

**Sociomathematical Worlds:  
The social world of children's mathematical learning  
in the middle primary years**

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## **ABSTRACT**

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This thesis presents the findings of a project that explored the ways in which primary school children developed understandings about mathematics, mathematical 'learning' and 'knowing' and themselves as learners of mathematics. The research aimed to describe the children's mathematical learning environments, to explore the ways in which children made meaning about mathematics through social interactions within these environments, and to identify elements of these environments that appeared to enhance or inhibit the children's learning of mathematics.

Located within the body of literature that takes a sociocultural view of teaching and learning, the study adopted the theoretical framework of symbolic interactionism because of its usefulness in explaining how, through the social interactions of everyday life, an individual constructs and reconstructs personal versions of 'reality', including a sense of identity. Through this lens, familiar objects, routine events and everyday language surrounding the teaching and learning of mathematics were examined for their significance to young learners.

The concept of the *sociomathematical world* was created and developed to describe the mathematical environment of the child as positioned within wider social networks. The sociomathematical world of the child was seen as the world of everyday life, the arena in which the child, through regular and routine interactions with others, negotiated meanings about, and made personal sense of, mathematics.

The research focussed on ten case study children – four girls and six boys – all attending different schools, and selected randomly from the primary schools in the Wellington region of New Zealand. For three years, from the beginning of their third year at school to the end of their fifth, the children were regularly interviewed and observed in their classrooms. Other key participants in their sociomathematical worlds were also interviewed, including families, teachers, principals, and classmates. Evidence of teaching and learning was also gathered from children's books and assessment records, and linked to local and global curriculum documentation.



A cumulative picture was compiled of the mathematical teaching and learning environments of these ten children. Originally intended to be presented as separate biographies, the data were instead collated and reported according to the four distinctive recurring themes that emerged from the findings:

- the emphasis of speed in mathematics teaching and learning;
- identification and differentiation based on socially constructed perceptions of mathematical 'ability';
- the establishment of 'doing maths' as solo written work;
- the presentation of mathematics as consisting of 'correct' and non-negotiable facts and procedures.

These dominant approaches to teaching and learning of mathematics were found to conform to deeply entrenched traditions, in which the learner was viewed as the passive recipient of, rather than an active participant in, education in general and mathematics education in particular. It was found that these taken-for-granted pedagogical cultures were not explicitly supported by the official curriculum.

Marked negative effects of these common teaching practices were commonly observed: alienation, marginalisation and impoverished learning. These impacts were experienced in varying forms and at varying times, by all the case study children, suggesting that changed views of mathematics and of mathematical teaching and learning are needed if the learning potential of all children is to be fully realised.

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## CONTENTS

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<b>Abstract</b> .....	ii
<b>Acknowledgements</b> .....	iv
<b>List of Figures</b> .....	ix
<b>List of Tables</b> .....	x
<b>Chapter 1 Introduction: The sociomathematical world of the child</b> .....	1
<b>Chapter 2 Learners' perspectives in mathematics education research:</b>	
<b>A review of the literature</b> .....	14
An overview of research trends.....	16
Researching attitudes.....	17
Attitudes and mathematics the lone learner .....	20
Limitations of traditional attitude research.....	26
The affective domain – the learner as a social being .....	28
Classroom culture – the learner as a culturally situated being.....	33
Social world theory, symbolic interactionism and mathematics – the learner as interactive sense-maker.....	38
Accounts of the learner – the New Zealand research .....	39
Student attitudes, affect and recent curriculum trends .....	44
Conclusion .....	48
<b>Chapter 3 Methodology: Investigating sociomathematical worlds</b> .....	51
<b>Theoretical frameworks</b> .....	52
<i>Symbolic interactionism</i> .....	52
<i>Sociology of everyday life</i> .....	57
<i>The social world of the child</i> .....	58
Range of measurement .....	60
<i>Webs of significance</i> .....	62
<i>Longitudinal ethnographic case study approach</i> .....	65

	<i>Classroom mathematics as ‘culture’</i> .....	66
	Methodological rationale and research approach .....	67
	<i>Setting up the research</i> .....	70
	<i>The sample</i> .....	72
	<i>Maintaining continuity</i> .....	74
	Gathering the data .....	76
	<i>Working with informants</i> .....	76
	<i>Archives</i> .....	87
	Analysis and interpretation .....	89
<b>Chapter 4</b>	<b>The speed pandemic: Creating mathematical ‘winners’ and ‘losers’</b> .....	91
	<i>Situation 1: Ritual Scene-setting</i> .....	91
	<i>Situation 2: The Silent Trial</i> .....	96
	<i>Situation 3: The Solo Performance</i> .....	99
	<i>Situation 4: Mathematical combat</i> .....	101
	Speed Activities .....	103
	<i>Speed activities as part of everyday life</i> .....	108
	<i>‘Making the familiar strange’</i> .....	112
	The children’s responses .....	114
	Speed activities and pressure .....	120
	The teachers’ perspectives .....	123
	School policy .....	125
	The home dimension .....	126
	Origins of the speed pandemic.....	128
	<i>Promotion of speed activities through commercial texts</i> .....	130
	<i>Speed in the official mathematics curriculum</i> .....	131
	<i>International perspectives on speed activities</i> .....	133
	<i>Exposing the taken for granted</i> .....	134
	<i>Speed activities, power and control</i> .....	135
	<i>Speed activities as part of everyday life: Resistance to change</i> .....	138
	Conclusion .....	140

<b>Chapter 5</b>	<b>Top, middle, bottom: The social construction of mathematical ability</b>	
	ability .....	141
	Mitchell: ‘Behind the eight ball’ .....	142
	Jessica: ‘Average is a nasty word’ or ‘A middle kid’ .....	154
	Rochelle: ‘A super smarty pants’ .....	160
	Children’s experiences of being sorted .....	164
	Teachers’ explanations of grouping systems .....	165
	Teacher’s views of the children .....	169
	Teacher’s beliefs about ability .....	171
	Parents’ views of their children .....	173
	Children talk about their ability.....	175
	Defining and identifying ‘ability’ .....	180
	Ability as a social construct .....	184
	School Policy.....	186
	Conclusion .....	187
<b>Chapter 6:</b>	<b>‘Doing maths’: establishing a mathematical work ethic.....</b>	<b>189</b>
	Children’s views of ‘doing maths’ .....	189
	Maths time .....	195
	Doing maths – the typical lesson .....	196
	Maths books .....	200
	‘Finishing’ .....	205
	Working Solo .....	207
	Working from textbooks .....	212
	Worksheets .....	220
	Working from the board .....	224
	Concrete materials .....	225
	School policy .....	230
	Homework .....	231
	The curriculum view .....	233
	Children’s responses to ‘doing maths’ .....	234
	Conclusion .....	236

<b>Chapter 7: The right/wrong dichotomy: making sense of mathematics</b> .....	239
The dominant approaches to new learning .....	239
Jessica's story: The tree diagram lesson .....	240
Dominic's classroom: Closed questions/right responses .....	243
Peter's story: One right way .....	246
Georgina's story: The algebra lesson .....	250
Reinforcing the 'right' way .....	254
Working form: the 'right' way to calculate .....	257
Right and wrong: creating high risk classroom environments .....	263
The universal language of mathematics? .....	267
Teachers' views of children's learning and knowing of mathematics.	270
Children's perspectives of learning and knowing mathematics .....	272
The place and purpose of learning mathematics.....	272
Children's views of the nature of mathematics .....	278
The home dimension .....	281
Teachers' views of mathematics .....	282
Mathematical identity .....	285
Conclusion .....	287
 <b>Chapter 8</b> <b>Conclusions: Interpreting sociomathematical worlds</b> .....	288
What does the sociomathematical world of the child look like? .....	288
The child's negotiation of meaning .....	291
Enhancing or inhibiting the child's learning of mathematics .....	293
Implications of the research .....	295
Limitations and suggestions for further investigation .....	299
 <b>References</b> .....	301
 <b>Appendices</b> .....	321
Appendix 1: Schools' information letter.....	321
Appendix 2: Sample summary letter .....	323
Appendix 3: Principal interview questions .....	325
Appendix 4: Parent interview questions .....	326

Appendix 5: Child interview questions .....	330
Appendix 6: Questionnaire sheet sample – <i>How I feel about maths</i> .....	333
Appendix 7: Teacher interview questions .....	334
Appendix 8: Child self-assessment sheets for classroom use – sample .....	337
Appendix 9: Basic facts speed test sample .....	338
Appendix 10: Children’s drawing sheet – sample .....	339
Appendix 11: Children’s self-rating – enjoyment and ability .....	340

### List of Figures

Figure 1: The sociomathematical worlds of the child within its wider context .....	64
Figure 2: Questions on the board, Pukeiti School, Mid Year 4 .....	91
Figure 3: Cards for speed game .....	102
Figure 4: Page from the back of Fleur’s maths book, Pukeiti School, Early Year 5 ...	111
Figure 5: Characteristic patterns of career orientation, strategies and adaptation (Reproduced from Pollard and Filer, 1999) .....	147
Figure 6: Jared’s drawing, Early Year 3 .....	190
Figure 7: Peter’s drawing, Early Year 3 .....	190
Figure 8: Toby’s drawing, Early Year 3 .....	190
Figure 9: Rochelle’s drawing, Early Year 3 .....	190
Figure 10: Georgina’s drawing, Early Year 3 .....	191
Figure 11: Dominic’s drawing, Early Year 3 .....	191
Figure 12: Fleur’s drawing, Early Year 3 .....	191
Figure 13: Jessica’s drawing, Early Year 3 .....	191
Figure 14: Mitchell’s, drawing Early Year 3.....	191
Figure 15: Liam’s drawing, Early Year 3 .....	191
Figure 16: Mitchell’s drawing, Early Year 4 .....	192
Figure 17: Peter’s drawing, Early Year .....	192
Figure 18: Dominic’s drawing, Early Year 4 .....	192
Figure 19: Liam’s drawing, Early Year 4 .....	192
Figure 20: Fleur’s drawing, Early Year 4 .....	192
Figure 21: Rochelle’s drawing, Early Year 4 .....	192
Figure 22: Jared’s drawing, Early Year 4 .....	193
Figure 23: Toby’s drawing, Early Year 4 .....	193

Figure 24: Georgina's drawing, Early Year 4 .....	193
Figure 25: Jessica's drawing, Early Year 4 .....	193
Figure 26: Timetable on the black board, Motu School, Late Year 5 .....	195
Figure 27: Sign on the Keeping Skilful Box, Bridge School, Mid Year 5 .....	208
Figure 28: Textbook exercises set by the teacher, River School, Late Year 5 .....	213
Figure 29: Tessellation worksheet, Spring School, Late Year 5 .....	222
Figure 30: Tree diagram , Roto School, Mid Year 5 .....	240
Figure 31: Tree diagram 2, Roto School, Mid Year 5 .....	241
Figure 32: Blackboard tasks, River School, Early Year 4 .....	243
Figure 33: Reflection of a trapezium, Beach School, Late Year 3 .....	248
Figure 34: Georgina's representation of counting numbers .....	251
Figure 35: Mr Solomon and the others' representation of counting numbers .....	251
Figure 36: Toby's 'piling', Mid Year 4 .....	258
Figure 37: Fleur's 'renaming', Late Year 4 .....	260
Figure 38: Rochelle's 'working form', Early Year 5 .....	260

### List of Tables

Table 1: The annual research timetable .....	76
Table 2: Speed activities experienced by the children, Years 3, 4, & 5.....	111
Table 3: Groupings experienced by the children, Years 3, 4, & 5.....	164
Table 4: Type and frequency of mathematics 'work' experienced by the children, Years 3, 4, & 5 .....	199
Table 5: Children's perceptions of work modes: frequency and preference .....	226
Table 6: School subjects the children believed to be the most important, Years 4, & 5.....	275



## CHAPTER 1

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### INTRODUCTION: THE SOCIOMATHEMATICAL WORLD OF THE CHILD

As I sat near a group of seven year olds who had just finished their mathematics lesson for the day, Georgina turned to me and remarked:

*I hate maths. I hate it because when we do tests, I only get three or four or five, because they're really hard.*

This was one of the first of my many research visits to classrooms, and at this stage I was not sure what Georgina meant by 'three, or four or five' but her statement conveyed in a powerful way, a number of significant insights into how this particular child *viewed* her own learning of mathematics:

- she was stating her *feelings* about mathematics;
- she was *linking* her feelings about mathematics generally, to her regular underachievement in a specific event in the mathematics lessons;
- she was *attributing* her failure in the event to its difficulty.

In speaking of 'three, four or five', she was assuming a *shared understanding* of a situation so apparently universal and mundane as to require no explanation.

This glimpse into one child's experience of the learning of mathematics, provided insights into her *social world*, a world in which she *constructed meanings* and *made sense* of *taken-for-granted* routines and patterns of *everyday life*. It was direct accounts such as these that were to guide the inquiry for the following three years of the research. What was it about her experiences of mathematics that had brought about such a strong sense of alienation in this young child?

The research was initially concerned with children's *attitudes* associated with the learning of mathematics. Connections between attitudes towards mathematics and achievement in the subject, had been of interest to mathematics educators and researchers for many decades, particularly where negative attitudes had been observed to accompany underachievement. In reporting the New Zealand results of the Third International and Mathematics and Science Study (TIMSS) which investigated the achievement, attitudes, and environmental factors in the mathematical learning of eight and nine year old children, Garden (1997) noted:

While a majority of students have positive attitudes to learning mathematics...beginning from a fairly young age, there is an increasing proportion of students having lost interest in the subject, with a concomitant decline in their achievement. This effect is considerably greater for girls than for boys. (p. 252)

It appeared that very few studies had attempted to explore and record in any detailed manner, why and how such attitude/achievement links might develop and why such a decline might be occurring. With this gap in understanding in mind, the current investigation was conceived. A random sample of ten children was chosen, and for a period of three years, from the beginning of their third year at primary school, to the end of their fifth, the mathematical learning of these children was investigated. This stage of their schooling was chosen for two reasons:

- Year 3 is a significant transition point from junior primary school mathematics to middle/upper primary school mathematics;
- The TIMSS report revealed that a proportion of children in their fourth and fifth years at school were already exhibiting 'negative attitudes' towards mathematics, so a longitudinal investigation would need to begin with children prior to Year 4.

Initially, the guiding questions of the inquiry were:

- How do children become aware of mathematics in their everyday lives?
- How and why do their attitudes towards mathematics develop and evolve over time?

The study aimed to compile intimate biographies of each child's emerging and changing feelings and beliefs about mathematics as they pursued their individual

'careers' as primary school students. It was hoped that those aspects of their mathematical experiences that contributed to the development of their 'attitudes' to mathematics might be revealed by documenting the routines and events of everyday life that were associated with mathematics.

From the outset, *attitudes* proved difficult to define and even harder to gauge or measure. How were attitudes to be distinguished from beliefs, feelings or values for instance? What questions or observations would most accurately reveal the child's attitudes? Why, in answer to questions about how they felt about mathematics, did the responses of the children change from visit to visit? Was *attitude* therefore the best lens through which to view the children immersed in their daily experiences of mathematics?

During the early stages of the research, classroom observations began to reveal a number of distinguishing features of the everyday delivery of mathematics across the wide range of schools included in the study. These characteristics comprised what appeared to be a shared ethos of mathematics teaching. This commonly held philosophy later came to be regarded as constituting *dominant approaches* to the teaching of mathematics.

While each of the children responded in unique and personal ways to their experiences of mathematics teaching and learning, their accounts exposed several important common themes. All of the children reported varying degrees of discomfort as a result of specific classroom practices, particularly those that placed the children under undue pressure or separated them by 'ability'. Their discomfort ranged from mild and occasional, to severe and frequent. Most of the children had developed restricted views of the nature and purpose of mathematics, views not considered as being consistent with the aims of *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992). For every one of the children, the formidable tasks of memorising their 'times tables' and formal written calculation procedures, loomed large in their mathematical worlds during these three critical years of their learning careers.

These responses were later categorised as *alienation*, *marginalisation* and *impoverished* learning.

Turning from its initial focus on attitude development, the inquiry was reframed to take greater account of the social environments of the children, particularly the daily routines and 'ritualised' actions from which they constructed meanings about what mathematics was, why it was taught at school, how it was learned, and their own and others' mathematical achievement. The research sought to describe the mathematical 'worlds' of the children, to explore the events and interactions within those worlds that were significant for the child, and to identify aspects of those worlds that appeared to assist or inhibit the child's learning of mathematics. These worlds were later named *sociomathematical worlds*. The redefined research questions were:

- What does the sociomathematical world of a child 'look' like, in particular, what are the significant interactions, objects and views of 'self' (mathematical identities) peculiar to that world?
- How do the interactions within that world contribute to the child's negotiation of meaning about that world?
- What aspects of that world appear to enhance or inhibit the child's learning of mathematics?

The thesis describes a significant period in the mathematical learning careers of these ten primary school children, careers that began many years before I met them, and that will continue in various forms, for the rest of their lives. The four girls and six boys, randomly chosen from ten different schools in the Wellington region of New Zealand, were visited regularly in their classrooms. They were provided with a broad range of methods by which they could express their ideas and feelings about their lives in general and about their experiences of mathematics in particular. By means of participant observation and videotapes, they were viewed in their classrooms during mathematics lessons, and other school activities. To enable the children to present personal accounts of their lives and the place of mathematics within them *in their own way*, they were interviewed using open-ended questions, asked to draw themselves doing maths and given a simple questionnaire to complete. Multiple perspectives of

the children and their environments were gained through formal and informal interviews with the children's teachers and parents and in all but one case, their homes were visited. Evidence of their mathematical learning was gathered from teachers, parents, work samples, classroom observations, and from the children themselves.

By the end of three years, I had become a familiar figure to these children, their families and the school staff. In many instances, the children's peers also came to recognise me and readily volunteered their feelings and beliefs about mathematics. As a result of this long period of contact, compelling evidence had been gathered identifying taken-for-granted everyday classroom practices as powerful signifiers in the processes by which the children constructed meanings and understandings about mathematics.

It was discovered, for example, that the *tests* to which Georgina was referring, occurred every day at the beginning of each mathematics session. These tests followed a similar format day after day. In addition, a formal school-wide speed test was administered each month. The daily questions were selected by the teacher, were mostly free of any meaningful context for the child, were called out at speed, and were expected to be answered without use of concrete aids or discussion with others. The marking process involved public exposure and great potential for embarrassment and humiliation. Right answers were rewarded with praise, ticks, stickers and admiration from peers, but the process of producing or justifying them, was not. For Georgina, this regular routine not only created for her a fear and a dislike of mathematics, but also resulted in impoverished learning since it made no discernible positive contribution to her memorisation or recall of the basic facts it was testing, nor to her understanding of mathematical ideas.

