand when discussing their views of mathematics. Given that much mathematics is dependent on a sound working knowledge of the number strand, this emphasis is not necessarily surprising, nor a problem. However, Caygill et al. (2013) noted: “The decrease in mean mathematics achievement among New Zealand students seems to be mainly due to a decrease in achievement on questions about statistics, and geometry and measurement” (p. 2). These findings from the 2012 TIMSS study, along with similar findings in recent PISA results (OECD, 2014), begin to raise some questions over the level of focus on the number strand in New Zealand’s mathematics education.

The findings of this research are in line with the early work of Frank (1988), followed by the later work of Wong, Marton, Wong, and Lam (2002) and Muis (2004) who all found that students primarily saw mathematics as computation. This focus has the potential to shape the students’ thinking about, and attitude to mathematics. If a student’s view of mathematics is predominantly number/computation and they have good computation skills, then they will likely have a more positive attitude to mathematics than a student who might have ability in geometry and measurement, but for whom computation is a challenge.

Student’s beliefs about mathematics shape their thinking, motivation and achievement within mathematics (Muis, 2004). A view of mathematics as number is likely to lead to students believing that mathematical problems can be quickly and easily solved. When put in an IBL problem-solving situation where that is not the case, students may lose motivation or interest (Muis, 2004). Considering that many participating students have achieved success in mathematics under the traditional approach to teaching and learning, their perceptions about mathematics as a subject may have contributed to a negative impact on their enjoyment of the subject, possibly contributing to the small decreases in enjoyment seen in the Year 4 and 6 classes. Furthermore, the sociocultural norms (2.5.2) of working together and assisting classmates to understand the mathematics also present a challenge for students who are used to quick easy solutions.

The study sample gives an ethnic breakdown for the participating students revealing that this research was conducted with a high level of Asian students. Anecdotal evidence from discussions with these students often reveals that in an effort to help with their learning their parents (and others) have them spend a lot of time practising algorithms and times tables. This approach fits with the contention of Kember (1996) who, in a discussion of the Asian approach to mathematics, notes “There has been wide spread anecdotal evidence of rote learning” (p. 341). This approach may have two effects: it may reinforce students’ views of mathematics as number; secondly these procedurally taught students often achieve high instrumental success (Skemp, 1976) within mathematics and come to expect that they will quickly and easily be able to solve all mathematics problems. When they encounter IBL, with the sociocultural norm of working together, discussing solution strategies, and creating multiple solutions, they may find the less procedural approach to teaching and learning more challenging and this may account for some of the drop in enjoyment. However, the participating students at All Saints School are overall very positive about mathematics and, as the teacher of some of them, it is my impression that as they become more familiar with the sociocultural and sociomathematical norms of IBL the small decreases in enjoyment will be regained.

With less time at school in which to be enculturated in the heavy focus on the number strand the Year 3 students may have been more positively influenced by the IBL approach to teaching and learning. They had just moved from the junior syndicate to the middle syndicate and were likely to be expecting things to be different. These students made great progress during the intervention and while they still viewed mathematics as number may have been more open to a wider range of thinking about how to go about mathematics which may have helped raise their achievement levels. These factors may also have positively influenced their enjoyment of mathematics.

Mathematics as number is a narrow view of the place and purpose of mathematics within the wider community. When it is combined with what can seem an endless stream of numbers on a worksheet, for struggling students this may have resulted in lower interest contributing to lower achievement. Exposing these lower achieving students to the wider aspects of mathematics through the IBL problem-solving tasks may have contributed to their raised achievement.

5.2.2 Attitude to Mathematics

Participating students at All Saints School began and ended this intervention with a high level of mathematics enjoyment, both at school and outside of school (4.3.1). Within this highly positive attitude there was a slight decline in enjoyment, as measured by the survey, from the Year 4 and 6 students to mathematics at school. While students' perceptions of mathematics as number may have contributed to this decline, as discussed above, another reason may simply have been the greater level of challenge associated with the problem-solving activities which were the focus of IBL. Students needed to work out what mathematics they were required to do, and then create, present, and justify a solution. IBL introduced an increase in complexity from the more traditional mathematics they were used to undertaking. The surveys were all conducted within the last three weeks of the school year and general tiredness and looking forward to the holidays may also have impacted on students' attitude to mathematics.

The response to the student interview question on their enjoyment of mathematics also revealed a highly positive attitude to mathematics (4.3.1). In the case of the interviews this highly positive attitude increased during the period of the intervention putting it more in line with the findings of Olander and Robertson (1973). This increase in positive attitude may have been the result of the students' reactions to the problem-solving tasks. While they were more challenging than previous work, they may also have considered them more interesting than just a page full of sums to answer. Further, they worked together on these tasks in small groups and this may have also contributed to an increased positive attitude. This puts the interview findings in contrast to the survey findings. Two possible explanations for this are: it may have been due to the interviewee telling the interviewer what they thought they wanted to hear; or it may have been that the interview was based around problem solving and the students' attitude to that may have been higher than their attitude to mathematics overall.

The findings of the present study differ from the results of some other researchers (e.g. Cotic & Zuljan, 2009; Higgins, 1997; Manswell Butty, 2001) who reported improved student attitude as a result of the introduction of IBL. Many studies that focused on the improved achievement of students also reported improved attitude, but without including before and after data for that attitude shift. However, Olander and Robertson (1973) tracked students' attitude during the period of the

intervention, as the present study also did. While the present study saw a slight decline for the Year 4 and 6 students, Olander and Robertson saw an improvement in students' attitude. The students' starting place in their attitude to mathematics provides a potential explanation for the difference in results. This study worked with students who began the study with a very positive attitude to mathematics. Olander and Robertson, by contrast, worked with students who began with a mean score in the mid-range of a 25-125 scale and saw a mean 9.26-point improvement over the period of the study. The difference between the starting points of the students offers a potential explanation for the different outcomes.