Because Georgina was identified as lagging behind her peers, she was placed in the bottom group, often with younger children. She was acutely aware of this marginalisation, and longed to be included with her age-group. Evidence was gathered of the *coping strategies* that Georgina employed in order to survive this everyday ordeal, and the *mathematical identity* she was developing as a result of her experiences. When Georgina transferred to another school at the beginning of Year 4,

her teacher permitted the use of a written array of the times tables during the daily basic facts tests. This small change significantly eased the pressure of the activity for her with a corresponding reduction of her feelings of alienation, and increased confidence. However, Georgina's father was frustrated at her inability to memorise mathematical facts since he recalled having no difficulty learning them as a child. He was keen for his daughter to succeed and tried to help her by testing her at home and playing games such as *Yahtzee*<sup>1</sup> with her, in the hope that this would develop her computational skills.

By the end of her fifth year at school, the gap between Georgina and her peers had not closed. Georgina had concluded that maths was generally 'hard', 'boring' and unpleasant, since much of what went on in the classroom, and reinforced at home, was for her inaccessible and excluding. It seemed to her that the chief purpose of learning mathematics was to enable adults to help their children answer mathematical questions. Georgina was not optimistic about her chances of success in mathematics in later schooling, or of the likelihood of choosing a career that required mathematical expertise.

Georgina is certainly not alone. Her story resonates with those who, as adults, recall their own uncomfortable or impoverishing, experiences of learning mathematics at school. This research illustrates that it is the interaction between the child's unique 'self' and the child's social experiences of mathematics in everyday life, particularly the classroom, that results in the construction of significant beliefs and feelings about mathematics, and the emergence of mathematical identity.

These findings, corroborated by the work of other researchers, have led to a number of conclusions. The thesis argues that dominant approaches to mathematics teaching, (Bishop, 1991; Dowling 1998, Ernest 1998a,) embedded within the traditional view of education as something we do *to* pupils, not *with* them, (Pollard & Filer, 1999) result in an impoverished, alienating, and marginalising learning experience for many young people (Boaler, 2000; Walkerdine, 1998; Zevenbergen, 2002). The thesis provides evidence for the view that mathematics teaching is commonly founded on an

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<sup>1</sup> *Yahtzee* is a game of probability and strategy using two dice. It requires addition and multiplication to calculate scores.

epistemology in which mathematics is believed to be an external body of knowledge, an absolute discipline consisting of facts and rules that subordinates process to outcome. This view is embraced and endorsed by the wider community, including parents, textbook designers and curriculum developers. The accompanying pedagogical practices favour transmission over inquiry, reward product over process and speedy response over careful deliberation, value abstract over concrete and numerical over spatial, view learning as hierarchical, and hold individuals ultimately responsible for their own achievement.

These practices fail to take account of the diversity of perceptions, needs, and feelings of children, at the same time contributing to children's constructions of meaning about mathematics itself, about how it is learned, and about their own mathematical capabilities. Such meanings begin to develop at a fairly young age and become entrenched with continued exposure.

The distinguishing characteristics of these dominant approaches have been identified as:

- pressure in the form of time constrained answering, tests, competition and public exposure routinely applied to motivate and/or manage learners (Anderson & Boylan, 2000; Forgasz, 1995);
- assessment based on an hierarchical view of learning stages leading to differential grouping and teaching by 'ability' where the concept of ability is reified and viewed as an innate and fixed quality of the learner (Boaler, 2000; Zevenbergen, 2002);
- compulsory learning modes consisting primarily of narrowly prescribed written exercises which each child was required to complete alone. (Doyle, 1988; Hiebert et al 1997);
- pedagogy based on transmission of non-negotiable rules and facts (Lave, 1988).

Mathematical learning is embedded within a prevailing *traditional view* of education in general as something that is done *to* pupils, rather than *with* them. The traditional view of education has been described by many researchers. (Davies, 1982; Goodnow



& Burns, 1985; McLaren, 1994; Pollard & Filer, 1999; Woods, 1983). Its distinguishing features are as follows:

- the teacher decides what is to be learned, based upon the directives of higher authorities who have deemed such learning to be in the best interests not only of the learner, but of the 'society' and the 'nation';
- the learner learns most effectively by attentively listening to and watching the teacher; learning is therefore the responsibility of the learner;
- learning is consolidated through repetitious *solo* practice on tasks set by the teacher;
- the learner's achievement and consequently ability, is judged, by both the learner *and* the teacher, on how faithfully the learner can reproduce the views, knowledge and methods of the teacher.

The dominant approaches to mathematics teaching are embedded within the traditional view of the learner in the following ways:

- the teacher decides what mathematical rules and facts the learner should learn, and when they should learn them. The learner is rarely consulted about this. (Dowling, 1998; Lave, 1988);
- the learner listens to and watches the teacher's explanations and demonstrations. Because the teacher already 'knows' the answers, the many questions the teacher asks during this process (I-R-F/triadic dialogue – Edwards & Mercer, 1987; Lemke, 1990) serve to test the children's reception of the rules and facts, and focus the learners' attention on the demonstration rather than stimulate genuine investigation or enhance *relational understanding* (Skemp 1978) of the mathematical ideas;
- the learner is given tasks by the teacher. These are designed to consolidate the learning - most often worksheets or pages of the textbook, sometimes games or practical activities. The learner's job is to work on these tasks alone. Seeking help indicates poor listening or lack of mathematical ability. Failure to complete the tasks indicates poor work habits, poor listening, or lack of mathematical ability;
- achievement is judged by 'right' (according to the teacher) answers and 'correct' (according to the teacher) procedures;



- pressure, through time constraints, competition and exposure is routinely exerted by the teacher throughout the learning/assessment process ostensibly to motivate, but also to manage students' learning.

This view was evidenced by the general lack of inclusion of children's perspectives in school planning and curriculum delivery, lack of attention to, awareness of, or concern about the social impacts of common classroom practices, and the lack of student choice in the learning process. Parents and policy makers were seen to embrace and endorse these views of education, of mathematics and of mathematical learning. Everyday classroom practices were seen to reflect school policy, community expectations and teachers' personal beliefs rather than the teaching approaches advocated by the national mathematics curriculum document, *Mathematics in the New Zealand Curriculum*.

Impoverishment, alienation and marginalisation are seen as the undesirable social and psychological impacts of these dominant approaches to mathematics teaching and the traditional view of the learner. They are identified as follows:

Indicators of *impoverished learning* in mathematics:

- the learner's understanding of mathematical ideas is limited to instrumental rather than relational understanding (Skemp, 1978);
- the learner has difficulty retaining mathematical ideas, routines, or facts, because of limited understanding (Harries & Spooner, 2000);
- the learner develops a narrow range of skills, e.g. computational procedures and recall of basic facts, and believes that mathematics is mostly concerned with numbers, facts, and rules;
- the learner's personal interests are not developed or built upon through mathematics lessons - i.e. contexts chosen by teacher or supplied by textbook;
- the learner is required to work in imposed learning modes that are not suitable or appropriate and therefore limit or inhibit learning e.g. lack of concrete materials, learner denied chance to discuss ideas with other learners, teacher or textbook use of unfamiliar mathematical language creating barriers etc;

- the learner believes that *doing mathematics* is answering externally imposed questions and either writing the answers down or calling them out;
- the learner believes that there is only one right answer to a question and only one right method to reach it;
- the learner is not interested in posing her/his own questions or investigating his/her own ideas.

Indicators of *alienation* in the learning of mathematics:

- the learner frequently experiences feelings of fear, nervousness, discomfort or boredom during mathematics lessons;
- the learner habitually lacks interest in, or expresses dislike of, mathematics;
- the learner attributes personal failure to the difficulty or lack of appeal of mathematics as a subject;
- the learner is unable to envision her/himself as needing, or wanting to use, mathematics in her/his life outside of school i.e. views mathematics as school-based and essentially purposeless;
- the learner does not express a wish to study mathematics in later schooling or at tertiary level.

Indicators of *marginalisation* in the learning of mathematics:

- the routine exclusion of some learners from certain groups because of perceived lack of ability, e.g. exclusion from extension group or denied the chance to work with friends or peers who are in a 'higher' group;
- the everyday exclusion of some learners from certain activities e.g. learners who are routinely denied use of *Early Finishers' Box*<sup>2</sup> or mathematics games because they are unable to complete set written tasks within the given timeframe;
- the placement of some learners in special needs groups with resulting stigmatisation;
- the selection of a limited number of learners to take part in mathematics competitions or extension group with resulting stigmatisation or elitism;
- the learner believes that only a few people can ever be really good at mathematics;

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<sup>2</sup> *The Early Finishers' Box* is a collection of creative mathematics activities that some teachers provide to cater for those children who finish their work quickly.

- the learner believes that they are stupid because they can't do mathematics;
- the learner attributes personal failure to lack of ability.

All of the children in the study were found to experience these impacts, at varying times, to varying degrees and in varying combinations. In some classrooms, a significant proportion of the children appeared to be highly disaffected with mathematics. Where all three impacts occurred simultaneously, as in Georgina's case, the net result was social and educational debilitation. It was little wonder that for three years she consistently rated mathematics as her least liked subject.

The following chapters will expand on the view of learners as marginalised, alienated and impoverished through their daily immersion in dominant pedagogical discourse and practice.

Chapter 2 locates this study within the body of research that has examined mathematics education from the learners' points of view. This review of the mathematics education literature examines the field of research on attitudes, student affect, and sociological studies of classrooms as cultural sites. It also looks specifically at New Zealand research in these fields. Curriculum documents are examined for evidence of the consideration of social and educational impacts of mathematics teaching and for direct accounts of students' points of view.

Chapter 3 describes the methodological rationale and procedures of the research. In the context of previous studies, it explains and justifies the choice of symbolic interactionism as the theoretical framework, and outlines its advantages and limitations.

In Chapter 4, the intensifying conflation of speed and competition in mathematics classrooms is examined for its alienating, marginalising and impoverishing effects. It is shown how beliefs associated with the learning of '*times tables*' have come to dominate the teaching practice of the middle primary years at school. It is shown how, because of the frequency of speed activities and the value placed on winning, 'speed' comes to be equated with 'ability' in the minds of many of the children. For

those who are not so fast, the daily pressure created by speed activities is the cause of great anxiety, and a reinforced sense of failure. For others who may be successful at speed activities, the intended mathematical learning outcomes are subsumed by the excitement and pleasure of 'beating' others.

Grouping practices and their effects are examined in Chapter 5. Methods of assessment and grouping were significant everyday practices by which the children gauged their achievement. For some, pre-testing and placement were the cause of considerable anxiety, resulting in disappointment and loss of confidence. For others this was a source of pride and satisfaction. Marginalisation through grouping is examined, including Peter and Jared's unhappy placement in special groups, and Jessica and Georgina's involuntary separation from friends and contemporaries. This study describes the experiences of Mitchell for whom mathematics had remained something of a mystery since he began school. A strong case can be made from this biography, for socially appropriate responses that support such children, their teachers, and their parents.

'Doing maths' at school, as described in Chapter 6, is defined through characteristic practices and routines, and presented as 'good work habits'. These include being able to work alone, recording answers neatly on worksheets or in a mathematics exercise book, and completing set tasks within the given time without requiring teacher intervention. It is shown how these practices privilege children who are able to read and follow instructions, work without the use of concrete materials or without discussing the mathematical ideas with others, and who possess the manual dexterity and concentration needed to produce 'neat work'. Independence, rapid completion and presentation are valued and rewarded through the daily practice of doing maths work, while mathematical learning itself is routinely overshadowed by the demands of the classroom work ethic.

Chapter 7 shows the ways in which everyday classroom practices defined mathematics for the children by excluding innovations or interpretations which were at variance with the teachers' preconceived 'right' responses and procedures. The children's ways of knowing were thus ignored, discouraged, 'corrected' or even sometimes publicly ridiculed. The right/wrong dichotomisation of mathematical

'knowing' resulted, it will be shown, in impoverished learning for the children. Inability to 'get it' led to feelings of alienation, and for those who perpetually failed to 'grasp the concepts' and were consequently marginalised by ability grouping, learning mathematics became a bewildering and disempowering experience.

In the concluding chapter, Chapter 8, it is emphasised how, through a process of individual sense-making, these everyday experiences produced for the children, particular feelings, beliefs and behaviours associated with mathematics. Woven together, these portrayals of the children's fascinating individual sociomathematical worlds provide a vivid picture of the child's-eye view of learning mathematics. While it is unwise to extravagantly over-generalise from these discrete and unique biographies, there are nevertheless a number of obvious and important common themes to be found. The chapter draws the themes together by providing an overview of the research in accordance with its sociological theoretical frameworks, and suggests that the evidence of these biographies has profound implications for the teaching of mathematics.

As Pollard and Filer (1999) note, in the recent rhetoric surrounding development and writing of curricula designed to raise educational standards:

'children' and 'pupils' are mentioned solely as an adjunct of the dominant discourse about standards. The implicit representation of children is thus rather like that of industrial raw material awaiting processing and the addition of added value...the assumed passivity of pupils is conveyed by the almost complete absence of any direct account of their perspectives, experiences or quality of life. Education ...is something which is done *to* children, not *with* children, and still less *by* children. (p. 23)

This research aims to provide such direct accounts. Much of what these children have to say challenges the dominant ethos of teaching in primary school mathematics and points to the need to examine the social impacts, both cognitive and affective, of many common and taken-for-granted everyday features of children's sociomathematical worlds.

The following chapter investigates previous research that has also sought direct accounts of learners of mathematics, and locates this study within the existing literature of sociocultural perspectives in mathematics education.

### LEARNERS' PERSPECTIVES IN MATHEMATICS EDUCATION RESEARCH: A REVIEW OF THE LITERATURE

As outlined in the previous chapter, this thesis is primarily concerned with the learner of mathematics. It argues that dominant approaches to mathematics teaching, embedded within the traditional view of education as something we do *to* pupils, not *with* them, result in an impoverished, alienating, and marginalising learning experience for many young people. The *learner* has been variously positioned in the minds of those concerned with the promotion and enhancement of mathematical learning including policy makers, curriculum developers, textbook designers, teachers, and researchers. Traditionally, as noted by Pollard and Filer (1999), *direct accounts* of learners' perspectives, experiences and quality of life have been largely overlooked by curriculum developers. This can be seen in the current rhetoric surrounding the issue of raising standards and enhancing achievement.

Correspondingly in mathematics education, the learner has been traditionally cast in the role of the passive receptor of a curriculum characterised by distinctive pedagogical and epistemological features, that so-called experts have deemed to be in learners' best interests. The belief that the worlds of learners of mathematics, consisting of their thoughts, opinions, feelings, beliefs, attitudes, and associated behaviours, may form an essential and indeed integral part of the process of learning mathematics itself, and that this process can and does produce disaffection, exclusion and limited learning, has in varying forms, informed a vast body of research in mathematics education.

It is the purpose of this chapter to consider the ways in which researchers have sought *direct accounts* of the life of the learner in mathematics education, and what these accounts have revealed in terms of students' cognitive and affective outcomes in their learning of mathematics. Almost invariably, research of this nature has been driven by the search for explanations of learners' success or failure in mathematics. Some easily

identified negative responses to mathematics are already well-recognised, both within popular culture and the world of mathematics education researchers for example 'maths anxiety' (Gierl & Bisanz, 1995; Hembree, 1990; Newstead, 1998; Wigfield and Meece, 1988). 'Enjoyment', 'motivation', 'enthusiasm' and 'confidence' have often been associated with success in mathematical learning. Such findings lend credence to the widely-held belief that student perspectives are worthy of investigation in mathematics education.

Firstly, the chapter outlines three major trends in mathematics education research in which learners' perspectives are of particular interest. Within each of these trends, the views of the learner, the perceived causes of success or failure, the connection between cognition and affect, and the research methods by which direct accounts of learners have been gathered and analysed, are examined. The major findings and also the limitations of this research are discussed. The chapter then reviews the body of research in which direct accounts have been sought from *New Zealand* students of mathematics. Finally, the chapter surveys curriculum documents for evidence of predominant views of the learner of mathematics, and the consideration, or otherwise, that is given to the social impacts of mathematical learning on the individual student. It is argued that recent curriculum developments in New Zealand and elsewhere, represent a waning belief in the importance of students' feelings and perceptions in the teaching and learning of mathematics, and a conviction that issues of impoverished learning, alienation, and marginalisation will vanish with the implementation of a highly structured and prescriptive curriculum.

Through this examination of previous investigations in which students' perspectives have been gained through direct accounts, it is demonstrated how the research presented in this thesis builds from the foundations of prior research. In light of the findings of other studies and the perceived limitations of prior inquiry, this investigation is placed within a contemporary context of mathematics education research and current trends in curriculum development. It is argued that the project provides much-needed depth in our understanding of the interactive dynamics of children's construction of meaning within their complex and multifaceted sociomathematical worlds, and illustrates the processes by which alienation, marginalisation and impoverished learning are cultivated.



### **An overview of research trends**

Three broad currents of thought have driven the research that has been designed to gather, by means of direct accounts, information about the lived experiences of mathematics learners:

1. Many decades of research have been devoted to exploring learners' *attitudes* in mathematics education. It is shown where attitudinal research originated, how it was developed and how it has been applied in the mathematics education context. The chapter explores the ways in which 'attitudes to mathematics' have been variously defined and measured by successive waves of researchers and how this research has been driven by an essentially psychometric view of the learner. Such views have rested on the belief that psychological characteristics of the learner such as personality, motivation, confidence, or natural ability, account for failure or success in mathematics. Such views separate cognition and affect, although an important link is believed to exist between them. Attitudes are assumed to be universal and undifferentiated. Alienation is well-recognised but marginalisation and impoverishment less so. The dominant epistemology of mathematics and its associated pedagogical practices are seldom questioned or challenged in this kind of research;
2. Perceived differential participation and outcomes in mathematics education believed to be linked to learner characteristics such as gender, ethnicity, socio-economic status, and learning disability has led to a broader view of the learner as a socially situated being. An outline is provided of the growing usage from the 1980s of the term '*affective domain*' in mathematics education, a belief in the importance of its inclusion in research into learning of mathematics, and the call for consistency and rigour in research methodology. Researchers with this sociocognitive view have tended to look to social factors in the immediate environment of the child as the reason for student failure, such as family, teachers, and peers as embodied within the formative domains of educational institutions. Within this view it is generally acknowledged that affect and cognition are closely connected, and that positive affect is somehow important in cognitive growth. The social surroundings and circumstances of the child including aspects of



pedagogical practice, rather than dominant mathematical epistemology are examined as causes of well-recognised student disaffection and marginalisation. Impoverished learning is less well recognised;

3. Researchers have more recently begun to adopt sociological views of mathematics education by examining the *sociomathematical norms* associated with mathematical activity inside classrooms, but reflecting widely-held beliefs about mathematics itself. From this perspective, the learner is regarded as a member of a community of learners, both inside and outside of classrooms, and situated within the broader social contexts of policy, institutional structures and popular belief. A review is provided of research where mathematics classrooms are viewed as cultural sites in which children are socialised and 'enculturated' into the routines and belief systems that characterise the learning of mathematics. Student failure or success is seen by these researchers as constructed within social environments in which the individual learner is embedded. Cognition and affect are viewed as inseparable. The dominant epistemological and pedagogical models of mathematics are deconstructed and alternative models are proposed. Alienation, marginalisation and impoverished learning are well-recognised.

### **Researching attitudes**

Research aimed at investigating the lived experience of the learner of mathematics education appears to have begun as studies of *attitudes* to arithmetic. Attitude research originated in the early twentieth century within the fields of psychology and social psychology. Its methods were later adopted by early researchers in mathematics education. The development of attitudinal research appears to have been driven by the belief that attitudes were predictors of behaviour. It was therefore considered important to be able to measure attitudes, and to study how they were formed. Attitude research in mathematics education became very popular in the 1950s and 1960s but limited generalisability and inconsistency of findings led researchers to question the validity or usefulness of its primary methodology, the self-report attitude scale. However, investigations of pupils' attitudes to mathematics are still being conducted to the present day using the quantitative approaches that were developed

half a century ago, their chief advantage being that they can be used with a large sample group.

In current usage the term 'attitude' can vary in meaning depending on the context, for instance, a 'kid with *attitude*' has come to mean a child of outstanding courage and determination. But to describe a child as having an '*attitude problem*' is to view them as suffering from some kind of self-limiting or deviant thinking. In both cases, *attitude* is regarded as a mind-set that individuals *possess* and that is discernible through their actions. Given the perennial usage of the concept of attitudes in mathematics education research, it is important to clarify just what successive generations of researchers have understood by the term attitude, and why it has become the focus of so much investigation.

Thurstone (1931) defined attitude as 'the affect for or against a psychological object' (p. 261). Thurstone devised an elaborate scale for measuring attitudes. Subsequent theorists expanded upon this definition. Allport (1935) viewed attitude as 'a mental and neural state of readiness, organised through experience, exerting a directive or dynamic influence upon the individual's response to all objects and situations with which it is related' (p. 810), recognising not only a 'readiness' aspect, but judgements which may result from such a mental disposition. Others such as Rosenberg and Hovland (1960) saw attitude as consisting of thoughts, feelings and behaviour. Himmelfarb and Eagly (1974) described an attitude as 'a relatively enduring organisation of beliefs, feelings and behavioural tendencies towards socially significant objects, groups, events or symbols' (p. 81). Ableson, (1976) who argued that attitude theory was 'a mess' (p. 40), because attitude measurement had failed to contribute to understandings of behaviour, provided the view that 'attitude toward an object consists in the ensemble of scripts concerning that object' (p. 41), a script being 'a coherent sequence of events expected by the individual, involving him either as a participant or as an observer' (p. 33).

In presenting an overview of the development of understandings about attitude from a social psychologists' perspective, Vaughan and Hogg (1998) conclude that 'most modern definitions of attitude involve both belief and feeling structures, and are much concerned with how, and, even if each can somehow be measured, the resulting data

will help to predict the future acts of an individual ' (pp. 81-82). Meighan and Siraj-Blatchford (1998) provide a sociological definition of attitude as 'a large number of habitual opinions about one central issue' (p. 181).

These definitions have common threads:

- an attitude involves an habitual, therefore relatively stable, personal orientation of the individual vis-a-vis an *object*– the self, others, ideas, events etc.

An attitude is composed of three main parts:

- *cognition* - beliefs and understandings about the object;
- *affect* - feelings about the object ;
- *conation* - behaviours or actions related to that object.

In spite of the difficulty in defining attitudes, there has been a prevailing belief that they are something that can and should be scientifically measured, consequently, numerous *instruments* or *tools* have been devised for this purpose. These tools mostly consist of some form of self-report scale by which subjects rate themselves. The scale developed by Thurstone (1931) relies on respondents' agreeing or disagreeing with a range of statements about an idea or object ordered from the most positive to the most negative. Likert (1932) devised a scale requiring respondents to designate the *extent* to which they agreed or disagreed with given statements. Commonly, five degrees of feeling about the statement are offered – agree strongly, agree, not sure, disagree, strongly disagree. Because of their relative ease of construction and use, variants of this scale have become widely used in the gathering of attitude data. The semantic differential scale developed by Osgood et al (1957) measured respondents' attitudes to a specified one-word concept by having them rate their responses, using a seven point scale, to pairs of antonyms relating to that concept. The Fishbein scale known as the expectancy-value model, incorporates *beliefs about* an object as well as personal *evaluations of* the qualities associated with that object (Fishbein & Azjen, 1975).

Attitude researchers have been particularly interested in how attitudes are formed and how they relate to behaviour. The difficulty of developing robust theories that link these two aspects of human life are outlined by Eagly and Himmelfarb (1974). Social psychologists such as Vaughan and Hogg (1998) maintain that there are three main

constituents of attitude development: personal experiences, interactions with others, and emotional responses. Frequency of exposure, they say, can greatly influence attitude formation – the more one is exposed to an object, idea or activity, the greater the likelihood of associated attitudes being formed.

### **Attitudes and mathematics – the lone learner**

Attempts have been made by successive waves of researchers to identify, describe, measure, quantify and compare students' attitudes to mathematics, and to link attitudes to a wide variety of variables in an attempt to determine their origins and development. Attitude research in mathematics has been considered important because attitude is believed to be a predictor of, and contributor to, achievement (Ma, 1997). Alienation, marginalisation and impoverished learning have, to varying degrees been recognised, investigated and explained by attitude research.

The approach of early mathematics educators was to view the learner as an isolated being. Johnson (1957) provides a view of mathematics, the learner, the learner's attitudes, and the role of the teacher that was typical of this era. Like many of his contemporaries, he believed that poor student attitude was to blame for lack of achievement. He saw the classroom as the primary contributor to the development of attitudes, maintaining that, 'learning involves emotional vectors such as attitudes,' and that, 'every hour in the classroom results in developing attitudes, desirable or undesirable' (p. 113). He asserted that teachers' attitudes were crucial in improving the mathematics curriculum and increasing participation in mathematics, and that the teacher's role was to identify and then modify through a process of conditioning, students' attitudes to mathematics. He compiled an attitude inventory intended as a tool by which teachers might assess students' attitudes to mathematics. Students were to rate their responses to a range of statements including: enjoyment of mathematics, appreciation of its importance, interest and motivation in learning, confidence, loyalty to the subject, to the mathematics teacher, and to fellow classmates, respect for the achievement of mathematical competence by colleagues, taking responsibility for one's own and others' achievement, commitment to extend learning beyond the course and an understanding of the generic nature of mathematical reasoning.

Johnson's identification of desirable attitudes reveals the subjective and socially situated nature of beliefs about attitudes.

Many early attitude investigations in mathematics education made use of some form of attitude measurement scale such as Johnson's inventory. Dutton (1951) developed a Thurstone-type scale by collecting a large number of prospective teachers' written statements about their feelings towards the 'psychological object' of arithmetic and selecting those which could be easily 'endorsed or rejected' by a respondent. Twenty-two of the statements were selected to represent a typical range of 'for', 'against' responses to arithmetic that became the basis of his attitude to arithmetic measuring tool. This scale was used by other researchers (e.g., Bassham et al, 1964). Aitken and Dreger (1961) developed a Likert-type scale to investigate the correlates of a number of variables to attitudes to mathematics, including course grades, intellectual ability, personality, temperament, and parental attitudes, while Anttonen (1969) developed a semantic differential scale. Variants of the Likert-type scale have been most widely adopted by subsequent generations of researchers of mathematics attitudes and are still being used (e.g. Utsumi & Mendes, 2000). Surveys based on scales such as these were usually administered to large groups of students, and statistical analysis used to determine the levels of correlation between the variables thought to be significant in the learning of mathematics. Scales of this type have typically included statements about enjoyment of mathematics, perceived difficulty, attributions of success, utility beliefs about mathematics and impressions of mathematics teachers. The scales have often been used to produce an aggregate 'attitude score' for each student, which could then be compared with achievement.

Literature reviews and meta-analyses show that great number of research studies of this nature have been undertaken since the 1950s (Aitken, 1976; Dungan & Thurlow 1989; Ma & Kishnor, 1997a), and that connections between a wide range of variables have been explored. A sample of these investigations provides an indication of some of the key questions that have been asked, some conclusions that have been reached, and areas of consensus or disagreement that exist within the research.

- At what age are significant attitudes to mathematics formed?  
A number of studies have concluded that attitudes to mathematics may be formed at any age, and provide evidence that they may begin at pre-school age (Wagner, 1980) and from the early years of primary school (Dutton, 1954; Fedon, 1958; Garden, 1997; Silverman, 1973). Negative attitudes to mathematics have been found to increase with age (Anttonen & Deighan, 1971; Flockton & Crooks, 1998; Neale, 1969; Stright, 1960; Swetman, 1995; Utsumi & Mendes, 2000; Wigfield & Meece, 1988), indicating that attitude formation is an ongoing process.
- Is there a causal relationship between attitude and achievement in mathematics and if so, what is it? Some researchers have detected a significant relationship between 'positive attitudes' to mathematics and achievement (Aitken & Dreger, 1961; Anttonen, 1969; Bassham et al, 1964; Cheung, 1988; Ma & Kishnor, 1997a; Wagner, 1980). Some researchers have come to the conclusion that attitudes to mathematics directly influence achievement (Hembree, 1990; Kulm, 1980). Others observe the concomitant nature of the relationship (Garden, 1997) or find evidence of causality in the reverse direction (Gough, 1954).
- Does student interest in and enjoyment of mathematics influence achievement? Some studies have indicated that interest in the subject is irrelevant to learning outcomes (Neale, 1969; Pollard et al, 2000), while others have found the opposite (Suydam & Weaver, 1975). Some researchers have reported a strong causal link between enjoyment of mathematics and achievement (Kloosterman & Cougan, 1994; Ma, 1997; Martin, 1997; Young-Loveridge, 1991), but this has been discounted by others (e.g. Pearce et al, 1998).
- Do students' self-efficacy beliefs influence achievement? Silverman (1973) found a direct relationship between positive self-efficacy and positive attitudes about mathematics. In their meta-analysis, Ma and Kishnor (1997b) concluded that self-concept declines with increasing student age, and that self-concept is linked to achievement. Meece et al (1982) maintained that self-efficacy declines with an increase in negativity towards mathematics. Davies and Brember (1999) noted a connection between self-esteem and attitude to mathematics. Links between

confidence and self-efficacy have also been recognised (Fennema & Petterson, 1985).

- Do students' beliefs about the nature and utility of mathematics influence attitudes and achievement? A number of researchers have found a significant link between achievement and the valuing of mathematics for its usefulness (Aitken, 1976; Cheung, 1988). Higgins (1997) found that a problem-solving approach to teaching mathematics produced an increase in positive beliefs about the usefulness of mathematics. Motivation has been linked to utility beliefs in a number of these studies.
- Do teachers' attitudes and beliefs regarding mathematics contribute to students' attitudes? Haladyna et al (1983) found that students were more motivated if they felt positive about the teacher. Anttonen and Deighan (1971) concluded that there was no relationship between teacher and student beliefs about mathematics, but Gilbert and Cooper (1976) found the two to be linked.
- Do parents' attitudes influence students' attitudes to mathematics? Poffenberger and Norton, (1959) found that parents have a strong influence on their children's attitudes to mathematics. Gottfried et al (1998) found in a longitudinal study that a stimulating home environment produces greater academic intrinsic motivation for students and that this effect continues through schooling.
- Does gender make a difference to attitudes? The findings of TIMSS research in New Zealand (Martin, 1996; Garden, 1997) and Young-Loveridge (1991) suggest that low-achieving girls are less positive about mathematics than boys, and that they were more likely to attribute success to hard work than ability. Some recent studies (Forgasz & Leder, 1996; Swetman, 1995; Utsumi & Mendes, 2000) have found girls to be more positive about certain aspects of mathematics than boys. A number of studies (e.g. Martin, 1996) have detected a decline in girls' positive attitudes to mathematics with increasing age. There can be little disputing the lower participation rates of females in mathematics at the tertiary level. (Alton-Lee & Pratt, 2000).



Since the 1950's, extreme aversive reactions to mathematics have been identified and viewed as a psychological condition variously called '*mathemaphobia*' (Gough, 1954; Winter, 1992), '*emotional block*' (Tulock, 1957), '*maths anxiety*' (Brush, 1981; Hembree, 1990; Newstead, 1998; Seaman, 1999) and '*number shock*' (Mandler, 1989). Such extreme negative reactions to mathematics have been linked to both reduced participation and diminished achievement. While the causal relationships between anxiety, participation, and achievement are debatable, Wigfield and Meece (1988) suggest that some degree of anxiety may actually enhance the mathematical performance of some students. The recognition of this apparent affliction has given rise to a particular branch of attitudinal research. Scales of measurement (e.g. MARS in Brush, 1981) have been developed and used to identify '*mathophobic*' students. Causes and *remedies* (Gough, 1954) have been sought and suggested for *patients* suffering from this condition (Tulock, 1957). Origins of maths anxiety that have been suggested are: a specific unpleasant learning experience, absence from school during a critical learning period, fear of failure, and too fast a learning pace (Gough, 1954), lack of confidence (Tulock, 1957), or the difficulty and high degree of abstraction of the subject matter (Brush, 1981; Seaman, 1999). The observation that females are more likely to report suffering from 'negative attitudes' including maths anxiety and maths aversion (Hembree, 1990) have led to speculation about what it is in the psychological makeup of the student that produces this 'phobic' state. Suggested *cures* have primarily centred on either:

- (a) intervention to correct a perceived deficiency in the student's mathematical orientation such as advocated by Aitken (1976) who notes 'promising' beginnings on the *treatment* of mathophobia by means of 'behaviour modification approaches' which he believes should be 'refined and extended' (p. 303), or;
- (b) application of teaching approaches that present mathematics in a more appealing and/or more readily assimilable form than traditional methods. The teaching methods promoted by the Lawrence Hall of Science (e.g. Downie et al, 1981) are one such example.

Both these response models view the learner as defective, therefore requiring either repair or special treatment. Walkerdine (1998) provides a view that the dominant epistemological constructs of mathematics itself, coupled with traditional views of



learning and ability, may lie at the root of this severe form of learner alienation and marginalisation.

Some recent research has compared classrooms with differing teaching approaches for the effect on students' attitudes to mathematics (e.g. Pearce et al, 1998), or before-and-after the implementation of alternative mathematical teaching approaches such as co-operative learning (Jacobs et al, 1996; Brush, 1997), and problem-solving (Cobb et al 1992; Higgins, 1997). Findings from these studies suggest that significant changes in attitudes such as confidence, perseverance, and belief in the utility of mathematics, can result. This would point to teaching and learning factors as being of greater significance than the individual attributes of students themselves, as was earlier believed (Aitken, 1963).

Attempts to synthesise and draw conclusions from the vast array of attitudinal research in mathematics education have given rise to a number of important considerations. Aitken (1963) suggested that attitudes toward mathematics were 'related to a broad constellation of personality variables indicative of adjustment and interest' (p. 479). Later (Aitken 1976) observed that changes in attitude towards mathematics involve a complex interaction among student and environmental factors. Dungan and Thurlow (1989) conclude that negative attitudes to mathematics do exist, that they are likely to lead to impaired learning, that they increase with age and that the effect is greater for girls. They suggested making mathematics more interesting and engaging for the students as a way of producing more positive attitudes. Ma and Kishor (1997a) provide a meta-analysis of attitude/achievement research in which they conclude that ability is a key variable in the relationship between attitude to mathematics and achievement in mathematics but suggest that more reliable quantitative methods be developed to produce a clearer understanding of the 'complex interactive nature' (p. 43) of these two aspects of student learning of mathematics. While reviews of the research have failed to yield any definitive and consistent causal correlation between any one variable, negative or positive student attitudes, and achievement in mathematics, much evidence points to a significant correlation between achievement and some key aspects of attitude to mathematics, such as utility and self-efficacy beliefs. However, researchers universally acknowledge the problematic intricacies of the attitude-achievement conjunction.

A distinguishing feature of the majority of attitude research is the faith that is placed in scientific method. Typically, a rigorous quantitative approach is applied throughout the research process including working from a hypothetical stance, gathering data by means of a well-tested measuring instrument, using accepted techniques of statistical analysis, providing detailed statistical reporting and drawing conclusions based on the statistical analysis. In his review of attitude research, Aitken (1976) commends its increasing sophistication and statistical methodology.

This overview shows that there has been wide diversity in mathematics attitude studies, and that such studies are most often narrowly focussed thus failing to explore the full range of significant dimensions of learners' perspectives and experiences, or to appreciate their complexities. These studies often attempt to establish links between variables that are difficult to define, they rely heavily on one-off written responses to questionnaires that give no indication of why any given participant has responded as they have, and they seek 'proof' through statistical analysis of large sample groups. Attitudes have been so variously defined, measured and analysed for co-variance, that findings have been difficult if not impossible to compare or rely upon. Because of the lack of consistent approaches, generalisable or reliable conclusions have been difficult to generate.

### **Limitations of traditional attitude research**

In attitude research, students are primarily viewed as isolate individuals rather than interactive, dynamic social beings, and as passive objects of scientific study rather than active participants. Their thinking and behaviour has been regarded as universal, measurable and predictable, hence the value of gathering quantitative data using large samples.

Since its beginnings, the questionable utility and legitimacy of *attitude* as a research domain has caused ongoing debate and controversy. Blumer (in a paper presented in 1955, reprinted in Blumer 1969) criticised what he viewed as a preoccupation with so-called 'scientific' attitude studies in social research of the 1930s to 1960s. He argued that 'the concept of attitude as it is currently held rests on a fallacious picture of

human action. Also it fails miserably to meet the requirements of a scientific concept' (p. 90). He reasoned that an attitude cannot be directly perceived, but rather 'pieced together through a process of inference,' and criticised the reliance either on personal impressions of what constitutes an 'attitude' or 'falling back on some technical device such as a measurement scale' (p. 91).

A number of early researchers of attitudes to mathematics have recognised the deficiencies that Blumer identifies. Dutton (1954) for instance, did not rely solely on self-report scales to explain student attitudes to arithmetic. By means of an open-ended question, he provided an opportunity for respondents to suggest in their own words, what it was they particularly liked or disliked about mathematics. He then grouped their responses. He concluded from their comments that 'unfavourable attitudes of significance are: not feeling secure in the subject, being afraid of word problems, and fear of the subject in general. Lack of understanding, teachers who punished students or used inadequate methods, difficulty in working arithmetic, and insecurity were the chief factors cited by the respondents as causing dislike for arithmetic' (p. 31). In this way, Dutton was able to identify some significant environmental factors, such as classroom practices, that students believed may have contributed to their formation of negative attitudes.

In recognising the limitations of attitude research in mathematics Neale (1969) questioned the interpretation of results that appear to support the view that favourable attitudes leads to improved achievement. He advocated for research that exposed the hidden curriculum that he believed produced certain behaviours in children in learning mathematics. Rather than simply attempting to make mathematics more fun and attractive, thereby creating positive attitudes, he suggested that learning institutions themselves needed greater flexibility in recognising and meeting the needs of individual children.