Students' attitude to mathematics in their own time did, however, improve over the duration of the intervention, with both the Year 3 and 6 students increasing their positive attitude, while the Year 4 students saw a small decline in their level of enjoyment. This increase in positive attitude was potentially due to the widening appreciation of the place of mathematics outside of school. The real-life rich mathematical tasks undertaken during the intervention may have widened the students' understanding of the place of mathematics in society. This being the case, it may have positively influenced their attitude to mathematics in their own time. The frequent reference to problem solving and other practical applications of mathematics in the time 2 student surveys suggests that the students have a growing appreciation for the place of mathematics in their lives outside the classroom. This widening appreciation of the usefulness of mathematics may have positively influenced students' attitude to the mathematics they encountered outside the classroom.

5.2.3 Attitude to Challenge and Struggle

Interviews with students make it possible to explore in more depth students' thoughts and feelings. Time 1 student interviews revealed a negative response to challenge, struggle, and having to work hard to solve mathematics problems. In a more traditional mathematics classroom ease of finding solutions can be seen as a sign of high mathematical ability and achievement. However, within IBL struggle and challenge are intended to be part of the process for all students (2.6). Boaler (2016) highlighted this aspect and encouraged both students and teachers to see struggle and challenge as positive things, because when struggle and mistakes are happening students are stretching and growing their mathematical ability. The school focus on

the work of Dweck (2006), as described in 5.1, is also relevant here; however, the earlier introduction of IBL provided an opportunity within the mathematics programme for the type of teacher talk advocated in Dweck's approach.

The positive attitude that students had towards problem solving, as seen in comments from the time 2 interviews, reflect Boaler's (2016) contention that "Students with a growth mindset take on hard work, and they view mistakes as a challenge and motivation to do more" (p. 7). For All Saints School's participating students, an increased level of challenge and struggle came with the problem-solving tasks associated with IBL mathematics. Initially this increased level of challenge and struggle may have been responsible for the small decrease in enjoyment seen from some of the Year 4 and 6 students. However, as students came to see the value in challenge and struggle their attitudes began to change and by the end of the study there was evidence from the second set of interviews that suggests that students were beginning to embrace struggle and challenge and see the value that this holds for their overall learning. As that happens it is my expectation these small decreases in enjoyment will be regained.

Research Question 1 focused on the impact of IBL on students' attitude to mathematics. For mathematics at school student attitude saw a small positive rise for the Year 3 students and a small decline for the Year 4 and 6 students. For mathematics outside of school there was a positive shift for the Year 3 students, a small positive shift for the Year 6, and a small decline for the Year 4 students. Data from the student interviews showed a rise in student attitude to mathematics. Overall, these results show a very small improvement which may or may not be the result of the introduction of IBL.

5.3 Confidence

Overall, the participating students in this study began and ended the year with a high level of confidence in their mathematical ability. This finding correlates with the National Monitoring Study of Student Achievement (NMSSA) (2015) which states: "The correlation between attitude scores and achievement was greater for students from high decile schools than for students from mid or low decile schools" (p. 58). The findings of the present study would seem to be in line with this finding from NMSSA as the students have an overall high level of confidence in their mathematical ability, a confidence which in most cases appears well placed as very

few students began the intervention year below the National Standard for mathematics, and even fewer ended the intervention below the standard.

Conversely, student problem-solving confidence at the beginning of the intervention was very low. However, by the end of the intervention their problem-solving confidence was at a similar level to their enjoyment of mathematics. Students' commitment to keep working at problems appears to have improved their ability to gain a solution, thus leading to a greater confidence in their ability. This outcome is in line with the TIMSS 2011 findings which showed that higher confidence correlated to higher achievement. "Average mathematics achievement was highest for the confident fourth grade students and lowest (by 75 points) for the students lacking confidence" (Mullis et al., 2012, p. 338).

Research question 2 asked what impact the introduction of IBL would have on students' mathematical confidence. There was little impact on students' overall mathematical confidence. However, the introduction of IBL has seen a large rise in student problem-solving confidence, suggesting a positive impact for the introduction of IBL.

5.4 Student Achievement

This section focuses on student achievement and the possible impact that IBL had on that achievement. When taken together the OTJ and PAT scale score data show an overall small positive gain in achievement. Each area of data will be briefly addressed before the findings are drawn together and the results discussed.

The OTJ data for all three participating classes show an overall positive variation for the intervention year (4.5.2). A positive result can be seen in the ten students whose OTJ increased by one place, including four underachieving students who went from *Below* to *At the standard* while the other six went from *At* to *Above*. A smaller negative result can be seen in the four students whose OTJ dropped by one place, three from *Above* to *At* and one from *At* to *Below*. This would suggest an overall positive impact from the introduction of IBL. However, some care needs to be taken as concerns have been raised over the dependability of OTJ (Ward & Thomas, 2015). With those concerns in mind we turn to a discussion of the PAT results to gain some triangulation.

PAT gave a more detailed analysis of achievement than was possible with OTJ. The greater detail allowed comparison with previous achievement, national

averages, and achievement between achievement level sub-groups within each class. The detailed information gained from this in-depth study of the PAT scale scores showed positive achievement gains which are most likely a result of the introduction of IBL into the classes' mathematics programmes. National averages for PAT results come from a sampling which covers schools of all decile ratings. High decile schools, such as All Saints School, would normally show achievement rates above the national average (Ward & Thomas, 2015).⁹

The Year 3 class had the largest mean scale score progress of the three classes with 160 percent of the national average, a statistically significant result. The Year 4 underachieving students showed achievement gains over both national average and the rest of their cohort to a statistically significant level. Given the size of the achievement gains made, and the fact that the most significant change for these classes was the introduction of IBL, the most likely cause of the achievement gains was the introduction of IBL.