In response to the failure of attitude research to produce convincing evidence, some researchers appealed for increased rigour in scientific method. Fennema (1989) noted the lack of a cohesive picture in research on affective variables, and proposed a theoretical model combining the traditional affective research methodology with that of cognitive science. She called for more precise definitions and greater clarity about

what was being studied, believing that a generic model might provide for consistency in research findings. Ma and Kishnor (1997a) also concerned at the lack of consistency and generalisable findings in attitude research, stated 'we would encourage researchers to use advanced statistical techniques, such as structural equation modelling (SEM) to investigate the bilateral relationship between ATM (attitude to mathematics) and AIM (achievement in mathematics) as a way of unfolding their complex, interactive nature' (p. 43).

While *attitude* studies may have served a valuable function in focussing researchers' attention on the broad spectrum of feelings, beliefs and opinions of learners in the teaching and learning of mathematics, they have been able to provide only very limited accounts of the students' lives as learners of mathematics. Understanding of learners has been difficult to obtain by means of the self-report questionnaire methodology of attitude research. Researchers of attitude have tended to overlook, fail to appreciate, or balk at, the complexity or the inseparability of the multitude of elements of the lived experiences of students and teachers, and the ways in which they are generated through dynamic social systems and processes.

McGuire (1986) in reviewing what he described as the 'vicissitudes' of attitudinal research, summarised the evolution of approaches: in the 1920s and 30s it was primarily concerned with what was assumed to be something that was relatively stable and therefore measurable. In the 1950s and 60s, attitude investigation centred on how attitudes are develop and how they change. In the 1980s and 90s attitudes were being increasingly viewed as composite and socially situated. It is this latter view that is examined in the following section of the chapter.

### **The affective domain – the learner as a social being**

The failure of attitude studies to provide convincing evidence of links between attitudes and achievement has prompted a number of reviewers of this body of research to question the underlying tenets and associated methodology of this line of investigation, and to call for different approaches. From the early 1980s, a change can be discerned in the discourse surrounding the personal responses of students in

mathematics education. The term *affect* (Fennema, 1989; Hart, 1989; Reyes, 1984) was increasingly used in place of, or inclusive of, *attitude*, and was increasingly viewed as a combination of a number of interrelated concepts such as beliefs, emotions and attitudes.

Along with the change in terminology came calls for changes in methodology that might more accurately convey the feelings, emotions and attitudes of individuals. While Kulm (1980) commended the self-report method of gathering attitude information about students by asking 'what better way is there to determine a subject's attitude toward maths than to ask a direct question?' (p. 364), he noted that most attitude researchers had relied almost exclusively up until that time, on self-report *scales*, and criticised this approach for its lack of 'attention to the characteristics of the attitude being measured' and 'combining in meaningless ways characteristics that ought to be considered separately.' He made two recommendations: that the researcher 'consider more common-sense, realistic views of assessing attitude' and that 'the researcher should consider direct observation of individuals rather than [surveying] groups' (p. 365). In summing up his view he stated:

Out of an apparent concern for expressing relationships concisely and numerically and the desire to perform statistical tests on means and variances, the most potentially fruitful approaches to individual attitude measurement may have been overlooked. The measurement of mathematics attitudes in the future should make use of many approaches, and researchers should not believe that scales with proper names attached to them are the only acceptable way to measure attitudes. (p. 365)

Similarly, Hart (1989) criticised the use of paper-and-pencil instruments and standardised tests, believing that more direct methodology such as think-aloud interviewing, although more time-consuming, provided 'a richer reflection of the student' (p. 44).

McLeod (1992) drew a distinction between what he termed the traditional paradigm of the study of affect, in particular 'the study of attitudes characterised by emphasis on definitions of terms, its preoccupation with measurement issues, and its reliance on questionnaires and quantitative methods' (p. 577), and a reconceptualised paradigm

placing greater emphasis on theoretical issues and more extensive use of qualitative methods such as interviews and 'think-aloud' student reporting. He noted the increasing attempts to merge cognitive and affective factors in learning theory, and emphasised the need to investigate the affective dimension in the development of mathematical understanding, particularly as it related to problem solving, which emerged during the 1990s as central to the pedagogy of mathematical inquiry embedded in constructivist views of learning. The work of Leron and Hazzan (1997) is an example of the researcher attempting to see learning from the point of view of the learner through the construction of 'virtual monologues' of students' thoughts, cognitive and affective, as they cope with problem solving tasks.

McDonough and Clarke (1994) also objected to traditional methods of attitude research commenting that, 'we find the term 'measurement' to have unfortunate uni-dimensional connotations and ...advocate the use by researchers and teachers of the term 'portrayal' in relation to the process of data collection regarding student affect' (p. 391).

More recently, Ruffell et al (1998) have also challenged the cause-and effect model of attitude studies in mathematics. Their attempts to 'measure' students' attitudes with a range of instruments from individual interviews, group interviews, individual questionnaire responses, and diary writing led them to suggest that *attitude* is a problematic construct, that many of the underlying assumptions of attitude research were difficult to support, and that there was a danger in interpreting student responses to research questions as *attitudes*.

In summarising their review of attitude research, Alton-Lee and Pratt (2000), state that 'given the complex relationship among attitudes, affect, ability and achievement, it could prove more fruitful to examine their development in-situ, triangulating students' own thoughts about confidence, liking and achievement in mathematics, with classroom processes, and standardised measures...studies that actually ask what the students' reasons are for participating or not participating in mathematics... provide information based on students' experience of mathematics in the classroom'(p. 129). Eisenhart (1988) suggested that one of the reasons for a reticence on the part of mathematics education researchers to use anything other than



quantitative methods to study students learning mathematics was what she saw as their insensitivity to 'the intersubjective meanings that might constitute the schools, classrooms, and instructional dyads they study...they assume they know the intersubjective meanings of the groups they are studying' (p. 111).

The social dimensions of student's responses to mathematical learning were becoming increasingly recognised during the 1980s. Mandler (1989) for example, acknowledged the importance of *social* and *cultural* factors in the feelings students have about mathematics in stating that 'they determine many of our expectations about: situations and tasks, about the social interactions that are important in the learning and teaching situation and about our successes and failures and their relevance to our social functions and our self-image' (p.17). Forgasz and Leder (1996) also believed that social environment might play a key role stating that, 'little is known about the relationship between affective variables and elements of the mathematics classroom' (p. 153).

The study of Anderson et al (1984) provides an example of changing views of affect and the learner as situated within the classroom. In the resume, it is stated that, 'this study introduces a novel methodology for examining affective experiences in the classroom.' By using sound and filming equipment, recordings were made of individual Grade 4 and 5 children engaged in everyday mathematics activities in their classroom. Immediately afterwards, the children were withdrawn and the recordings were replayed, during which time the children were able to pause the recording at any point and report on the feelings they had been experiencing at the time of filming. Of the thirty-two children interviewed in this way, five reported having experienced positive feelings only during the lesson. Of the others, more negative feelings were reported than positive. 80% of reported feelings were negative - angry (35%), unhappy/bored/dumb (25%), anxious (20%) - while 20% of reported feelings were positive - happy/amused/hopeful/relieved. Feelings of anger were associated with wrong answers or negative interactions with friends, feelings of unhappiness were related to lack of understanding, incompleteness, and wrong answers, and anxiety was linked to incompleteness and task difficulty. Positive feelings were associated with completion, right answers and positive interactions with friends. By means of this methodological approach, the researchers were able to gain a view of the children

operating within their classroom environment. They noted that children were primarily engaged in producing written answers to textbook questions. These had been individually assigned by the teacher to suit the children's needs, thus precluding task collaboration. In light of the children's responses, the researchers speculated on the effects of classroom pedagogy, particularly the compulsory and solitary nature of this kind of mathematical activity.

This study has been reported in some detail because it represents a significant departure from the scientific measurement methodology of previous attitude research and heralds a changing view of the learner. Here the learner is located within an everyday social learning environment and students' responses are reported not through self-rating on a pre-worded scale but in their own words and in relation to a real experience. Affective factors are *situated* and linked to specific circumstances. Herrington (1998) used this 'stimulated recall' technique to compare Year 6 students' responses to mathematics and social studies lessons. He found that the students' responses indicated that the beliefs they held about the subject of mathematics and how it is learned, were quite different to those they held about social studies.

McDonough and Clarke (1994) challenged what they saw as 'the dichotomization of learning into cognitive and affective...in which cognitive beliefs and the associated enactment of these beliefs have been privileged in comparison with affective beliefs' (p. 393). They report on a number of studies designed to access student affect, in which a variety of techniques were used, including regular written self-reporting in response to open-ended questions and statements, such as, 'How could we improve maths classes?' belief questionnaires, personal dictionaries of mathematical concepts, and completion of the statement 'Maths is like...'. They found that students' comments included both cognitive and affective dimensions and this led them to postulate that 'belief' was the unifying construct in which both affective and cognitive responses were developed and manifested.

In an investigation of gender and affect in the mathematical learning of 12 and 13-year-olds, Forgasz and Leder (1996) aimed to determine whether a relationship existed between students' beliefs, and specific aspects of the classroom environment. A multi-method approach was used to gather data about the students' experiences and



feelings. The tools they used included a questionnaire based on a Likert-type scale, open-ended questions including students' perceptions of their own ability and responses to typical lessons, interviews with students and the teacher, and videotapes of lessons. It was found that key factors linked to beliefs about competence were teachers' personal interest in the students and promotion of participation in the classroom. Teachers' and classmates' behaviour, the learning activities themselves as well as their contextual settings, assessment and an emphasis on competition, were also found to be significant factors. Through this kind of investigation, affect could be linked to real experiences through students' direct reporting as well as self-rating on traditional scales.

This sample of research that investigates the affective domain demonstrates the evolutionary broadening of attitude research through the use of socially situated and multifaceted methodological approaches. These approaches emphasise *portrayal* (McDonough & Clarke 1994) but may also be supported by *measurement*. They are able to more successfully access the thoughts and responses of learners, and relate these to real experiences. They recognise, and therefore attempt to understand, the complexity and interconnectedness of influences within the environment of student learning.

### **Classroom culture – the learner as a culturally situated being**

The concept of mathematical learning as an essentially social and cultural activity (Abreu, 2002; Atweh et al, 2001; Cobb, 1995), has changed the focus of mathematics education research over the past decade. Bishop (1991) has reasoned that mathematics, a value-laden cultural construct, is learned within the classroom, a cultural site, in which the learner is *enculturated* into a particular mode of thinking and doing known as *mathematical*. From a cultural point of view, alienation and marginalisation in mathematics education can be explained as processes of exclusion and inaccessibility for those learners who are unwilling or unable to operate successfully within the classroom code of accepted cultural practices. Mathematics classrooms have been increasingly viewed as possessing a distinctive culture. Voigt (1998) for example describes the mathematics classroom in the following way:

...the culture of the mathematics classroom appears to have a life of its own...Everyday [sic] the participants in the classroom develop unreflected customs and stable habits that enable them to cope with the complexity of classroom life while functioning as a resistance to educational reform. (p. 191)

The view of mathematics as cultural, has been endorsed by those researchers who have adopted the term *ethnomathematics* (Barton, 1993; D'Ambrosio, 1997) to describe the culturally-mediated ways of *mathematising* that are found in the everyday life of people everywhere, not only within classrooms. Mathematical knowledge, in this view, exists not as an absolute and immutable body of truths, but as social practices associated with the numerical, spatial and temporal dimensions of human existence. Studies comparing the in-school and of the out-of-school mathematical facilities of children (Abreu et al, 1997; Nunes, Schliemann & Carraher, 1993), indicate that children's out-of-school mathematical knowledge was not transferred to, or legitimated within, the school setting. Anderson (1997) argues that the Eurocentricism embedded within the dominant mathematical discourse, is characterised by racist and sexist biases that cause alienation of learners. From an ethnomathematical point of view, there is no such thing as the 'deprived' child for whom the existing school mathematics curriculum must somehow cater, rather it is the 'deprived curriculum' of school that fails to reflect the multiple realities of its learners.

Through a cultural lens, some researchers have viewed the student as belonging to a 'community' of learners (Cobb, et al, 1992; Lave, 1988; Lave & Wenger, 1991; Wenger, 1998). Cohen (1985) describes *community* as a group of people with something in common, whose 'membership distinguishes them in a significant way from the members of other putative groups' (p. 12). Yackel and Cobb (1996) adopt a symbolic interactionist view of classroom as communities with particular social *norms*. These norms are seen as the 'regularities in patterns of social interaction' (p. 460). The norms specific to the mathematical classroom activity are termed *sociomathematical norms*. Researchers with this point of view (Cobb et al, 1992; Heibert et al 1997), direct their attention towards changing the traditional norms of mathematical pedagogy to better fit with a philosophy of teaching of mathematics with understanding.

In contrast, Winbourne and Watson (1998) have described the learner as situated not within a community of practice, which they believe implies a cohesive and unified body of norms that are understood in the same way by every community member, but as situated in the classroom viewed as an intersection of a multiplicity of mathematical practices, local and further afield. Within this local community, the learner attains an identity or 'becomes' through participating within the multiplicity of practices. They claim that through aggregated narratives of students' experiences in the classroom and school setting, the processes of mathematical 'becoming' might be discerned.

Ernest (1998a) also views the social context of the school mathematics classroom as, 'a complex, organized, and evolving social form of life (or sets of forms of life)' (p. 231). The ingredients of this social context he identifies as:

1. the aims and purposes of the activities;
2. the participants and their relationships, including roles and modes of interaction;
3. the discourse of school mathematics including the symbols, concepts, conventions, definitions, symbolic procedures and linguistic presentations of mathematics, modes of communication and rhetorical styles, and discourse of social regulation;
4. the material resources of the classroom including exercise books, textbooks, technological tools, displays, location, space and routinized times. (p. 231)

Using individual microphones, Alton-Lee et al (1993) recorded the public and private talk of children in the classroom. These conversations provided a vivid and telling window into classroom life as perceived and interpreted by children, and illustrated the ways in which peer culture can operate independently of the 'official world of the teacher's agenda' (p. 50). Walshaw (1999) used similar techniques to study three secondary girls' experiences of mathematics classrooms. However, such studies have been rare in mathematics education research. Many early researchers who have taken a cultural approach to the study of the teaching and learning of mathematics, have used descriptive methods to record and analyse classroom interactions, and interviews with children to gain an insight into their *mathematical* cognising rather than their individual perceptions and reflections about the nature and purpose of their mathematical experiences as they are engaged in mathematics in the classroom.

Hart and Alleksaht-Snider (1996) note the increasing influence of anthropological approaches to mathematics education research and advocate that researchers 'include elaborated descriptions of not only the microlevel classroom processes but also the larger sociocultural contexts in which their studies are situated' in order to identify factors that 'build a sense of belongingness and counteract student resistance to mathematics learning' (pp. 17-18).

Zevenbergen and Lerman (2001) situate the classroom within a wider social context when they maintain that 'the mathematics classroom is a cultural representation of the values of the dominant culture – in terms of mathematical knowledge but also with social values. Mathematics is taught within the social practices of a Western middle-class value system' (p. 576). They note that the different backgrounds and discourses that learners bring to school mediate their experiences of the classroom. By interviewing secondary school students, Zevenbergen (2002) found clear differences between the comments of the children in the 'high' and 'low' streams, whose feelings and beliefs about their learning of mathematics reflected the marginalising effects of grouping by ability.

From the viewpoint of classrooms as one of many cultural sites, and learning as situated (Even & Tirosh, 2002), *knowing* also becomes contextual and situational, therefore variable (Lave, 1988). Boaler (2000) states this perspective clearly:

Students do not just learn methods and processes in mathematics classrooms, they learn to *be* mathematics learners and their learning of content knowledge cannot be separated from their interactional engagement within the classroom, as the two mutually constitute one another at the time of learning. (p. 380)

Researchers who have sought first-hand accounts of learners within these sites have made extensive use of qualitative methods. Boaler (1997a) for example, describes her comparative research of students of mathematics within two schools with differing teaching philosophies, as 'in-depth, longitudinal ethnographic studies of students working within their own school environments' (p. 3). Her report of this study includes many excerpts from her interviews with the students. By using their own words, she is able to 'take the reader some way towards the worlds of school mathematics as they experienced them' (p. 3). She found that at Amber Hill school

where mathematics was taught in a very traditional manner, a high proportion of the girls became disaffected by, and disillusioned with, mathematics at school. The top stream of students reported that the speed and pressure they experienced during classes had a negative affect on their learning. At Phoenix school where mathematics was taught in a less pressured and more flexible, open, and reflective style, a much greater proportion of the girls reported enjoying mathematics. Boaler (2000) supports research that is based on situated perspectives of classroom communities. She describes theories of situativity as being characterised by their, 'focus on interactive systems that are larger than the behaviour and cognitive processes of an individual agent' (p. 380), and uses this view in a 4-year study involving 1000 students. The study aimed to 'determine the impacts of differing teaching methods and ability grouping on the students' perspectives of the mathematics classroom and their subsequent attainment' (p. 381). She described the schools in the study as having a traditional approach to teaching mathematics, with the dominant model consisting of teacher demonstration, followed by student practice of closed questions from textbooks or cards. From the students' comments, the major themes of monotony, lack of meaning, and learning as an individual rather than a group exercise, emerged as significant alienating features of classroom mathematics.

Anderson and Boylan (2000) investigated the effects on teaching and learning of the pedagogical approaches advocated by National Numeracy Strategy in England. They found, from their classroom observations and interviews with Year 4 children, that the direct questioning that is encouraged through the Numeracy Strategy, produced anxiety in the children through pressure to produce right answers when exposed to peers, and noted that the pace of questions appeared to increase such anxiety. Anxiety was found to be widespread and not confined only to those students who had trouble answering the questions, but also to those who were able to produce the right answers. They suggest that such anxiety may inhibit student learning.

**Social world theory, symbolic interactionism and mathematics – the learner as interactive sense-maker**

The sociological works of Mead, (1934) Goffman, (1959), Blumer, (1969), Schutz, (1970) and Berger and Luckmann (1966) have attracted the attention of some recent mathematics education researchers because they provide theoretical frameworks that help to describe the socio-cognitive processes of the individual learning mathematics within social settings. The life-world, or the world of everyday life, described by Schutz<sup>1</sup> (1970) has inspired the work of several researchers. Brown (1997) for example using phenomenological theories states 'in particular I examine Schutz's framework used in describing how an individual experiences their world, as an approach to understanding how the student experiences the mathematics classroom' (p.132). He analysed children's engagement in mathematical tasks for signs of personal sense-making and noted the ways in which individual learners acted on expectations born of prior experience (schemes) and adjustments through new experiences.

In her year-long qualitative study of eighth grade students' learning of algebra, Ivey (1994) compared the social worlds of three students, concluding that their world views strongly influenced their cultural expectations of newly-reformed classrooms, and that when they experienced 'metaphorical dissonance' between their own world views and those of the new classroom culture, they responded either by rejecting the new classroom expectations or contesting them in an attempt to effect a return to the familiar. Ivey argues that the ways in which the students perceived the world was critical to the ways in which they created new meanings in the changed environment.

Roth et al (1999) used the 'life-world' approach to investigate Year 12 students' experiences of learning physics. They found from interviewing the teacher and students, that the students and their teacher experienced the same physics class as different worlds or 'worlds apart', exacerbated by the limited dialogue during lessons.

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<sup>1</sup> Life-World: 'The total sphere of experiences of an individual which is circumscribed by the objects, persons and events encountered in the pursuit of the pragmatic objectives of living. It is a 'world' in which a person is 'wide awake' and which asserts itself as the 'paramount reality' of his life.' (p. 320).

Boylan (2002) has used the concept of life-world to investigate the practice of teacher questioning in the classroom. The students were asked to rank statements about possible situations that might occur during teacher questioning, for example, 'The teacher does not ask questions,' to which the students reacted with incredulity. Boylan discovered that the patterns and practices of the everyday life of the classroom conveyed strong messages to pupils and that these messages were interpreted through each child's life world filter.

Because research from an interactionist perspective is concerned with the ways in which the individual participants perceive and make sense of their shared experiences, deeper understandings of what have hitherto been called the 'hidden curriculum' might be better understood. Sierpiska (1998) describes the possibilities of such an approach for mathematics education research: 'The interactionist approach opens up avenues for research that are fascinating for mathematics educators because they take so much of the experientially known reality of the mathematics classroom into account. It focuses on what for us as teachers is the main occupation and the biggest challenge: our interactions with students in and about mathematics' (p. 58).

### **Accounts of the learner – the New Zealand research**

In the report on the Second International Mathematics Study (SIMS) (Department of Education, 1987) which surveyed the achievement of Year 9 and Year 13 students, the model for the study is described as resting on four major assumptions, which include:

Student achievement and attitudes in mathematics are affected directly by characteristics of the students themselves, by the coverage of the subject in class, and by the mathematics experiences they have had in previous years.

The teaching methods, and the content to be taught by the teacher, are influenced directly by the nature of the students in the class and by the levels of competence and attitudes to learning that they have on entry to the class. (p. 16)

This view of the students' attitudes to mathematical learning as both influenced by, and influencing, the teaching/learning process, point to a belief about student attitude formation as involving some form of reciprocal interplay within classroom



environments. The study measured student attitudes by employing a Likert-type response model to statements about feelings towards mathematics in general and specific topics within the mathematics curriculum, about the nature of mathematics and its utility outside of school. It was found that Year 9 students considered topics involving a high degree of numerical and algorithmic manipulation more important than spatially represented topics, that their enjoyment of the topic was enhanced by reduced level of perceived difficulty, but was not related to perceived importance. No significant correlation between achievement and enjoyment was reported. On average, Year 9 students were found to feel slightly more positive about mathematics than neutral, and teacher opinions seemed to have little affect on those of students, but no analysis was reported on the relationship of strongly positive or negative responses to variables such as sex, ethnic group or socio-economic background. While mathematics was viewed as necessary and useful by the teachers and students, there appeared to be uncertainty about its nature – whether it was a fixed body of rules and facts, or a dynamic discipline requiring creativity and conjecture.

Young-Loveridge's study of the development of children's number concepts from the ages of five to nine years (1991) included, in the final year, questions about attitudes to mathematics. Subject preference was measured by children's selection of their favourite three subjects from a choice of seven. A little over half the students selected mathematics as one of the three, with 37% of boys and 17% of girls choosing it as their favourite subject, demonstrating mathematics' significantly greater appeal to a much larger proportion of boys than girls. Subject preference was then related to achievement, with only two of the low-scoring children choosing mathematics as a favourite subject. While lower achieving students of both sexes were less likely to select mathematics as a preferred subject, of the high achieving students, 92% of boys and 45% of girls included mathematics in their list of three. The nine year olds were also asked to comment on their feelings about mathematics, how they knew whether or not they were good at it, to attribute a cause for their achievement in mathematics, and to specify if and why they thought mathematics was important. Although many of the children said they enjoyed mathematics, the positive attitude seemed mostly linked to getting lots of answers right. Self-perception of ability was mostly based on teacher feedback in the form of grades and ticks on their work. While most of the



children viewed maths as important, low achieving girls in particular, demonstrated concerning negativity towards mathematics, with little faith in their own ability.

Wylie and Smith (1993) charted the progress of thirty-two children through the first three years of their schooling. They compiled pictures of life at home and in the classroom through parent and teacher interviews, classroom observation and by direct accounts from the children themselves in response to open-ended interview questions. The study showed that by the end of their first year at school, children were aware of mathematics as a subject, and were able to tell whether or not they were good at it, and why. By the end of their third year at school, they demonstrated well-developed beliefs about what counted as maths, what maths work entailed, their own mathematical ability, and whether they liked maths or not. There were signs that lack of achievement in mathematics was already linked to a diminished liking for the subject. Their reported comments such as '*I'm not good at maths because sometimes we get crosses*', '*I'm not good at maths because I get the times tables wrong*', '*I'm good at maths because I've got brains and I work hard*', provide insights into the social worlds of these young children as they come to grips with the task of learning to be pupils at school and learning to be students of mathematics.

In the Third International Mathematics and Science Study (TIMSS), conducted in 1994, students' 'attitudes' to mathematics feature prominently. One of the key research questions of the study was: 'What mathematics and science concepts, processes and *attitudes* have students learned?' [italics added] (Chamberlain, 1997, p. 8). In the section of the report relating to the students, (Martin, 1997) it states that:

'As noted in previous IEA, national and international studies, the achievement of students is related to their personal characteristics and strongly influenced by aspects of their home environment. Attitudes to schooling, subjects under study, and features of the school environments are also important in determining levels of achievement.' (p. 137)

In order to determine students' attitudes towards mathematics, *response statements* were incorporated into the 'student background' questionnaire. The statements related to enjoyment of mathematics, degree of difficulty, and causal attribution in achievement in mathematics. The study then linked student responses to their performance of mathematical tasks. A strong correlation was demonstrated between

children's reported enjoyment of mathematics and their perceived ability. In the research report (Garden, 1997) it was stated that 'while a majority of students have positive attitudes to learning mathematics...it appears that beginning from a fairly young age there is an increasing proportion of students having lost interest in the subject, with a concomitant decline in their achievement. This effect is considerably greater for girls than for boys' (p. 252).

When the TIMSS research was repeated in 1998/99, it was reported (Ministry of Education, 2000) that, in response to the Positive Attitudes Towards Mathematics Index (PATM):

New Zealand students in general held positive views on the utility of mathematics and science and their enjoyment of them as areas of learning. New Zealand students did, however, tend to hold more moderate views than some of their international counterparts. The proportion of New Zealand [sic] that had very positive attitudes towards mathematics (ie, at the high level of the PATM Index) was 34 percent. This compared with, for example, the considerably higher proportion of Malaysian (74%), Chilean (45%), and Singaporean (45%) students in this category. Ten percent of New Zealand students held very negative attitudes towards mathematics (i.e., at the low level of the PATM Index) which was comparable to the international mean proportion of 11 percent...Within...most countries, including New Zealand, attitudes towards mathematics and science were related to achievement. (p. 13)

The TIMSS results for Year 8 and 9 students (Martin, 1996) also demonstrated a strong link between enjoyment and achievement. The report stated that, 'A notable feature of the statistics is the decrease from Form 2 (Year 8) to Form 3 (Year 9) of the proportion of girls harbouring positive thoughts about learning and using mathematics' (p. 136). It showed that while the girls' perception of their achievement tended to decrease, that of boys increased between those years at school.

In their evaluation of the *Beginning School Mathematics* resource (BSM) developed by the New Zealand government and introduced to schools nationwide in 1986, Visser and Bennie (1996) included pupils' attitudes to mathematics as an indicator of the effectiveness of the programme. Their research, in which 199 students were interviewed at age 5 years, and 155 of the same children interviewed eighteen months later, revealed that while 83% of new entrant children in the study expressed positive feelings about school in general, only 50% felt positive about mathematics. Eighteen

months later, positive feelings about school had increased to 92%, and about mathematics to 64% of girls and 68% of boys. Three symbols representing different facial expressions were used for the children to indicate their feelings in response to each question – one with a smiling mouth, one with a 'neutral' mouth, and one with a sad mouth. It is difficult to draw conclusions from this research given the very young age of the children and given that they were not asked to explain their choices, but it seems possible that their enthusiasm for school in general was not matched to the same extent by their liking for mathematics. However, the report cites the research of Bennie et al (1990) in which teachers had remarked upon the improvement of young children's attitudes to mathematics since the introduction of the BSM resource.

In the National Education Monitoring Project report of a study of children's learning of mathematics in Years 4 and 8, Flockton and Crooks (1998) state that 'students' attitudes, interests and liking for a subject have a strong bearing on their achievement' (p. 61). The *Mathematics Survey* (student questionnaire) used in the study consisted of items in which students rated themselves according to how they felt about mathematics, how good they, their parents, and their teachers thought they were at mathematics, how willing they were to take risks in mathematics, whether they did mathematics outside of school, and whether they wanted to pursue learning in mathematics when they were adults. From a list of typical classroom mathematics activities they were asked to select up to three preferences. The study included a range of open-ended questions in which the students were asked to nominate up to three things they thought they were good at in mathematics, to comment on what they thought people needed to learn in mathematics, or what they had to do to be good at it, how they coped with 'really hard' mathematics and what they considered to be, 'some interesting maths things you do in your own time' (p. 63). The results showed that 52% of Year 4 students were highly positive about mathematics, compared with 25% of Year 8 students. This decline in subject preference was accompanied by a marked decline in students' reported perceptions of mathematical ability from 40% of Year 4 students being very positive, to 4% of Year 8s. It could be argued that by Year 8, students had developed greater awareness of their comparative ability. However, the very low self-efficacy ratings of the Year 8 students are worthy of investigation.

All of these studies include an attitudinal or affective dimension of student learning based on the belief that the views and feelings of learners are an integral part of the learning process. Collectively, they indicate that a proportion of New Zealand children become alienated and marginalised through their learning of mathematics, and this phenomenon would appear to begin early in primary schooling. The children's perspectives on the nature (basic facts and numerical calculations) and purpose of mathematics (to get a job, be better at schoolwork, help their own children when they grow up) and their most often cited signs of success (right answers) would also suggest that learning mathematics is an impoverishing experience.

This survey of mathematics education research in New Zealand as it reveals aspects of the world of the learner, supports the contention of this thesis that from a fairly young age, many students are becoming either disenchanted with mathematics, losing faith in their ability to learn mathematics, or gaining limited and limiting views of what constitutes legitimate mathematical activity.

### **Student attitudes, affect and recent curriculum trends**

McDonough and Clarke (1994) noted that 'while mathematics curriculum developers have attended carefully to issues of content coverage and even to models of cognition, the same attention has not been accorded to student affect. The planning of instructional and assessment practice seldom includes consideration of such factors as student motivation, confidence, satisfaction or self-esteem' (p. 391). A review of national curriculum statements over the past two decades upholds this view, although the stated aims sometimes include general statements that incorporate some aspects of student affect, as the following examples show:

The National Council of Teachers of Mathematics (1991): 'Mathematical power ...involves the development of personal self-confidence...Students' flexibility, perseverance, interest, curiosity, and inventiveness also affect the realization of mathematical power.' (p. 1)

The National Council of Teachers of Mathematics (2000): 'All students should have access to an excellent and equitable mathematics program that provides solid support for their learning

and is responsive to their prior knowledge, intellectual strengths, and personal interests.' (p. 13)

Australian Education Council, (1990): 'An important aim of mathematics is to develop in students positive attitudes towards mathematics and their own involvement with it, and an appreciation of the nature of mathematical activity. The notion of having a positive attitude towards mathematics encompasses both liking mathematics and feeling good about one's own capacity to deal with situations in which mathematics is involved. Both are desirable but the latter is essential!' (p. 31)

Department of Education, New Zealand (1985): 'Aims ... to help children to develop a positive attitude towards, and a continuing interest in, mathematics... appreciate that mathematics is important in the world around them.' (p. 6)

Ministry of Education, New Zealand (1992) 'aims to: help students to develop a belief in the value of mathematics and its usefulness to them, to nurture confidence in their own mathematical ability, to foster a sense of personal achievement, and to encourage a continuing and creative interest in mathematics.'(p.8) 'The development of more positive attitudes to mathematics and a greater appreciation of its usefulness is the key to improving participation rates for all students.' (p. 13)

Ministry of Education, New Zealand (1994a) 'The learner must be at the heart of all planning.' (p. 4)

Overtly stated concerns about student affect and student beliefs regarding mathematics are conspicuously absent from more recently-developed mathematics curricula such as *The National Numeracy Strategy* of the UK (Department for Education and Employment, 1999), in which student enjoyment, confidence, subject relevance, meaningfulness and valuing of mathematics, scarcely feature. Ernest (1998b) suggests that the aims of a back-to-basics numeracy approach are congruent with the political thinking of radical 'New Right' conservatism, where learner differences are viewed as fixed and biologically determined. From this perspective, student affect is of little consequence.

Recent research in mathematics in New Zealand commissioned by governmental agencies, is notable for the complete absence of learners' perspectives. For example The Education Review Office (2000) study, which, in the wake of New Zealand's

relatively indifferent showing in the international TIMSS and TIMSS-R studies, compared the intended, implemented and attained curricula for mathematics and science of five countries - Korea, Singapore, The Netherlands, Ireland and New Zealand – did not include student affect in its investigation. At no stage were classroom environments explicitly or implicitly compared for their effects on children's beliefs about mathematics, or their feelings of confidence, motivation or enjoyment. The only allusion to attitudes came in the comparisons of societal expectations. Given that a significant proportion of the TIMSS research, and NEMP studies quoted in the introductory section of the ERO report, was devoted to gathering data on student affect, and given that both of these studies noted the link between students' negative attitudes and reduced achievement, it would seem important that the views of students be considered in any comparative study. The fact that students' perspectives were overlooked indicates a view of the learner as passive, and the paramount end of education as measurable achievement. In this view, issues of accessibility or comfort are believed to have negligible effect on student learning.

It may be inferred that policy makers' attention to learners' perspectives – the way students feel, and what they believe, about mathematics – has been subsumed in the sudden and dramatic implementation of 'numeracy' initiatives of the late 1990s and early 21<sup>st</sup> century. The political rhetoric surrounding children's mathematical learning has moved from an emphasis on enjoyment, confidence, meaning and accessibility to one of utility and achievement. In countries such as the UK and New Zealand, where relatively low scores in the TIMSS research have caused national embarrassment and consternation, the response has been the rapid implementation of rigorous, hierarchical, and intensive mathematical learning programmes to remedy what has been identified as poor teaching of mathematics, evident in the following statements from the Department for Education and Employment, 1999:

Numeracy is a key life skill. Without basic numeracy skills, our children will be disadvantaged for life. (Foreword)

Over the past few years an accumulation of inspection, research and test evidence has pointed to the need to improve standards of literacy and numeracy...the Framework illustrates the intended range and balance of work in primary mathematics to make sure that pupils become properly numerate...its purpose is to help primary and middle schools...set appropriately high



expectations for their pupils and understand how they should progress through the primary years'. (p. 2)

New Zealand's *Early and Advanced Numeracy Projects* (Ministry of Education, 2001a) is almost entirely devoid of explicit references to the lives of students. A search of the teacher training manuals reveals only one passing allusion to affective dimensions where it is asserted that 'the use of meaningful contexts, a key message of MiNZC<sup>2</sup>, is crucial if children are to see the relevance of mathematics to their lives' (Section C, p. 2). Copious in their prescriptions of assessment, learning stages, children's strategies and teaching activities, the manuals do little to either provide, or to guide the teachers' selection of, contexts that may be particularly meaningful or relevant for particular demographic groups of learners for whom 'meaningfulness' may vary significantly from that of others, such as boys/girls, rural/urban dwellers, Maori, and Pacific Islanders, socio-economically advantaged/disadvantaged students or English/non-English speakers.

Initial analysis of the early implementation of the numeracy project in New Zealand claims that, 'most children in the studies have made greater gains than expected in number knowledge and strategies, regardless of gender, ethnicity, or the school's decile<sup>3</sup> rating. There is greater enthusiasm for mathematics learning. In fact, children are having fun!' (Ministry of Education 2001b, unnumbered page of foldout pamphlet). This suggests that project evaluators assume that the methods advocated by the project are appropriate for *all* children regardless of their vastly varying backgrounds, dispositions and interests. It appears, both from this official report, and from the fact that children's views of, and feelings about, mathematics are not considered in the stated outcomes of the project, that *children's increased enthusiasm* and *having fun* are regarded more as pleasing by-products of the project, than as principal aims in themselves, or even as motivators or vehicles for the enhancement of mathematical learning. The project therefore assigns far greater value to children's measurable 'achievement' than to their social worlds of feelings, beliefs and behaviours as learners of mathematics. It could be argued that student affect is

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<sup>2</sup> *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992)

<sup>3</sup> Schools in New Zealand are rated on a scale from 1 to 10, by means of a calculation used to determine the mean socioeconomic status of their students. A rating of 1 indicates a school with the most economically disadvantaged student population, and a rating of 10, the most advantaged.

minimised and sidelined through this form of curriculum development. This view is strengthened by the fact that, in two early evaluation reports of the exploratory phases of the project, (Higgins, 2001; Thomas and Ward, 2001) no terms of reference have been included to gauge the impact of the initiatives on the classroom experiences of the learners, nor have the reports included teachers' comments on the effects of the project on children's lives. Learners feature solely in terms of statistically-represented, measurable learning 'gains.'

The lack of reference to learner affect in New Zealand numeracy project documents supports observations made by Pollard and Filer (1999), that pupils appear in current education reform as passive objects. Similar concerns have been raised regarding the numeracy strategy in England (e.g. Anderson and Boylan, 2000) suggesting that the creators of such initiatives do not believe in the existence of close links between achievement in mathematics, and the perceptions, feelings, and quality of life of the learner in the mathematics classroom. The re-emergence of this traditional view of education in which affect is regarded as irrelevant in the learning process, heightens the need for greater understanding of the inseparability of cognition and affect in learning, and of the inseparability of learner and social context.

## **Conclusion**

This chapter has shown how the research that documents learners' perspectives of their learning of, and about, mathematics has centred for the most part around the belief that a strong relationship exists between the learning of mathematics and learners' personal responses within and around that learning. The body of existing research that provides direct accounts of pupils, demonstrates convincingly that issues of affect must be addressed if students' mathematical learning is to be fully and fairly nurtured, and if students' participation in mathematical activity is to be encouraged at secondary school and beyond. Deep understandings are needed of learners' experiences of the everyday practices associated with mathematics within the classroom and elsewhere, while recognising the position of classroom practice within wider contexts.



It appears that many mathematics educators have come to accept student disaffection, marginalisation and impoverishment in mathematical learning as 'normal' since they appear to be so commonly experienced. They have tended to explain these unfortunate side-effects of mathematical teaching and learning as deficits in the learner, the home, or the teacher. Such views have led to the implementation of interventions to offset or correct these so-called defects. Recent literature indicates that this view is at best, limited.

The following chapter will explain the methodology developed to explore the sociomathematical worlds of the ten children in this study, and examine their interpretations about, and responses to mathematics as experienced in their everyday lives. These worlds include teachers, classmates and friends within the classroom and school setting, family and home, the routines and *objects* associated with learning mathematics, and the indirect influences and constraints of the structures, expectations, and requirements of the mathematics curriculum designers, government educational agencies, school, community, and wider society.