The Year 6 class mean progress was 120 percent of the national average which when tested with *t*-test, did not reach the level of statistical significance. When this is compared with the previous year's achievement of only 80 percent of the national average, this represents a large increase in mean student achievement. Several possible factors as outlined in 5.1 may have influenced this result. However, the most significant change to occur for this class was the introduction of IBL, suggesting that IBL might have been the cause of the students' improved achievement.

The mean achievement score for the Year 4 class at just 102 percent of the national average, when coupled with the drop in achievement over national average gains made in the previous year, may suggest some in this cohort did not respond well to the introduction of IBL. With this class being especially procedurally focused, due in part to a high proportion of Asian students, the change in teaching and learning style to IBL may have contributed to the decline in performance for some students.

These results are in line with those of other researchers (e.g., Boaler, 2008; Erbas & Yenmez, 2011; Hiebert & Wearne, 1993; Mistretta, 2005) who all saw achievement gains as a result of the introduction of IBL, and are strongly suggestive

⁹ Unfortunately, at the time of writing average scale score progress by decile band was not available meaning exact comparisons were not able to be made.

of a positive impact on achievement for the introduction of IBL in the participating classes.

Research Question 3 asked what impact the introduction of IBL would have on overall student achievement. As outlined above, the students had a new teacher with a new teaching style, and this may have affected some students positively and others negatively and is therefore unlikely to have been responsible for these changes in achievement. The mindset work may have influenced these results but given the much shorter timeframe for its introduction, this is less likely to be an influence. IBL was the major change in the teaching and learning programme during this year and marks a significant change in the way students encountered and undertook mathematics. Instead of being taught process and practising it, they were given a problem and asked to work out a solution. All solution attempts were accepted and errors were accepted and viewed as positive learning opportunities. Given the length of time given to the intervention and the significance of the change in teaching approach the results reported here are most likely to be the result of the introduction of IBL.

An important aim of this research, posed as research question 4, was to discover whether the introduction of IBL would assist with closing the achievement gap between the higher and lower achieving students. The TIMSS and PISA studies highlight the large gap between New Zealand's lowest and highest achieving students: "New Zealand is counted among the 10 PISA countries and economies with the widest spread of achievement in mathematical literacy" (MOE, n.d.c). The results discussed below show that in this setting IBL has contributed to a closing of that achievement gap for the participating students at All Saints School.

Sub-groups of each class (4.5.5) were examined showing an increase in achievement for the lowest achieving students against both national averages and the rest of their cohorts. The Year 3 lowest achieving students did not show the same achievement gains as the Year 4 and 6 students. The Year 4 lowest achieving students made 218 percent of the national average progress, and 269 percent of the highest achieving students' average progress a statistically significant result. Although not statistically significant, the Year 6 lowest achieving students made 184 percent of the national average progress and 165 percent of the highest achieving students' progress. Generally, my experience as a classroom teacher suggests that students who are underachievers show a pattern of underachievement over a period

of time, as was the case with the underachieving students reported in this study. The fact that these students made the greatest gains strongly suggests a positive impact on these students and the achievement gap for the introduction of IBL.

There are several aspects to IBL which may have assisted with this achievement gain for the lowest achieving students. These low achieving students were assigned to work with their higher achieving classmates, a specific sociocultural norm of IBL. This experience exposed them to the thinking and mathematical strategies of their higher achieving classmates. Such exposure may have broadened the range of approaches to solving mathematical tasks that these low achieving students were able to employ. Such learning through participation and engagement with more experienced members of the community fits well with the sociocultural theory that underlies IBL. The time and emphasis on whole class discussion meant that not only were the lower achieving students being exposed to the thinking and strategies of the higher achieving students in their group but of the entire class, again a key sociocultural norm fitting with the underlying sociocultural theory. In a more traditional mathematics classroom students were given *the* strategy for solving the problem. Even within the Numeracy Development Projects' environment where learning a range of strategies is advocated, students were rarely, if ever, shown more than one strategy at work on the exact same problem; they were shown one strategy and given practice activities for that strategy. Here in the IBL environment these low achieving students (along with all the others) were exposed to a large range of ways of solving the same problem. Third, all students were expected to be able to explain the solution and solution strategy their group created. This meant all group members were responsible for ensuring all other group members' success. This expectation created a much more cooperative working environment than traditional teaching and learning in mathematics, a key sociocultural norm for IBL. The lower achieving students may have benefited from this level of assistance from their higher achieving classmates. Further, being assigned the same mathematics task to work on as the highest achieving students raised expectations of achievement for these low achieving students. Being given the same task as recognised high achieving students may have suggested to these students that their teacher believed they could manage these tasks and may have assisted in increasing the effort they put into their mathematics. A key aspect of IBL is the expectation that problem solving activities undertaken require persistence and effort to solve, but that

it can be done, which may have convinced the low achieving students of the value of persistence resulting in higher performance (Boaler, 2016). Finally, the Numeracy Development Project approach where problems are broken down into their simplest form and then gradually increase in complexity is a quite different approach to IBL where the complex real life problem is the focus. This focus on reality may have assisted the lower achieving students to see the purpose in mathematics, creating a higher level of interest for the low achieving students, resulting in greater effort and therefore greater achievement.

The PAT, which were used as an important tool in determining student achievement, are multiple choice tests and so very different from the IBL tasks and their benefits listed above. However, a greater range of strategies, greater persistence, the expectation of success, and the actual success they achieved may have all increased the ability and motivation of these low achieving students, enabling them to achieve success in a wider range of mathematics, not just problem solving (Boaler, 2016). The success in raising the achievement of four of the five *Below the standard* students to *At the standard* and the three Year 4 students who went from *At* to *Above* would further reinforce this contention of IBL assisting in boosting wider mathematical ability. However, it must be noted that three students saw a decline in their OTJ assessment with one Year 4 going from *At* to *Below*, and two Year 6s going from *Above* to *At*.