This approach takes the view of the learner as a *socially interactive sense-maker* of and about mathematics, within and through processes of interaction with others. Given the limitations of previous studies, the longitudinal ethnographic approach of this research has attempted to provide richer insights into how children, situated within complex social contexts, experience the learning of mathematics in their everyday lives. Student failure or success from this research perspective is portrayed as a social construct brought about through the everyday interactions of learners within their sociomathematical worlds and defined and explained within the dominant educational discourses and practices of current teaching.

### **Research directions**

In his review of mathematics education research, Niss (2000) called for investigation that not only describes but also *shapes* teaching practice. In order to shape, it can be argued, we need to understand as fully as possible the current state of teaching practice, including its history and complex dynamics, before we can begin to understand *why* the situation needs to be shaped and *what* shaping might be required.

Bauersfeld (1997) cited in English (2002) in commenting on mathematics education research in the 1990s observed that 'too often, the choice of a research agenda follows actual models, easily available methods, and local preferences rather than an engagement in hot problems that may require unpleasant, arduous, and time-intensive investigations'(p. 3).

The *how* of shaping must be built on a thorough appreciation of the 'hot problems' of the current state and the processes that drive and maintain that current state both at the macro and micro levels. This research aims to provide the kinds of understandings of the mathematical learning of children that may inform the 'shaping' of teaching practice. It has sought to cast light on the real issues of mathematics education as experienced by children in their everyday lives. To gain rich and useful insights into the dynamics of learning mathematics through compiling the lived biographies of individuals, it was considered necessary and indeed unavoidable, to employ arduous, intensive, and time-consuming, longitudinal, ethnographic methods. While the sample is small and localised, it is hoped that the generalisable methodology and findings of the research will add to the literature in this area.

### **METHODOLOGY: INVESTIGATING SOCIOMATHEMATICAL WORLDS**

As explained in the previous chapter, this investigation began as an exploration of children's 'attitudes' towards mathematics. It was shown how research in this area developed because of growing concerns that negative attitudes towards mathematics were prevalent at all instructional levels and that a link was believed to exist between attitude and achievement. It was also shown how 'attitudes towards mathematics' came to be viewed as a complex dimension of the learner's life consisting of a broad range of beliefs and feelings embedded within the individual's world of experiences associated with mathematics.

This chapter describes the theoretical framework and the methodology used to investigate this aspect of children's lives.

Because the original research questions were, 'How do children become aware of mathematics as part of their lives?' and 'How and why do their attitudes towards mathematics, including their feelings and beliefs about mathematics, develop and evolve over time?' it seemed appropriate to:

- focus the study on a number of individual children as they learned mathematics;
- extend the study over a significant period of time;
- begin the study sufficiently early in the primary schooling of the children to 'catch' them before their attitudes to mathematics had become either well-developed or overly entrenched;
- 'measure' and 'document' the children's developing 'attitudes' towards mathematics over the study time and to provide, where possible, explanations for the attitudes including any noticeable changes.

From the outset, it became clear that each child in the study was immersed in a complex social environment of which mathematics learning was an integral part.

While initial data-gathering revealed distinctive features common to most of their mathematical environments, individual children were responding in a range of different ways. Selecting only those aspects of each child's life that could be classified and measured as 'attitudes to mathematics', seemed unworkable and indeed, undesirable. If 'attitudes to mathematics' were to be in any way understood, there appeared to be a compelling need to make sense of the complex interplay between child and environment and interpret it in ways that were more all-encompassing than an approach that sought only to define, isolate and dissect individuals' 'attitudes.' It seemed far more appropriate and meaningful, therefore, to refocus the research towards describing the sociomathematical environments of the children and documenting each child's experiences of mathematics within those environments.

### **Theoretical frameworks**

With 'social context' firmly at the heart of the inquiry, it became necessary to choose useful theoretical frameworks to guide the research process. An initial survey of relevant disciplines included educational psychology, social psychology and sociology. Sociological theories seemed to be the most promising. One of these, *symbolic interactionism*, was to prove extremely helpful. Of particular relevance was the branch of symbolic interactionism known as the *social world of everyday life*. A growing understanding of these theoretical perspectives contributed to a methodology that evolved as the study progressed. This will be described later in the chapter. A combination of these interrelated frameworks was eventually adopted for the middle and later phases of the research procedure and analysis, as reflected in the final choice of title for this work, *The Social World of Children's Early Learning of Mathematics*.

### **Symbolic interactionism**

The theory of symbolic interactionism developed along with other forms of interpretative sociology such as phenomenology and ethnomethodology, all of which sought to go beyond earlier 'scientific' positivist and empiricist approaches that were based on the belief that society was some kind of organic whole that could be studied objectively or 'scientifically'.

The origins of symbolic interactionism can be found in the work of sociologists such as Mead and Dewey who, in departing from the influential works of Marx and Durkheim, turned their attention away from the large societal structures to the relationship of the individual with society. Mead (1934) maintained that each of us comes to view our 'self' in two ways: as 'I' (self seen from within) and as 'Me' (self seen from without, or as others might see us) and that these selves are continuously defined and redefined for us through our actions and interactions with others.

This perspective was developed by later sociologists, in particular Blumer (1969) who coined the term *symbolic interactionism*<sup>1</sup>. He identifies three basic premises of symbolic interactionism:

- human beings act toward things on the basis of the meanings that the things have for them;
- the meaning of such things is derived from or arises out of the social interaction that one has with others;
- these meanings are handled in, and modified through, an interpretive process used by the person in dealing with the things he encounters. (p. 2)

Blumer describes symbolic interactionism as being founded on a number of *root images*. The most important of these is *social interaction*. He contends that the human groups we term 'societies or 'cultures' exist only in action, and must therefore be viewed *in action*. By *action* he means, 'the multitudinous activities that individuals perform in their life as they encounter one another and as they deal with the succession of situations confronting them' (p. 6). He argues that it is useless to study the concept of attitude, for instance, as though it were a distinct and measurable part of an individual, and that only through an examination of the individual's social world of interactions, can any sense at all be made of concepts associated with individual attitude such as motivation, confidence and causal attribution for achievement.

Another root image of symbolic interactionism is its view of *objects* in our lives.

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<sup>1</sup> Herbert Blumer first used this term in an article he published in E. Schmidt (Ed.) (1937) *Man and Society*, New York: Prentice-Hall.

According to Blumer, an object is 'anything that can be indicated or referred to' (p. 11), so objects can be either physical, ( e.g. chairs and desks), social (e.g. 'pupils' and 'teachers') or abstract ideas such as moral, ethical or doctrinal principles (e.g. logic, ability, independence). Objects, Blumer believes, are given meaning and *understood* only through *interaction* with others. For instance, a mathematics exercise book becomes an object imbued with meaning for a child only through interactions with others. Through its everyday use in the classroom setting, as a prop in the daily performance of interactions between classmates and teacher, (Goffman, 1959) the child comes to know its label and its expected use. If the teacher says, 'Get out your maths books, turn to the back, put the short date and number one to ten in the margin,' the maths book may be viewed as a 'social' object with a particular social purpose and associated with particular social activities. Without such social interaction, the maths book would have little or no meaning. Although everyone in the class uses it, the *meanings* associated with the maths book will differ from one child to another. For some, it may be associated with feelings of dread, for others, excitement or boredom. It may be an object viewed with pride or disgust, as important or of little consequence. The meaning of the maths book for the teacher will be quite different again. She may view it as a source of burdensome 'marking', as a sign of children's progress or failure, as an indispensable tool, or even as a control mechanism in the teaching of mathematics. As we shall see in later chapters, the familiar 'maths book' is one of many *meaningful* and therefore *symbolic* physical objects in the social world of classroom mathematics. It is the interplay between *social interaction* and the *symbolism of objects*, which gives symbolic interactionism its name and its meaning.

A third root image of symbolic interactionism is the individual's development of, through interaction and response with others, a sense of '*self*', whereby the self can also be seen as an object. Through a process of role taking in which we attempt to see ourselves as others see us, we assume an 'identity' as we come to understand what it is to be who we are. A person can therefore interact with her/himself as an object by means of internalised 'talk.' Individual actions are believed to be a product of those factors the individual is taking into account at the time, and how these are being interpreted, while collective action is an interlinkage of the separate actions of individuals. It will later be shown how children in this study develop a sense of their

'mathematics identities' based largely on external indicators generated by interactions with classmates and others, and reinforced by self talk.

Symbolic interactionism was overshadowed by other theories during the 1980s perhaps because it was seen to focus too narrowly on individuals' processes of sense-making while appearing to neglect the wider and, many would claim, more important, influences of societal structures, thus failing to adequately take into account or explain the effect on individuals of broad social phenomena such as poverty, racism or gender issues.

Symbolic interactionism has enjoyed a revival in recent times, particularly among researchers in education. The work of Pollard (1985) and Pollard and Filer (1996, 1999) for example, convincingly uses a symbolic interactionist approach for a series of studies based on longitudinal case biographies of a cohort of children passing through 'Greenside' primary school. The authors describe how symbolic interactionism has been a strong influence in their research. Because 'it is focused on the creation of meaning as people interact together using both verbal and non-verbal communication ...' it could be used to demonstrate 'how classroom understandings and tacit rules are developed as teachers and pupils negotiate and cope with the challenges which each poses for the other' (Pollard and Filer 1999, p. 4). These authors have been greatly influenced by the work of Woods (1983) who summarises the theory in the following way:

At the heart of symbolic interactionism is the notion of people as constructors of their own actions and meanings. People live in a physical world, but the objects in that world have a 'meaning' for them. They are not always the same objects for different people, nor are the situations interpreted in the same way...People interact through symbols. A symbol is a stimulus that has a learned meaning and value for people, and man's response to a symbol is in terms of its meaning and value...Language is one such symbol, as are gestures and objects. People learn through interaction, an enormous number of symbols. The meaning of these are of course shared, and this enables smooth social interaction. (p. 1)

Furthermore, Woods provides a strong argument for a symbolic interactionist approach to educational research:

Because it is firmly located within the real world of teaching and its issues more obviously relate to teachers' day-to-day concerns, such as effective teaching, classroom control, and pupil deviance, interactionism speaks more clearly to practitioners than do some forms of sociology with higher degrees of abstraction and wider, system-related, concerns. But it is not unrelated to these. Thus it can serve as a bridge to more inaccessible pastures. (p. xii)

Meighan and Siraj-Blatchford (1998) write 'for symbolic interactionists it has been the interactions between individuals that have provided the best explanation for their actions. Individuals have been seen as active in accepting, modifying and resisting the influence of others' (p. 303). Cohen and Manion (1994) believe that such a view sits well with the kinds of analysis needed to make sense of the localised and intense interactions found in educational settings such as the classroom.

Swingewood (2000) describes the symbolic interactionists' view as one where 'meaning is not fixed but fluid, open, continually reconstituted in relation to the world of objects, a process by which individuals bring to every situation a stock of knowledge and symbols inherited from previous patterns of interaction on which they can draw to develop shared cultural definitions and further forms of action' (p. 171).

For mathematics education researchers too, symbolic interactionism is becoming a more widely used theory. Erna Yackel (2000) for example, explains that she is interested in symbolic interactionism because it can be used to examine sociomathematical norms and negotiated meanings that contribute to taken-as-shared understandings in the classroom.

Symbolic interactionism proved extremely useful as a framework for this research project because, while it recognised the individuality and therefore uniqueness of each person (case studies of children learning mathematics), it also acknowledged the importance of social interactions with family, peers and teachers, and environmental objects (books, teachers, work), with which, between which and within which each individual negotiates personal meanings through interactions with others (classroom activities/teacher pedagogy, school policy, homework).



### Sociology of everyday life

Building on Blumer's theories, Erving Goffman in his work *The Presentation of the Self in Everyday Life* (1959) turned sociological researchers' attention to 'everyday' interactions. He was primarily concerned with the relationship between the individual and the group and likened these micro-interactions to a theatrical play, within which each person 'acts' in defined 'roles', modifying and remaking such roles according to interactions with others.

Berger and Luckmann (1996) devote the first part of their book *The Social Construction of Reality* to examining the foundations of knowledge in everyday life – the reality of everyday life, social interaction in everyday life and language and knowledge in everyday life.

Among the realities there is one that presents itself as the reality *par excellence*. This is the reality of everyday life...I apprehend the reality of everyday life as an ordered reality. Its phenomena are prearranged in patterns that seem to be independent of my apprehension of them and that impose themselves upon the latter. (p. 35)

Douglas, in *Understanding Everyday Life* (1971) asserts that 'all of sociology necessarily begins with the understanding of everyday life, and all of sociology is directed either to increasing our understanding of everyday life, or more practically, to improving our everyday lives' (p. 3).

Weigert (1981) in *Sociology of Everyday Life* explains the nature of everyday life, and the challenge facing the researcher of everyday 'realities':

The structures which support our lives and processes by which we put these structures into action are always at work in our taken-for-granted world...One of the difficulties in analyzing everyday life is the hiddenness of the structures and processes which generate our 'of course' world. To 'do' a sociology of everyday life, then, we must see the ordinary as strange, the routine as new, and the unquestioned as doubtful. That is, we must switch from the natural attitude which supports our sense of everyday realism to a sociological attitude which sees into underlying structures and processes. (p. 48)

In order to understand a child's learning of and about mathematics, then, it would seem imperative to 'see' and describe the 'taken-for-granted', the 'ordinary', the 'routine' and the 'unquestioned' in the everyday world of that child.

In conjunction, the theory of symbolic interactionism, and its subsidiary theory of everyday life, were used to provide the necessary guiding principals by which to uncover and 'make strange' the features of a child's social world of learning mathematics, and to describe and explain the child's interactions, interpretations, sense-making and negotiating of meanings within that world. The 'attitudes' of the original research questions were seen as enmeshed within the social world of the child but quite inseparable from it.

### **The social world of the child**

From a symbolic interactionist's point of view, a child's world can be described as a complex, multidimensional web of interactions and experiences continuously woven and rewoven between actors and objects. Within such worlds, the child is negotiating, through interactions with others, a sense of 'self' or identity, and an accompanying understanding of and relationship with the self, with others and with objects.

A symbolic interactionist approach to the examination of children's learning of mathematics requires that the '*social worlds*' of individual children as they are learning mathematics in everyday settings be observed and described in great detail. Such observations must take into account the *social interactions* within those worlds, the *objects* that exist within those worlds and the sense-making through *reflections* of the children and of others, about those worlds.

A case study approach continued to be the most appropriate choice for this investigation, and it seemed that if sufficient depth, 'thickness' or richness were to be achieved in describing the social worlds of each case study child, then a longitudinal investigation would be the most productive. As Verma and Mallick (1999) note, 'the greatest advantage of this method is that it endeavours to understand the individual in relation to his or her environment' (p. 82), and, 'one of the strengths of the case study is that it allows the researcher to focus on a specific instance or situation and to

explore the various interactive processes at work within that situation...its prime value lies in the richness of the data that are accumulated and that can only be acquired as a result of long and painstaking observation and recording followed by subsequent analysis' (p. 114).

The work of Loughran and Northfield (1996) provides a good example of this kind of research. They attempted to discover the multiple realities of life for students in one Year 7 classroom and state that 'by exploring students' views of schooling, a complex array of factors begin to be uncovered which start to explain the diversity of individual responses in the classroom. Each student is developing a self-image and classroom experiences are both affected and shaped by their past and contemporary experiences' (p.64). They explored the learners' views of success and failure, the purpose behind learners' actions even where they may have been at variance with the intended learning outcomes of the teacher, and the ways in which the students attempted to reconcile the teachers' expectations with 'well-formed perceptions of the personal and institutional demands of school' (p. 89). They instance two students who introduced themselves in terms of their mathematical self-image or identity:

Kathy:	I'm Kathy and I'm no good at maths.
Rhonda:	My name is Rhonda and I can't do maths and I'm not much better at other subjects. (p. 64)

Reay and Wiliam (1999) and Winbourne (1999) also explore the growth of mathematical identity in the learning of mathematics.

While case studies are necessarily limited, rendering generalisations of dubious validity, Pollard and Filer (1999) who compiled what they termed 'strategic biographies' of individual children as they pursued their 'pupil careers', reason that 'the stories of children are likely to resonate with the experiences of others because they provide examples of fundamental processes in human experience, through which individuals develop and act in society' (p. 267). Weigert (1981) also argues that biography 'is the proper source of unity in human existence'(p. 62).

In choosing a starting point for the case studies, it was essential to consider what part of a child's early life might have the most significant effect on their growing awareness of mathematics as an entity or *object* as Blumer would put it. The extent to

which children are exposed to mathematics *before* they reach school will depend, it would seem, upon their early childhood experiences including input from parents, other individuals in the child's life, experiences at early childhood centres, and exposure to media such as *Sesame Street* and *Playschool* television programmes, books that include mathematical ideas and equipment such as *Duplo* and computer software that may encourage the development of certain kinds of mathematical thinking. But engaging in these kinds of 'mathematical' activities is different from being aware of 'mathematics' as a distinct part of the social world. Although some children may hear the word 'maths' used by parents, siblings or early childhood teachers, it is unlikely that they will have developed a concept of maths as associated with particular kinds of human activity, before they enter the social world of the primary school.

Familiarisation with the fundamentals of mathematics *as a social activity* would appear to begin in the early years at school (Wylie and Thompson, 1998). When Visser and Bennie (1996) interviewed five year olds who had not long been at school and asked them '*How do you feel when the teacher says 'it's time for maths?'*' each child was asked to respond by pointing to the symbol of a face whose expression most matched their own feeling. Whether the children's responses indicated if, or to what extent, they understood what was meant by the term 'maths', was not explored in the study. The responses of these same children eighteen months later appeared to have become much more definite. Visser and Bennie suggest that the children's longer experience of school might explain these changes. It can be assumed that at some stage during their first year of compulsory education, children are introduced to the term *mathematics*, or more usually in New Zealand, *maths*. They will come to associate it with various activities that in turn give them a 'feel' for and understanding of, what others mean by this term. As demonstrated in later chapters, the word *maths* may or may not be part of the everyday language of the home, playground or media, but it quickly becomes part of the language of the classroom.

### **Range of measurement**

As discussed in Chapter 2, previous research has tended to concentrate on limited aspects of attitude. The methods that researchers have most frequently used to gain

information about attitude are pencil-and-paper surveys based on questionnaires, often using some form of scale and often administered to large sample groups. Statistical analysis has been used to demonstrate whether a link exists between variables such as enjoyment of mathematics and achievement in mathematics. It appears that little previous research has taken an ethnographic longitudinal case study approach to investigate attitude to mathematics. This study attempted to identify features of the children's environments that appeared to contribute to each child's many 'attitudes' to, and beliefs about, mathematics.