When the achievement of the lowest achieving students is considered, the results of this study are in line with the results reported by researchers such as Boaler (1998, 2008), Higgins (1997), Hiebert and Wearne (1993) and Mistretta (2005), who show positive gains for low achieving students from the introduction of IBL. This strongly suggests that the introduction of IBL can result in positive achievement gains when introduced into high SES high achieving primary schools. Further, there will be students in all primary schools who are finding difficulty in achieving success in mathematics and these results suggest that the introduction of IBL into the mathematics programme has the potential to assist those struggling students to be successful in mathematics.

Research Question 4 addressed the issue of the achievement gap between highest and lowest achieving students and was a key motivation behind the introduction of IBL in this project. These results clearly suggest that the introduction of IBL has contributed strongly to a closing of the achievement gap.

5.5 Summary

In this setting the introduction of IBL had only minor impact on overall student attitude to mathematics and it did not show much impact on overall student mathematical confidence; however, it did show a large shift in students' problem solving confidence. The data showed some statistically significant gains in overall achievement for the Year 3 students from the introduction of IBL. However, the largest impact on the students at All Saints School from the introduction of IBL was on the achievement gains seen by the lowest achieving students, especially for the Year 4 underachieving students who gained to a statistically significant level. The Year 6 students made large gains but not to a statistically significant level. Both the Year 4 and 6 results saw a closing of the achievement gap between highest and lowest achieving students. The implications of these results are discussed in the final chapter.

Chapter Six

Conclusion

6.1 Introduction

The motivation for the intervention on which this research is based came from the desire to raise the achievement of the students within All Saints School who were finding success in mathematics a challenge. The intervention brought IBL into the mathematics teaching and learning programme within the participating classes. The study design was mixed methods with data collected from PAT, OTJ, student surveys, and student interviews.

Drawing on the results of this study, implications for classroom practice are discussed, limitations to the results highlighted, opportunities for further research outlined, and some general comments about mixed-methods research discussed, before concluding remarks are made.

6.2 Implications of this Study

The gains made by the lower achieving students during this research study suggest that the introduction of IBL into the mathematics teaching and learning programme should be considered as one instructional approach to assist in raising the achievement of lower achieving students. There would be few, if any, schools including high achieving, high decile schools, such as in this study, where there is not some percentage of students who are finding reaching the national standard in mathematics a struggle. In the case of the students in this study that was a very small percentage, with just 10 percent of the students beginning the study with PAT stanines of less than 5. In this high achieving situation, the introduction of IBL had a positive impact on the achievement level of the lowest achieving students; therefore, the introduction of IBL into a wider range of schools could see the raising of student achievement levels on a wider scale.

Initial teacher training would benefit from the inclusion of a section on the IBL approach to mathematics teaching and learning. Student teachers could be introduced to a range of research literature that outlines how to go about implementing this method into the classroom and the potential benefits of this

approach. Given the challenges to implementation outlined in the literature review, student teachers would benefit from opportunities to both view IBL in action and to participate in IBL mathematics sessions.

Teacher professional development programmes could be developed to assist current classroom teachers to introduce IBL into their mathematics teaching practice. The PLD undertaken with the classroom teachers in this study was created for these teachers in this school setting and may not be ideal in all settings. It does, however, offer an approach which PLD providers may be able to adapt and further develop for wider use.

A number of the challenges to the implementation of IBL in mathematics already identified within the literature were encountered in this research also. These implementation challenges, including the time required for the students to learn and get comfortable with a new approach to teaching and learning, argue for an extended timeframe to be allowed for PLD if it is to be successful in embedding IBL and producing the increased achievement levels desired.

The apparent success of IBL suggests that real life rich mathematical tasks should be included widely in mathematics teaching programmes. The situating of mathematics in action, rather than the classroom, offers potential benefits in improved understanding of the place of mathematics within the wider society and for improved student attitude to the subject.

6.3 Limitations

As outlined in Chapter Three this study was conducted with limited numbers due to the size of the school in which the research project was based. The limitation in numbers and the further limitation of a single researcher, means that the results need to be treated with some care.

The fact that the research was all conducted within one school, due to the researcher being employed in that school as a classroom teacher, places some limits on the generalisability of the results. However, as Cohen et al. (2007) suggest, it is possible to consider the typicality of a situation, thereby suggesting that results may be generalisable to similar settings. This suggests that the results found here may be generalisable to other high decile urban schools within New Zealand.

The situation of research studies in schools allows researchers to determine how specific aspects of both existing and proposed new approaches to teaching and

learning affect students, teachers, the classroom programme, and student achievement in the real life day-to-day hustle and bustle of the busy classroom. When the researcher is one of the school's permanent classroom teachers this can have both potentially negative and some positive effects. The main limitation of having a school staff member as the researcher is that they may not notice or think to focus on certain aspects of what is happening due to familiarity blindness: 'this is just how it is around here', or 'just the way things are done at xyz school'. On the positive side, undertaking research within your own classroom and school allows classroom teachers to step back from the day-to-day busyness of teaching and consider specific aspects of their programme and teaching style and determine how they impact on the students and student achievement. Undertaking research also allows teachers to continue to update and improve their skills, and try new things, all while taking an in-depth look at the influence these things have in the classroom environment, on teaching and learning, and on student achievement.

6.4 Opportunities for Further Research

The small numbers of participants involved in this study and the single school setting mean that the research questions focused on in this study would require further investigation across a wider range of high SES high-achievement schools. Further research should also be undertaken on a wider number of schools with a range of SES and achievement characteristics.

Given that IBL was new to both the teachers and the students, together with the potential challenges to the successful implementation of IBL (2.5.3), it would be beneficial to conduct follow-up studies to determine the impact of IBL on participating students at All Saints School over a longer time period. As both students and teachers improve their familiarity and confidence with IBL and develop their skills in using IBL in their mathematics programme, there is the possibility that this could create greater gains in the coming years and this warrants further study.

This study documented a very large rise in students' problem-solving confidence. It would be instructive to have a further study addressing whether this rise in confidence is in fact accompanied by a corresponding rise in problem-solving ability.

The use of mixed ability grouping and its impact on the achievement levels of the high achieving students in an IBL approach also warrants further investigation.

The marked difference in the achievement levels of the lower achieving students needs to be examined to ensure that it is not happening at the expense of their higher achieving classmates and that these achievement gains can be sustained over time.

6.5 Mixed Methods Research

Undertaking a study using a mixed-methods design allows for the gathering of a greater range of data. The use of both qualitative and quantitative methods within the one study allows for the collection of both achievement and attitudinal data giving a fuller, richer understanding of the impact of an intervention. In the current study the use of just the quantitative data would have left the researcher without the understanding and insights about how positive the students are towards mathematics. Also missing would have been the evidence of the improving attitude of the students towards challenge, struggle, and mistakes. To have undertaken the research without the quantitative data would have left the teachers without the valuable insight into the significant progress made by the underachieving students during the intervention. Again, this is valuable data for the classroom teachers and allows them to continue to develop programmes that will benefit the students who have previously found mathematics success a challenging goal to ascertain.

6.6 Concluding Comments

This intervention was undertaken in response to the learning encountered by the researcher during postgraduate study. The desire was to see a rise in the achievement levels for the students who were having difficulty succeeding in mathematics. In respect of closing the gap between highest and lowest achieving students the results exceeded the expectations of both the researcher and the participating classroom teachers. This study further demonstrates the value of the inclusion of IBL for underachieving students in schools that already have high overall achievement statistics. The study adds to the growing wealth of literature which argues for the inclusion of Inquiry Based Learning within the mathematics programmes in all schools.

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Appendices

Appendix A

List of studies examined for the impact of IBL on Student Achievement

Author	Student Grade	Date	Student numbers	Achievement Gains for IBL Y/N
Olander & Robertson	4	1973	374	Y
Carpenter, Fennema, Peterson, Chiang, & Loef	1	1989	480	Mixed
Hiebert & Wearne	2	1993	135	Y
Villasenor & Kepner	1	1993	144	Y
Fennema, Carpenter, Franke, Levi, Jacobs, & Emerson	1-3	1996	14 classes	Y
Newman, Marks, & Gamoran	4-10	1996	2128	Y
Higgins	6 & 7	1997	18	Y
Boaler	Year 9-11	1998	310	Y
Schorr	Not Specified	2000	12	Y
Manswell Butty	10 & 12	2001	364	Mixed
Mistretta	3-6	2005	578	Y
Boaler	14-18 year olds	2008	700	Y
Cotic & Zuljan	9 year olds	2009	179	Mixed
Rakes, Valentine, McGatha, & Ronau	Middle school – high school	2010	22,424	Y
Shayer & Adhami	1	2010	275	Y
Erbas & Yenmez	6	2011	134	Y
Taylor & Bilbrey	5	2012	1210	Y
Kogan & Laursen	College	2014	3212	Y
Morgan, Farkas, & Maczuga	1	2014	13,393	N

Appendix B

PAT Scale Score Average Progress

Darr, Neill, Stephanou, & Ferral (2009)

Scale Score Progress							
Mathematics							
<i>Average Scale Score - Term 1</i>							
Yr 3	Yr 4	Yr 5	Yr 6	Yr 7	Yr 8	Yr 9	Yr 10
21.4	30.6	38.9	45.1	49.6	55.0	60.6	65.4
 <i>Average Progress</i>							
Yr 3-4	Yr 4-5	Yr 5-6	Yr 6-7	Yr 7-8	Yr 8-9	Yr 9-10	
9.2	8.3	6.2	4.5	5.4	5.6	4.8	
<i>Scale Score (patm) from Table 6 p.30, Teacher Manual</i>							

Appendix C

Non-routine problem solving example

1. Three Dice

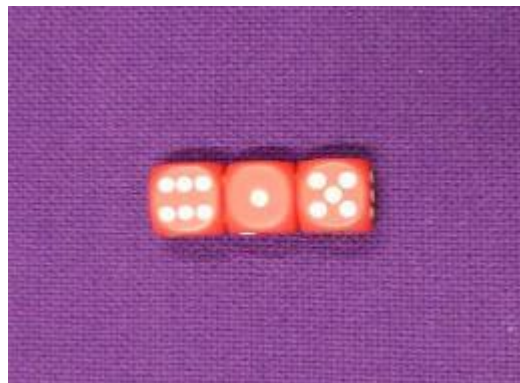
(From the nrich website)

Stage: 2 ★

Take a look at some ordinary dice.

What do you notice about the way the numbers are arranged?

Now look at these three dice in a row:



The numbers on the tops of the dice read 6, 1 and 5.

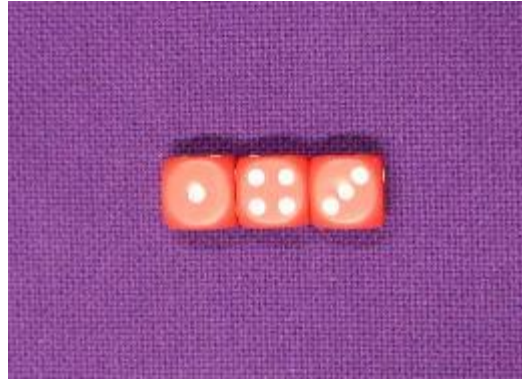
What do the numbers on the top add up to?