'Attitudes' to mathematics cannot be considered as discrete, rather as interwoven threads forming a web of interrelating ideas. Key attitudes have been grouped below, accompanied by the kinds of questions used by previous researchers, and a number that have been specially developed for this research:

- *Beliefs about mathematics*

What is mathematics? (subject awareness and beliefs about its nature)

How does a person learn mathematics? (subject knowledge acquisition)

How good am I at maths? (self-efficacy/ confidence)

- *Feelings about mathematics*

How do I feel about it? (subject enjoyment)

Which bits do I enjoy most/least? (mathematics activity enjoyment level)

How hard/easy do I find mathematics? (perceived difficulty)

How do I feel about my progress? (achievement awareness)

What makes me want to learn mathematics? (motivation)

- *Opinions about mathematics*

How important is mathematics? (comparative subject value)

Why do we learn it? (perceived subject value)

Which parts of mathematics are the most important to learn? (mathematics skills/concepts comparative value)

- *Beliefs about self and mathematics*

How good at mathematics am I? (mathematical 'identity' )

What makes me learn mathematics best? (learning style attribution)

Why am I no good/OK/ good at it? (achievement causal attribution)

Why are other people no good/OK/good at it? (subject achievement attribution)

Why do I enjoy/not enjoy it? (affect causal attribution)

From a symbolic interactionist view, children make sense of mathematics in their social world by means of negotiated meanings, and expressed through understandings, feelings, opinions and beliefs, via social interaction through shared activities, events, and experiences that are, both collectively and individually, significant. Where these activities, events and experiences are recognised by others as constituting identifiable and anticipated routines and rituals, they may become symbolic. Symbols as explained by Cohen (1985) do not simply represent something else, they 'give us the capacity to make meaning' (p. 15), and while the symbol, for example 'teacher' or 'mathematics' may be shared, the meanings are not, having been 'mediated by the idiosyncratic experiences of the individual' (p. 14). Consciously or subconsciously, individuals construct their versions of sociomathematical 'reality' through interaction with others. The research needed to look for evidence of all these facets of 'attitude' in order to discover what 'symbols' might have contributed to the children's perspectives.

### **Webs of significance**

The children themselves, those with whom they interacted and the environments in which these interactions took place, constituted the children's sociomathematical worlds – their worlds of mathematics in everyday life. These worlds consisted of their families, friends, classmates and members of the wider community, including people they 'met' through the media of television and computer technology. The key constituents of these worlds are listed below:

#### *The child:*

- temperament
- self-esteem
- interests
- previous experiences
- relationships (parents, siblings, friends, peers, teacher).

*Home and out-of school contacts:*

- parental beliefs about mathematics
- parental expectations of, and aspirations for, the child
- parents' own mathematics experiences and expertise
- out-of-school access to mathematics concepts through construction toys such as *Lego* and *Connex*, puzzles such as *Tantrix*, games such as *Snakes and Ladders*, *Monopoly*, cards and *Yahtzee*, computer software that develop logic and spatial visualisation such as *The Logical World of Zoombinis* and *Tetris* and computer mathematics packages such as *Cluefinder*.
- involvement in regular and irregular activities which develop mathematics skills such as cricket (run rates and batting averages), swimming (measuring time and distance), baking, sewing, carpentry (dividing dough and laying out on tray, measuring length, volume, capacity and area)
- parental input into child's mathematics learning such as help with homework and 'testing' child's basic facts knowledge

*Peers:*

- friends' perspectives on mathematics including if and how mathematics is discussed
- peer views of who is and isn't good at mathematics and why

*Teachers:*

- training
- qualifications
- teaching style
- experiences with mathematics as a learner and as a teacher
- mathematical competence
- beliefs about mathematics
- assessment methods and expectations of the child

*School:*

- mathematics policy e.g. homework, assessment, participation in external competitions
- expectations of children
- communication with parents



While these form the key elements of the child's experience of mathematics in everyday life, this life is embedded within wider societal structures which may have a strong bearing on what happens at school and in the home. In this study, the child's experiences of mathematics in everyday life have been viewed as constituting a part of the child's social world. This dimension of the child's life world has been named the *sociomathematical world*, represented diagrammatically in Figure 1.

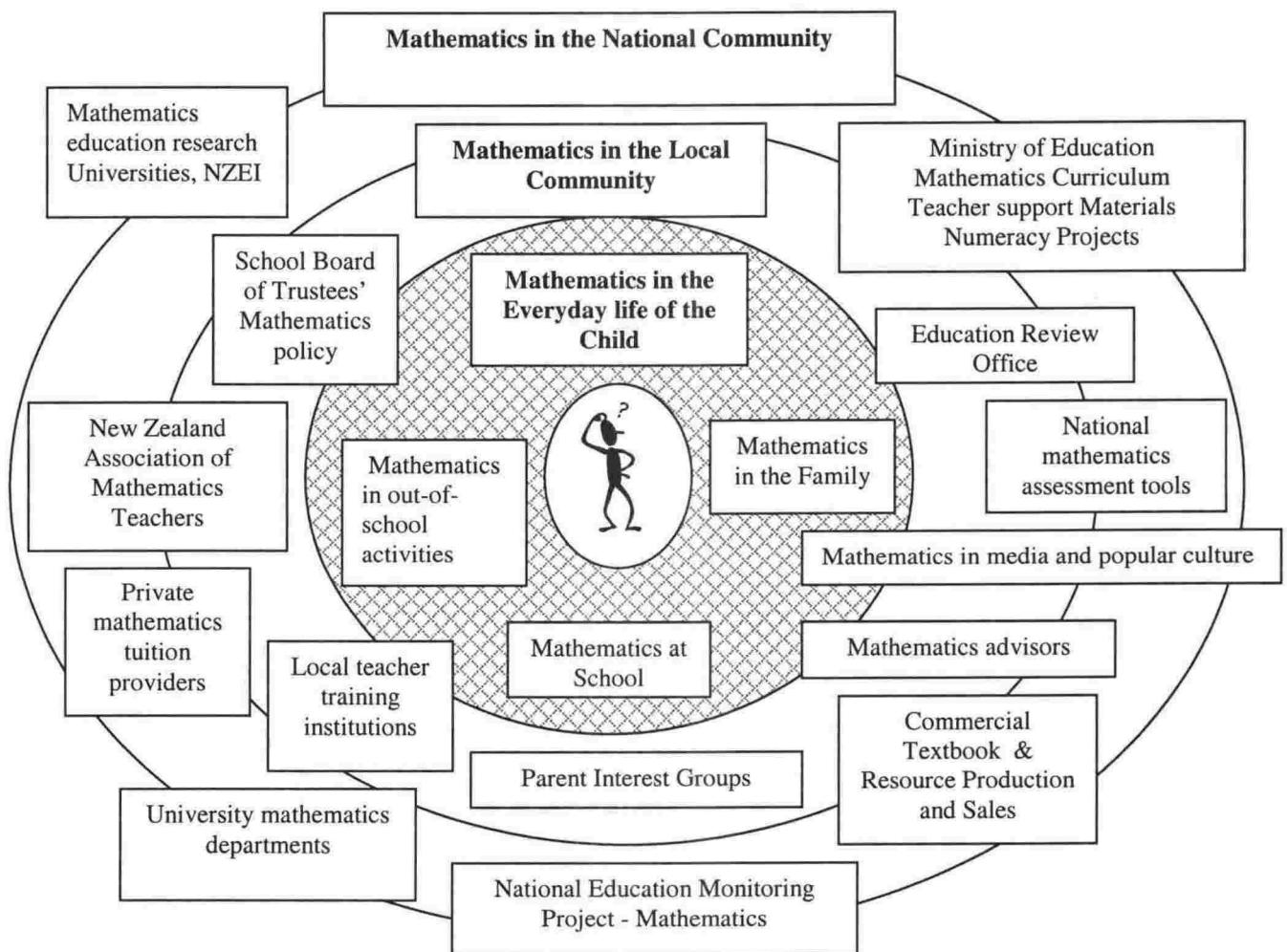



Figure 1:  The sociomathematical world of the child within its wider context

The sociomathematical world of the child is the world of everyday life, the arena in which the child, through regular and routine interactions with others, negotiates meanings about, and makes personal sense of, mathematics.

### Longitudinal ethnographic biographical case study approach

Woods (1983) argues that everyday life is best studied through participant observation:

'The key method of interactionist research is that of participant observation. It involves taking part in the ordinary everyday life of the group or institution under study in an accepted role, and observing both the group and one's own self... close observation and sympathetic interviewing over a lengthy period - a popular time span is a year - and in a variety of contexts can bring us close to an appreciation of that interpretive work, that construction of meanings that is at the heart of social life. (pp. 16-17)

Few longitudinal ethnographic biographical case studies have been undertaken in the specific area of learning in mathematics, although Pollard and Filer's (1996) research focused on five case studies of children's learning of reading and mathematics over a four year period. Their study however, failed to distinguish or analyse in any convincing way, the significant features of the learning environments of these children that may have contributed to their understandings, feelings, opinions, beliefs related to *mathematics* as separate from *reading*, and their concomitant achievement in these subject areas. It requires the sharply-focused eye of the mathematics education researcher with an understanding of the subject itself as well as familiarity with mathematics education from historical, political and pedagogical perspectives, to discern those elements of home, playground or classroom environments that may be contributing to a child's sense-making about *mathematics* in particular within those social contexts.

It would be expected that rich and detailed (thick) descriptions of individual children within these contexts should provide evidence about each child's position and sense-making within those environments. Subsequent analysis might suggest possible contributors to attitude formation. While much statistical evidence exists to link socio-economic factors, parental expectations and background, teaching approaches, peer influences (e.g. Garden, 1997), etc to attitude and mathematical achievement at school, there is a dearth of studies that have attempted to examine these links in any depth. An ethnographic case study approach might therefore be expected to shed greater light on some of these issues.

### Classroom mathematics as 'culture'

Nickson (1992) reviews the approach to education research which views the learner as situated within a 'culture'. She defines classroom culture as 'the invisible and apparently shared meanings that teachers and pupils bring to the mathematics classroom and that govern their interaction in it' (p. 102). She also warns of the danger of assuming that there is only one such culture. This research shows that classroom 'cultures' can vary markedly from classroom to classroom, that they change over time and that they differ from subject to subject. They are, in other words, fluid, dynamic and mutable. I will also argue, however, that significant congruence exists in the features of mathematics teaching practice encountered across a broad spectrum of classrooms. The similarities are found in the patterns of everyday routines, tasks and procedures that are associated particularly with mathematics. Therefore one of the tasks of this study was to describe the classroom 'cultures' as constituting the everyday processes in which each of the study children were immersed. Levitas, cited in Nickson (1992) describes culture as follows:

Every child in every society learns from adults the meanings given to life by his society; but every society possesses with a greater or lesser degree of difference, meanings to be learned. In short, every society has a culture to be learned though the cultures are different. (p. 101)

This study then, needed to focus on the lives of a number of children with the purpose of defining and describing their social worlds, in particular their *sociomathematical* worlds. It needed to uncover the meanings given to the activities within these worlds and look for ways in which these meanings might contribute to the formation of children's 'attitudes' to mathematics. Biography has been used as the key instrument in sociological investigation (e.g. Berger and Berger, 1972; Denzin, 1989). Smith (1994) suggests that 'biography, with a concern for the way a specific individual perceives and construes the world...moves the sociological interpreter towards the subject's point of view rather than that of the observer' (p.299). As Weigert (1981) argues, biography 'is the proper source of unity in human existence' (p. 62).

The research questions were modified accordingly to:

- What does the sociomathematical world of a child 'look' like, in particular, what are the significant interactions, objects and views of 'self' (identities) peculiar to that world?
- How do the interactions within that world contribute to the child's negotiation of meaning about that world?
- What aspects of that world appear to enhance or inhibit the child's learning of mathematics?

In order to answer these questions, the research needed to:

1. provide a picture of the 'everyday life' of each child by:

- describing the child's environments (home, school, classroom, peer group, community, popular media);
- constructing a picture of the child within each of these environments;
- tracking change over time.

2. provide a window into the 'everyday mathematics life' of each child by:

- describing the child's 'mathematics environments' (home, school, classroom, peer group, community, popular media);
- constructing a picture of the child within each of these environments;
- tracking change over time.

3. interpret the interplay of child and environment over time including gathering evidence of influences that may be significant contributors to children's developing and changing feelings about mathematics.

### **Methodological rationale and research approach**

Data gathering methods were required that would describe as fully as possible, the sociomathematical worlds of the children. An ethnographic approach was the obvious choice. Anderson (1990) describes ethnography as involving 'participant observation, description, a concern with process and meaning and inductive analysis'

(pp. 148-149). Because 'they go looking, rather than go looking for' (p. 150), the detailed questions emerge after the researcher becomes immersed in the situation.

Anderson lists the main sources of data in ethnographic research as:

- the physical setting;
- situations or events ;
- informants;
- archival material.

Because the study sought to understand as fully as possible the sociomathematical worlds of the children, it drew strongly on all of these sources, outlined as follows:

*The physical setting*

- classroom
- home
- school environment

*Situations or events*

- everyday mathematics sessions in the classroom
- special mathematics events such as tests
- special needs classes

*Informants*

- target children
- teachers
- parents
- other children in the classroom
- principals

*Archival material*

- children's mathematics exercise books
- children's self-assessment records
- children's mathematics work on classroom display

- teacher's records
- school manuals and reports

The following methods were used to collect data from these sources:

#### *Physical settings*

- Written descriptions in the form of field notes were used to record the significant features of the settings, particularly in the classrooms
- Classroom video footage provided a more accurate record of important aspects of the physical setting such as children's seating arrangements, wall displays and mathematics equipment storage.

#### *Situations or events*

- Written descriptions in the form of field notes were used to record situations and events during classroom observations
- Photographs were occasionally taken as a record of a special event such as a game, or of an everyday situation such as the teachers' recording of mathematics instructions and exercises on the whiteboard
- Video recordings

#### *Informants*

- Interviews and informal contact with children
- Interviews and informal contact with parents
- Interviews with teachers
- Interviews with principals
- Informal discussions with classmates

#### *Archival material*

- Children's mathematics exercise books
- Children's work samples
- Classroom mathematics task sheets
- Children's self assessment sheets
- Teacher's assessment records
- School policy documents

## Setting up the research

### *The Sample Size*

Initially, a complete case study sample of four children appeared to be a manageable end goal, given the complexity of material that would be gathered, and the time and resource limitations placed on the researcher. Because the research was to extend over a period of three years, however, it was necessary to start with sufficient sample size to allow for dropouts. A sample of ten children was therefore chosen as a starter group.

### *Choosing Schools and Making Contact*

A random sample of twenty schools was taken from all the state and integrated<sup>2</sup> primary and contributing<sup>3</sup> primary schools in the Wellington region. In early February 1998, the principals of the first ten schools on this list were contacted by letter, describing the nature and purpose of the research and seeking participatory consent. Of the ten, eight consents and two refusals were received. School A refused on the basis that, while the principal was keen for the school to participate, there was no teacher of Year 3 students at the school who was willing to be part of the study. School B stated simply: 'Sorry, we are unable to commit ourselves to this. All the best for your research.' The next two schools on the list were then contacted and both of the principals agreed to their schools' participation.

### *Initial setting up*

An information package for teachers of Year 3 children was then sent out to the ten consenting schools (Appendix 1). Some consent forms were returned quickly, but others required a follow-up telephone call. Within a few weeks, all schools had found a consenting teacher. The schools were then asked to provide, from that teacher's class, a list of all the Year 3 children whose seventh birthdays fell within 30 October 1997 to 28 February 1998. These parameters were chosen so that the sample children would be reasonably close in age and therefore comparisons, if any, could more easily

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<sup>2</sup> An integrated primary school is one that has been established as a private religious school, but has later been integrated into the state system, while retaining much of its original character.

<sup>3</sup> A contributing primary school caters for students from Years 1-6 pupils, contributing students to a local intermediate school for Year 7-8 students.



be made. This stage of the process proved difficult in some cases. Several schools had trouble accessing this information and in the end, provided me with children who were not of the requested age. In one case, this was not discovered until the parents pointed it out and another child had to be selected. In another case, this was not discovered until the study was under way, by which time it was too late to change. Another child who was found to be slightly older than requested was retained because of the time delay involved in finding a replacement.

#### *Selecting the children and contacting parents*

A child was randomly selected from the list provided by each school. The school was then sent the information package for the parents of the selected child, and asked to forward this to the family home in the envelope provided. This was done in order to safeguard the privacy of the parents. In every case, the parents agreed to participate, although some required a follow up telephone call to elicit a response. In one case, many such calls were necessary. The parent in question was unable to respond because she worked long hours, was managing a family of five children and simply couldn't find the time. Some parents were clearly excited by the fact that their child was part of the study and viewed it as a privilege, one saying, 'It's like winning lotto!' or as one parent wrote 'Thank you for the opportunity, Fiona. We will be more than happy to assist.'

Each school was allocated a pseudonym. Geographical features were chosen for these school code names because, not only were they simple to use, they sounded like places that could possibly exist, while in no way resembling any of the names of schools in the sample area. The code names were chosen carefully so that there would be no connection between the location of the school and its code. Where a child changed school, the new school was named by using a Maori version of the English name initially allocated to the school, as follows:

Bay School / Whanga School

Beach School

Bridge School

Cliff School / Pari School

Hill School /Pukeiti School  
 Island School / Motu School  
 Lake School / Roto School  
 Mountain School  
 River School  
 Spring School

Each child and teacher was also allocated a code name, and these were chosen for their dissimilarity to the original names, again to prevent possible identification.

### **The sample**

#### *The children*

The final random sample was consisted of 4 girls and 6 boys who were given the following pseudonyms:

<b>Girls</b>	<b>Boys</b>
Fleur*	Dominic*
Georgina*	Jared
Jessica*	Liam*
Rochelle	Mitchell
	Peter
	Toby

#### *Family structure*

Of these children, six were from intact two-biological-parent families (marked with\*). The remaining four children lived with their mothers, the natural fathers no longer a full time part of the family. In one case the father had died the previous year. In another instance, the father was in prison and had lost all contact with the children. In another, the father was living overseas but visited and maintained contact, and in another, the father lived close by with occasional contact with the child. All of these mothers were living with another male partner.

One child was an 'only' child. The others had one or more sibling/s. In at least two cases, the siblings were of different fathers.

#### *Ethnicity*

Eight of the children were entirely of European descent. Toby's father was Chinese/Malaysian and Rochelle's father Maori. Liam's father was an Irish/English immigrant. Jessica had Polish grandparents who maintained close contact with the family. All the children were born in New Zealand and all spoke English as their first language.

#### *Parent employment*

All the children were living in families where one or both of the parents worked full time. In several cases, a parent became unemployed or changed jobs during the study. In two of these cases, the male caregiver was transferred to work outside of the Wellington area and the families consequently relocated. There was a wide spread of socioeconomic circumstance among the study families.

#### *Absence from school*

During the first year, two of the children spent some time away from school because their parents had taken them on holiday outside New Zealand. Liam was absent for seven weeks, and Fleur for two. Because Fleur was away during a significant learning period, this absence had a marked impact, as discussed in Chapter 7 (p. 274).

#### *The schools*

The schools varied in size and character. Several were 'full primaries' (Years 1-8) while the others, 'contributing primaries' catered for Years 1- 6 only. Two were Catholic schools integrated with the state system, one was a private single sex school, and one an International School in a non-English-speaking country. This school was incorporated into the study when one of the children left New Zealand with his family.