Can you use what you found out about the way the numbers are arranged to say what numbers are on the bottom of the dice? Were you correct?

What is the sum of the numbers on the bottoms of the dice?

Let's try that again.

This time the numbers on the top read 1, 4 and 3.



Can you work out the total?

Can you work out the numbers on the bottom and their total?

Try out some arrangements yourself. Each time record the sum of the numbers on the top and the sum of the numbers on the bottom.

Do you notice a relationship between the 'top sum' and the 'bottom sum'?

Can you explain it?

I experimented with arrangements where the top sum is a multiple of three, and find that in each case the bottom sum is also a multiple of three. Is it always true?

I try to arrange the dice so that the top and bottom sums are both multiples of four, but can't seem to be able to do it. Can you? Can you explain what you find out?

On the other hand, if I arrange **four** dice in a row it is easy to make the top and bottom sums both multiples of four. Can you arrange four dice so that the top and bottom sums are both multiples of three? Can you explain what you find out?

An Olympic Event

Preparing for your event

1. **Plan** a one day Olympic style event for your school. **Check** the Olympic.org sports web-site for a full list of sports at an Olympic Games. *How many sports are there at the games?* Now **chose** 3 or 4 events for your Olympic style event: there should be at least 1 running and 1 throwing event in your 3 or 4 chosen events.
2. **Draw a map or plan** of the area you will use for your event.
3. **Allocate a space** for each event so that they will all fit. Make sure you think about how much space you have got to make sure you can have all these events happening at the same time.
4. You might like to organise your event so that each year group is split into different countries so you can compete country against country. You could make flags and banners for your event, have team colours and wear your team colour on the day.

Mathematics Problems for you to solve:

1. How are you going to measure the throwing distances? Try some different ways and **find the most accurate** way of measuring.
2. Given that your running event will need to have heats and a final, **calculate** how long it will take for each year group to complete that event? (Remember some Year groups will have more people in than others and they might need more heats.)
3. How will you work out how long each Year group will need at each event to make sure everyone gets a turn at each event?

Briefly **explain** how you solved the problem (1 to 2 minutes) to the rest of your class explaining the maths you did and how you worked it out. This should be done after you have solved each of the problems in creating your Olympic event (e.g. 3 problems = 3 explanations).





Be **prepared to** answer questions about what you did and how you did it.

Appendix D

All about Maths Survey

Year 3

(Note: questions 12 & 13 in italics were added for the Time 2 survey)

	1) Not at all 	2) A little 	3) Some 	4) A Lot 
1. How much do you like doing maths at school?				
2. How much do you like doing maths on your own time?				
3. How good do you think you are you at maths?				
4. How good does your teacher think you are at maths?				
5. How much do you think maths is useful to you outside of school?				
6. How important do you think maths will be for you when you finish school?				
7. How much do you like helping other people with their maths?				

8. Who helps you with your Maths when you are having trouble? Tick all the people who help you:
- a. Your teacher
 - b. Your friends
 - c. Your classmates
 - d. Your mum or dad
 - e. Older brothers or sisters
 - f. Someone else _____

9. Put a cross on the line to show what you think about maths:

Fun 1 _____ 2 _____ 3 _____ 4 Dull

Easy 1 _____ 2 _____ 3 _____ 4 Hard

Difficult 1 _____ 2 _____ 3 _____ 4 Simple

Exciting 1 _____ 2 _____ 3 _____ 4 Boring

Messy 1 _____ 2 _____ 3 _____ 4 Tidy

Loud 1 _____ 2 _____ 3 _____ 4 Quiet

10. What things do you like doing in maths at school? (tick up to 4)

Using equipment		Problem solving	
Working in my maths book		Explaining my maths ideas	
Maths tests		Listening to others explain their ideas	
Maths games		Maths puzzles	
Working from a text book		Working on my own	
Helping someone else with their maths		Worksheets	

11. Would you like to do more, the same amount, or less maths at school?

(Chose one)



More



The same



Less

12. *What is maths?* _____

13. *Why do we do maths at school?* _____

All about Maths Survey

Year 4 & 6

(Note questions 12 & 13 in italics were added for the Time 2 Surveys)

	1) Not at all	2) A little bit	3) Some	4) A Lot
1. How much do you like doing maths at school?				
2. How much do you like doing maths on your own time?				
3. How good do you think you are you at maths?				
4. How good does your teacher think you are at maths?				
5. How much do you think maths is useful to you outside of school?				
6. How important do you think maths will be for you when you finish school?				
7. How much do you like helping other people with their maths?				

8. Who helps you with your Maths when you are having trouble? Tick all the ones that apply:

- a. Your teacher
- b. Your friends
- c. Your classmates
- d. Your mum or dad
- e. Older brothers or sisters
- f. Someone else _____

9. Put a cross on the line to show what you think about maths:

Fun 1 _____ 2 _____ 3 _____ 4 dull

Easy 1 _____ 2 _____ 3 _____ 4 hard

Difficult 1 _____ 2 _____ 3 _____ 4 simple

Exciting 1 _____ 2 _____ 3 _____ 4 boring

Messy 1 _____ 2 _____ 3 _____ 4 tidy

Loud 1 _____ 2 _____ 3 _____ 4 quiet

Complicated 1 _____ 2 _____ 3 _____ 4 uncomplicated

Confusing 1 _____ 2 _____ 3 _____ 4 straight forward

Useful 1 _____ 2 _____ 3 _____ 4 waste of time

What things do you like doing in maths at school? (tick up to 4)

Using equipment		Problem solving	
Working in my maths book		Explaining my maths ideas	
Maths tests		Listening to others explain their ideas	
Maths games		Maths puzzles	
Working from a text book		Working on my own	
Helping someone else with their maths		Worksheets	

10. Would you like to do more, the same amount, or less maths at school?

(Circle one) More Less The same

11. What things do people who are good at maths do or know that helps them to be good? _____

12. *What is maths?* _____

13. *Why do we do maths?* _____

Appendix E
All about Maths Interview

1. Rate how much you like maths out of ten, if 1 is lowest and 10 is highest.

2. What things do you like about maths? _____

3. What things do you not like about maths? _____

4. If you could change 1 thing about how maths is done in your class what would that be? _____

5. What could your teacher do to help make maths easier for you? _____

6. If you could tell your teacher anything about maths or how you do it in class what would you like her to know? _____

7. Have a look at this maths problem solving question. I don't want you to work out the answer today, but what I would like you to do is tell me if you think you could work out the answer to this question. Why or why not? _____

8. *What do you do if you get stuck in maths?* _____

9. *What is maths?* _____

10. *Why do we do maths? What is the purpose?* _____

Appendix F

Ethics

Information and Consent Forms Students and Parents Victoria University of Wellington

Information for students

Kia Ora,

My name is Mr Shallard and I have been a teacher in your school for over 5 years now, so you will have seen me around the school. You may have me as your teacher this year, or have had me as your teacher in the past.

To help make sure I am the best teacher I can be I am also going to school myself at Victoria University. As part of my learning I want to find out about your learning in mathematics. I want to compare your PAT mathematics score from last year, this year and next year, and I want to see what your teacher says about your mathematics in your end of year report last year and this year.

I will ask you to write your answers to some questions about your learning in mathematics. I might also ask you some questions with a small group of students from your class. I will pick some names out of a hat to choose who I will talk with. I will explain to you what will happen so you can decide if you would like to talk to me about your learning. I don't want to forget anything you tell me. If I talk with you I will record what you say on an audio tape. If you are not in my class this year I will not tell your teacher what you say. Nobody will be worried if you choose not to join in at any time. If you change your mind after we have started you are quite free to say that you "don't want to do this anymore" and nobody will be upset about that. If you do change your mind anything you have said or work you have done that I have collected for my study

will be deleted so that it cannot be used, even by accident. I will have to write a really big recount of what I do, which will be read by my teachers. When I do that I will not tell them your name or school. When I have finished with the information I will delete it from my computer and any papers will be shredded so no-one else can find and read what you have said and done.

Your teacher and Principal are happy to have me do this with you and your class. You do not have to take part. Neither I nor your teacher will be upset if you choose not to.

Mr Shallard

Victoria University of Wellington Consent to Participate in Research

Learning through Inquiry

Student Consent form

Please tick the boxes you agree with.

I am ok to write some answers about how I feel about maths at school.	
I am ok with Mr Shallard talking to me about my learning.	
I am ok with Mr Shallard making an audio (sound) recording in our class and when I talk with him.	
I am ok with Mr Shallard collecting some information about my learning in Mathematics. I know nobody else except Mr Shallard and his teacher will see it.	
I know I can change my mind and not be involved, even after I have started, if I want to. I know if that happens it won't affect how I do in maths and information about me and my maths work won't be included in Mr Shallard's work.	

My name is:

Victoria University of Wellington

Information for parents

Learning through Inquiry

Parent Information Sheet

We are inviting your daughter/son to participate in a research project and asking for your support.

Researcher Introduction

My name is Steve Shallard and most of you will know me as I have been teaching at your child's school for over 5 years now and I may well have taught, or currently be teaching your child. As some of you will be aware, I am the school's mathematics lead teacher and in that role have been working hard to assist our students to achieve to the best of their ability in mathematics. As part of my own ongoing learning and development as a teacher I am undertaking postgraduate study at Victoria University of Wellington.

Project description and invitation

Mathematics is one of the government and Ministry of Education's focus subjects as considerable effort is being directed into resolving student underachievement and raising overall achievement levels across the country. At your child's school we are continually evaluating how we teach in an ongoing effort to ensure the best possible learning outcomes for every student.

As part of that process I am leading this project which focuses on further developing our mathematics teaching and learning programme. This project proposes to introduce an approach to mathematics teaching and learning which has proved highly successful for all students where it is already in use, both within New Zealand and internationally. The proposed research will involve adding (not replacing) a 'learning through inquiry' approach to the teaching style already in use in your child's class. Your child will still be taught every strand of the mathematics curriculum, as they have been since starting school. This project will run for the 2015 academic year. This study will be investigating the effectiveness of this addition to the programme.

Your son/daughters participation in the study at class level will involve:

- Analysis of data from your child's normal PAT taken at the beginning of 2014, 2015 and 2016
- Analysis of National Standards data from the end of 2014 and 2015
- Use of your child's PAT and National Standards data to assist in the selection of students from a range of achievement levels for interviews about their attitude to maths
- Completing a mathematics problem solving task at the beginning and end of the school year, 2015 designed to measure students' self-efficacy.
- Completing a survey on their attitudes and beliefs about mathematics and mathematics learning at the beginning and end of the year, 2015.

We are asking your approval for your daughter/son to participate in all the above aspects of this research and for permission to use data, as identified above, about

your child's achievement that is routinely collected by the school, in analysis for the project.

All children in the classes involved in the study will continue to receive the best possible teaching we can provide across the entire mathematics curriculum. If you choose not to give consent to your child's involvement this will in no way whatsoever impact on either the teaching they receive or the grades that they will be given in their mid-year and end of year reports.

If you give consent for your child to be involved and at a later point change your mind and wish to withdraw your child from the study you will be able to do this by simply informing me or my supervisor of your desire to do so. At that time your child's involvement will end and any data or information given by your child, or about your child, will be permanently deleted from the research information.

In addition, we would like to find out how children with varying degrees of achievement in mathematics respond to the use of this new approach. To assist in this aspect of the study, up to six students from each participating class will be selected, based on PAT results, to participate in audio taped interviews, again at the beginning and end of the year. Interviews will inquire into children's beliefs about their ability in mathematics, how they think they best learn mathematics, and about the usefulness of mathematics to them outside of school. If your child is selected as a potential participant for interview you will be informed of their selection given a further information sheet related to the interviews, told why they have been chosen and asked for your consent to their participation in the interviews.

Participant identification

The research will be confidential and the participants and school will not be identified by name either in result summaries or in the researcher's thesis work. In all analysis and reporting pseudonyms will be assigned to participants, classes, teachers and the school to ensure confidentiality. All data and responses will be stored in password protected files and secure locations and will be destroyed at the completion of the research project.

Dissemination of results

The findings will be used by the researcher in the preparation of a Masters Degree thesis. As a requirement for that degree the thesis will be made available through the university library. There is also the possibility that the findings may be used in the publication of articles in academic journals. I will provide the school and yourselves, the parents, with a summary of the findings.

This research has been approved by the Faculty of Education Human Ethics Subcommittee under delegated authority from the Victoria University Human Ethics Committee, (Reference number, 21444). If you have any ethical concerns about the research you should contact Dr Allison Kirkman (Allison.kirkman@vuw.ac.nz), ph 04 463 5627, Chair of the Human Ethics Committee, Victoria University of Wellington.

Signed

Steve Shallard

Victoria University of Wellington Consent to Participate in Research

Learning through Inquiry

Parent [care givers] Consent Form

I have read the information about the research project: Learning through Inquiry
I understand what would be required if I agree for my daughter/son to participate in the research.
I understand that my child is participating in this research voluntarily and that he or she may withdraw from it at any time.
I understand what would be required if I agree to my daughter/son being involved in an interview.
I understand that my child's and the school's identity would be protected and audio recordings will not be used in any presentations.
I understand that all data would be stored in password protected files and locked cabinets and will be destroyed on completion of the research project.
I understand that my child's responses during interviews may be used anonymously in the thesis.
I understand that the data may be used anonymously in the publication of results in academic journals.

Please tick the box as appropriate:

I consent to my daughter/son participating in this research	Yes	No
I consent to my daughter/son participating in a short interview	Yes	No

Signed: _____

Date: _____

Name: _____

Appendix G

Problem Solving Tasks used in Student Interviews

Year 3

Well, Well!

Freddo has had a nice swim in the bottom of the well and decides



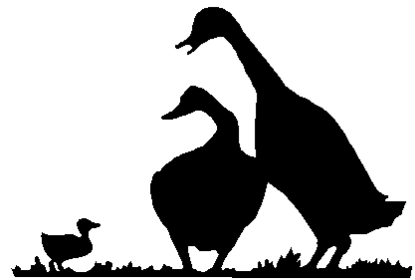
that now is the time to get out. Freddo climbs 3m up the wall of the well and then rests. But the wall is slippery and he then slips down 1m. He is so tired

he goes to sleep for the rest of the day. The next day he does the same thing. Climbs up 3m, slips back 1m, and goes to sleep. In fact, he does this every day until he gets out of the well. Now the well is 13m deep. How long does it take Freddo to climb out of the well?

Year 4

Pigs and Ducks

Jennie the old
sheep dog



is lazing around in the
paddock near the house.
She counts the number of
animals in the paddock.
There are 11 of them, pigs
and ducks. Then her eye
runs over the legs. She sees
28 legs. How many ducks
are there?

Year 6

Lollies!

On Monday, Sam and Sylvia shared some lollies that their Mum had



given them. Sam got 2 lollies. Sylvia got 4 lollies.

How many lollies did they have to share?

If their Mum gave them each the same number of lollies every day up to (and including) Wednesday, how many lollies did they each get?

Appendix H

Student Shift Data Sample

The charts explained

These charts show the student movement between time 1 and Time 2. The bold data is the percentage of students who selected the same answer at Time 2. The other data in each column shows the percentage of students who changed their response and what they changed it too. The numbers in parentheses show the standard error of the proportion. The bottom row shows the number of students who selected that response at Time 1.

How much do you enjoy maths in your own time?

Table I 1.1 (Figure 4.2 & Chart 4.1)

Year 3		Percentage of Students rated at Time 1			
		Not at all	A little bit	Some	A lot
Percentage of students rated at Time 2	Not at all	0	25 (0.217)		
	A little bit	67 (0.271)	0	50 (0.354)	100 (0)
	Some	33 (0.271)	25 (0.217)	0	
	A lot		50 (0.25)	50 (0.354)	0
	<i>n</i>	3	4	2	1
Year 4		Percentage of Students rated at Time 1			
		Not at all	A little bit	Some	A lot
Percentage of students rated at Time 2	Not at all	0	25 (0.217)	-	-
	A little bit	-	0	-	12.5 (0.115)
	Some	-	75 (0.217)	67 (0.192)	50 (0.177)
	A lot	-		33 (0.192)	37.5 (0.171)
	<i>n</i>	0	4	6	8
Year 6		Percentage of Students rated at Time 1			
		Not at all	A little bit	Some	A lot
Percentage of students rated at Time 2	Not at all	0		12.5 (0.115)	
	A little bit	60 (0.219)	60 (0.219)	37.5 (0.171)	
	Some	40 (0.219)	40 (0.219)	37.5 (0.171)	25 (0.217)
	A lot			12.5 (0.115)	75 (0.217)
	<i>n</i>	5	5	8	4