### **Maintaining continuity**

As other researchers have found (e.g. Pollard and Filer, 1999; Young-Loveridge, 1991), longitudinal studies present problems because of the inevitable changes that occur in people's lives. New Zealand has a relatively mobile population, and so a significant dropout rate from the original sample was expected.

In order to retain participants' interest in and commitment to the project, a letter was sent at the end of the first year and second years to all participating principals, teachers, and parents, summarising some of the early results of the research (Appendix 2) and thanking the participants, children included, for their support, interest and cooperation. One principal was grateful for the feedback commenting that it had been shared with the staff and points that had been raised had been useful in the school's ongoing planning of its mathematics programmes. Where possible, contact was made with the teachers who would be working with the target children the following year.

During the three years of the study, there were a number of changes of school and teacher. After the first year, Fleur and Mitchell were transferred to new schools because the family had moved house. After the second year, Georgina was shifted to a neighbouring school, and Toby's family relocated overseas. In each case, it was necessary to contact the principal of the new school in order to request consent for the continuation of the research. Fortunately, all of these principals readily agreed.

In all cases, the children changed classroom and teacher for the second and third years of the study. Each of these teachers was informed of the research and consent to participate was requested. Once the study had been under way for some time there was pressure to comply, but wherever anxiety or reluctance was detected, teachers were fully informed of the research purpose and reassured that the research methods had been found to be comparatively non-threatening and unobtrusive. Prospective participant teachers were encouraged to discuss the process with previous year's participants to gain a sense of what to expect. All Year 4 and 5 teachers consented to become part of the study.

At the end of the first year, the teachers were asked to provide personal impressions of the research process. Comments were universally reassuring. As one teacher explained 'At the beginning I nearly backed out because I just thought, "Oh no, I don't want to be observed," you know. Then I thought I've got nothing to hide, you know, because I'll know it'll be fine and you're not exactly the most threatening person'.

Two families moved out of the Wellington area during the study. By the end of the first term in the second year, Fleur's family let me know of their pending relocation to another New Zealand city. From the outset they had been keen participants, and assured me that they wished to continue. We discussed ways that this could be achieved and it was decided that I would travel to the new school every few months in order to keep track of this child. Four such visits were made to this school. During the final data collection phase, I was unable to visit the school but interviewed all participants by telephone. The teacher also sent me evidence of the child's achievement in mathematics.

Towards the end of the second year of the study, Toby's family informed me of a likely move to Europe. They too were committed to their child's continuing participation. While this was obviously going to pose problems, we talked of ways in which some of the research could continue. During the final year of the research I was able to make one visit to Toby's new home and the International School he was then attending. Other interviews were conducted with Toby by telephone and with his mother and teacher by email.

The parents' demonstration of a genuine sense of ownership towards the project was heartening. In every case where significant changes were imminent, the parents kept me informed.

## Gathering the data

During the first year of the longitudinal ethnography, a workable timetable was developed (Table 1). It was successfully continued for the three years of the data-phase, with modifications where necessary.

**Table 1: The annual research timetable**

**Key:** 😊 = face to face, ☎️ = by telephone, 📖 = written material, ✉️ = letter

	Early (March)	Mid-Year (June/July)	Late (November)
Child interview	😊	😊	😊
Parent interview	😊	😊	☎️ or 😊
Teacher interview and viewing of child's work samples	😊📖	😊📖	😊📖
Principal interview	😊		
Classroom observation	😊	😊	😊
Classroom video recording		😊	
Participants' progress report			✉️

## Working with the informants

### *Gaining Entry - Talking with the principals*

At the beginning of the study I made an appointment with each of the principals to inform them in person of the purpose and procedures of research and to gain background information about mathematics teaching at their school (Appendix 3). When some of the children in the study changed schools, it was necessary to make contact with the principal of the following school firstly to gain consent to continue the research and after that to make a time for the interview. Principals were often very interested in this research and gave generously of their time to talk with me. While I generally kept to my short list of questions I found that I also had to be prepared for the many questions the principals would often ask me. These initial meetings were essential in gaining and maintaining access. Because they served to

establish trust and rapport, they were critical to the smooth running of the project over the three years.

### *Talking with the Parents*

Very early in the research, meetings were arranged with the parents of each target child. Parents were offered the options of meeting either at their home or at their child's school. In all but one case, parents opted for home visits. A list of questions had been prepared for the initial interview (Appendix 4). These were designed to find out about the parents' own experiences with mathematics at school, and as adults, about their opinions relating to mathematics in their child's schooling, and about their perspectives on their child including the child's temperament, interests, and progress at school. The interview was also designed to provide information on the qualifications and expectations of the family, their beliefs about learning, teachers and schools and their hopes for their child. It gave insights into family routines and resources. Information was also sought on the kinds of activities that families engaged in with their child, particularly those that might involve mathematical thinking such as games requiring problem solving and counting skills, measurement, and calculation. Many studies suggest that these factors are significant in children's achievement at school and the link between the mother's experiences and beliefs have been shown in a number of studies of student achievement, to be of particular significance (Garden, 1997; Wylie & Smith, 1993; Wylie & Thompson, 1998).

### *Talking with the children*

Hadfield and Haw (2000) stress the importance of providing young people with a 'voice'. They say 'Young people are in the best position to talk about being young...we should listen because it is the voice of experience. Only young people can know how it is to be a young person at a particular time, in a particular community...as a pupil in a school or within a family.' Connolly (1997) cautions that in search of the 'authentic' voice of the child, adult interviewers inevitably influence their subjects, suggesting that critical reflexivity is required to acknowledge and offset researchers' blindness to infiltration of their own values and assumptions into the research process. While this study aimed to maximise opportunities for the children to communicate in their own ways, the meanings they created about mathematics



within their individual and unique worlds, there were obvious constraints on children's willingness and/or ability to do so, given the methods that were used.

The children were interviewed on each of the visit days, either immediately before or immediately after the mathematics lesson, whichever fitted best with the classroom programme. On our first meeting, I introduced myself and clarified the purpose of my visits. I explained that I wanted to find out about children learning maths, what they thought and felt about maths, and how their ideas might be able to help teachers make maths learning better for children. This approach is in line with Schatzman and Strauss (1976) who stress that it is vital when interviewing, to 'communicate the idea that the informant's views are acceptable and important' (p. 74). In an attempt to put each child at ease, I began the interview by asking some general questions about school, friends, and favourite playtime activities.

While conducting the interviews, I sat alongside each child, usually at a table, with the tape recorder placed in front of us. The tape recorder was an unfamiliar device to some of the children, and several of them asked me to demonstrate how it worked.

The interview was based on a prepared list of questions (Appendix 5). I made sure that the questions were visible to the children, and as they grew older, some took a real interest in them. Because many of the questions were open in nature, this allowed for the children to use their own words and ideas to describe their sociomathematical worlds. A self assessment questionnaire sheet was included in the interview - *How I Feel About Maths* from Beesey and Davie, 1991 (Appendix 6). This was used at every child interview throughout the study. Sometimes the children completed this themselves, sometimes I recorded for them, whichever was most appropriate or expedient at the time. Because the sheet provided the children with scales for rating themselves in terms of how they felt when doing mathematics, and how good they thought they were at mathematics, this provided valuable quantitative data tracked over the three years.

Location proved to be an important factor in the success of the interviewing. Privacy was essential, yet sometimes impossible to achieve or maintain because of limited

available places in primary schools for this kind of activity. Teachers were asked to suggest suitable places for interviewing and usually recommended a particular site. Occasionally these areas were found to be unsuitable. There was rarely a room already arranged for us, even though the teachers had been reminded beforehand that I would be interviewing the child. Commonly-used sites were the medical room, the library, the reading recovery room, resource room or interview room. Sometimes the principal's office was offered. Often, however, the staff room was the only space available but interviews there were subject to frequent interruptions. Sometimes other children would come in to work on projects such as cooking. This would create both background noise and a potential audience, thereby inhibiting free communication. Whenever others were present, there was a noticeable clamming-up effect on the child interviewee. At such times, the interview was paused until the 'intruders' had departed. If that was impracticable, the child was asked whether he or she felt comfortable about continuing, or whether it was preferred that we move to another location. This occurred on several occasions. Where peers were party to what was going on, as happened on an occasion when we were directed to use a see-through withdrawal room adjacent to the classroom, it proved impossible for the child to relax or maintain focus.

Anderson (1990) says that because ethnographers 'go looking, rather than go looking for' (p. 150), detailed questions will often emerge only after the researcher becomes immersed in the situation. This proved to be the case. The interview questions required constant review, undergoing regular changes, additions or deletions. Such alterations were necessary for a variety of reasons:

- as the study progressed, my view of what was important expanded and shifted and the investigation was re- focused from 'attitude' to 'social world' and 'meanings';
- some questions proved more fruitful than others;
- the children themselves changed, for example, drawing pictures of themselves doing mathematics was appropriate at the beginning of their third year at school but with their increasing self-consciousness was found to be more difficult by the beginning of the following year, and discarded in the final year;

- it appeared that the children sometimes become bored or frustrated with repetition of questions, or had prepared pat answers in advance. For this reason, new questions were added, the old reworded, or acknowledgment made of the 'same old question' with explanations of how helpful it was in finding out whether their ideas had changed or not, and why.

It was found that there were limitations in adhering too rigidly to the list of questions and their precise wording because opportunities for gaining further information were sometimes lost through having done so. Communication was greatly enhanced by a flexible 'chatty' approach. The process became more relaxed over time as the children became more familiar both with the interview process and with me. As the questions became more familiar they could be used more flexibly, often as discussion starters rather than as a precise 'tool'. The duration of the interviews averaged about 30 minutes depending on how fully the child responded to the questions, and the length of time that was available.

The success of the interviewing varied. From the outset, Fleur, Georgina, Jessica, Dominic and Toby were extremely easy to talk with because they were able to articulate their thoughts and feelings freely and fully. They appeared to be at ease with adults. Rochelle and Liam were more reticent though they did well in what for them was clearly a somewhat unusual and therefore uncomfortable situation. They grew more talkative over time. By the third year of the study, they were much more forthcoming. It was hard to tell whether this was because I had become more familiar, whether they had developed greater facility with oral language, whether they had become more self-aware and self-confident generally, or whether they had had more experiences to talk about. It was most likely to have been some combination of these factors.

Generating conversation was difficult with three of the boys. Jared in particular made very little eye contact, which was disconcerting. However I was not alone with this problem. His teachers and even his family talked of how difficult it was to know what Jared thought or felt about things. As his sister said in an exasperated tone during my final family interview, 'Can you *ever* get anything out of him?' Peter

displayed extreme difficulty in accessing or sharing his thoughts and feelings, which was a real test of my patience and ingenuity when faced with a string of 'don't know' responses. It was a formidable task to find ways to gain his trust and to provide him with opportunities to communicate what he could not or would not say! It was found that he did best when presented with scales on which to rate himself, rather than having to generate his own ideas. His responses were clearly based on careful thought. This experience demonstrated that an open question approach might not suit every child. Mitchell also posed a communication challenge because of the limitations of his concentration, his comprehension of the questions, and his verbal skills. For this reason, some of the questions were omitted during his interviews.

Even when communication was hard work, all the children conveyed a real sense of how the world appeared to them, and what objects, symbols and everyday happenings were of particular significance. In spite of the challenges, the child interviews yielded without a doubt, the most rich and compelling views of the children's beliefs and feelings and how and why these changed over time.

#### *Talking with the teachers*

The initial interview with each teacher, based on a structured list of questions (Appendix 7) was recorded on audiotape. The interviewing usually took place before or after school. In several instances, although it had been made clear that the interview would be recorded, the teachers seemed to prefer to be questioned in the staffroom during morning interval. At such times it was necessary to adopt an informal approach, even abandoning the question list and just 'going with the flow'. Had I insisted on our moving to a separate room, those teachers could well have felt threatened. The need to establish a good working relationship with the teachers was paramount. In spite of the informality of some of those interviews, and set against the background noise of the staffroom, it was still possible to elicit important data relating to the teachers' own beliefs about mathematics, and the teaching and learning of mathematics. It was also possible to establish how the teacher typically organised mathematics sessions and insights were gained into the study child's progress from the teacher's point of view.

Subsequent interviews were very informal. Teachers would often just chat freely about how the child was progressing and recount events that they thought might be of significance. They were very helpful in sharing with me their assessment records and child's work samples. Specific questions were asked only when necessary. This seemed to be the most unobtrusive approach. Because of time constraints and pressure on teachers particularly later in the academic year it was necessary to minimise the impact of the visits.

### *The setting*

Careful note was taken of the general physical environment of the classroom including the type and arrangement of furniture, the wall displays, and the availability and storage of equipment. It was hoped that this would reveal clues as to what was valued by the school and the teacher and the visual messages to which the children in the classroom, and in particular the study child, were being exposed.

Physical evidence of mathematics teaching and learning was of special interest and proved revealing. The *visible mathematics* in each classroom might include:

- mathematics equipment and how and where it was stored;
- mathematics in the class timetable;
- wall displays of children's mathematics recording;
- evidence of class mathematics activities;
- mathematics-oriented activities and equipment.

### *Events and situations*

Each teacher was carefully observed operating within the classroom. Of particular interest were classroom organisation, classroom discourse, patterned routines and signs of what was valued in mathematical activities. Observations included:

- the teachers' relationships with children in general and the study child in particular;
- the teachers' issuing of instructions and implementation of classroom procedures;
- the teachers' questioning styles;
- the teachers' responses to children's answers.

Decisions about time spent in classrooms posed a dilemma. On the one hand there was the need to spend sufficient time in the classroom to gain a full sense of the everyday happenings there and on the other hand, the need to limit visits so that the teacher and child were not placed under excessive scrutiny. A delicate balance had to be negotiated and maintained and much depended upon the relationship that I was able to develop with the teacher and child.

Another dilemma arose from the need to achieve a workable 'status' in the classroom. While I was aware that I was a privileged guest in the classroom, I also wanted to become part of the 'fabric' of the class environment in order to get sufficiently close to the action to really see and hear what was going on, hence a tension between achieving minimal interference while taking the role of participant observer.

Observation visits were arranged well in advance and usually slotted into a carefully organised timetable allowing for all observations of a given phase of the study to take place within the span of a few weeks. This was important because schools tended to be doing similar things at given times of the year, for example conducting Progress and Achievement Tests (P.A.T.s) in the sixth week of the first term of the school year, and participating (or not) in Maths Week<sup>4</sup> events or the Australian Mathematics Competition<sup>5</sup>.

This enabled comparisons to be made between school 'cultures'. While the visit arrangements were initially made directly with the teachers, the parents and principals were always informed of pending visits and where possible the teachers were also reminded the day before, just in case of a last minute change of timetable subsequent to the setting of the appointment. On one occasion the teacher could not be contacted so a message was left. Because this was not passed on, I arrived to find that the teacher had forgotten about my visit and was about to take the class on an excursion. Rather than disrupt the planned programme for the study child, I rescheduled the visit.

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<sup>4</sup> In New Zealand, an annual national Maths Week, promoted by the New Zealand Association of Mathematics Teachers, is usually scheduled for the first week of July.

<sup>5</sup> The Australian Mathematics Competition is usually run early in the 4<sup>th</sup> term of the school year. New Zealand schools can opt to compete. They usually select only their most 'able' students to take part.

Minimising intrusiveness always took precedence when arranging and conducting the visits.

Visits were often arranged so that I could arrive at school before the start of the teaching day. It was useful to spend a few minutes in the classroom before school so that I could freely and systematically explore the room and unobtrusively record evidence of mathematics teaching and learning. I would also look for displayed examples of the study child's work in mathematics and other curriculum areas. Often I would discover wall displays of children's goals or personal profiles, especially at the beginning of the year. This would provide extra data on the child's beliefs and feelings about him/herself, likes and dislikes, and perceived strengths. It was also at these less formal times that children would often approach me to enquire about who I was and what I was doing in their classroom. The most common question asked of me was 'Are you our teacher today?' When they heard that I was there to see how children learned maths, they would often volunteer their candid thoughts on the subject. Such interaction helped to establish and maintain rapport, dispel fears, or dissipate excessive curiosity before the formal daily classroom programme began, as well as providing valuable accounts of children's views of mathematics in those classrooms.

Weigert (1981) describes appropriate tools for undergoing ethnographies of everyday life, saying that the best way to understand 'what is going on and who is doing it' is through naturalistic observation and description of concrete situations. He also suggests that through observing and interpreting people's reactions when their everyday routines are disrupted, we can come to understand more about what those everyday routines 'mean' for the participants. This *disruption factor* was experienced in some instances as an unintentional result of my visits, but through the responses of the children, the meaning of 'everyday' routines was clarified, as discussed in the concluding chapter (p. 229).

To begin with, it seemed that a detached observer role would be desirable in order to avoid possible 'contamination' of the everyday routines under observation, and to reduce the burden for the teachers and children who were being observed. As the



study progressed, it seemed that not only was absolute detachment impossible to achieve, but that 'detachment' often equated with 'distance' from the subjects of the research so that important opportunities to 'see' or make sense of events in the classroom were being missed.

During the teaching time, I positioned myself near the target child in a place where I would not be seen to be interfering in what was going on. This was usually at the back or to one side of the room. In the form of field notes, I recorded as much as I could of all that I saw and heard. Because classrooms are sites of enormously complex social activity where so much is happening at once, this was often difficult. Inevitably, choices had to be made during the busiest times as to what was most important to record. While my priority was the target child, I also needed to gather information that would help me to understand the social context of the classroom. This might include interactions between children and teacher, children and children, everyday routines of the mathematics lessons, and the teacher's specialised use of language when teaching mathematics. Details such as teacher instructions and explanations to the whole class, teacher comments to other children that could be heard by the whole class, or children's incidental talk near or around the target child, were also recorded where possible. So rich were the data that I found I was writing almost constantly during classroom observations.

It was a concern that the children and teacher might feel inhibited by such concentrated recording, but only in a few cases did it appear to disturb those being observed. Other children sometimes asked me what I was doing. I always replied that I was wanting to find out how children learn about maths and how they feel about it. This response was invariably met with interest and would often prompt some significant comments from children, helping considerably to flesh out my picture of cultures within the mathematics classroom. Two of the teachers chose to formally introduce me to the whole class and explain why I was there. In one instance, this clearly embarrassed the study child. In most cases however, I was accepted into the classroom with a minimum of disruption and, as I became known to the children, with friendly greetings.

Once the study child was working on a designated task either alone or in a group, I would watch for a while, and then, particularly in the second and third year of the study when the children knew me well, I would often approach the child and ask them to tell me about what they were doing, how they felt about this particular activity, and how difficult it was for them. It was also a good opportunity to check with them whether this session was typical or not. It sometimes happened that the lesson was far from usual.

Occasionally I would set my notebook aside and 'engage' in the mathematics activity alongside the child, asking questions about the task, and the approaches they were using. Such interaction added to my awareness of the child's mathematical learning processes and understandings which helped me to make sense of how the child was feeling about mathematics and why. When I stepped out of the 'detached observer' mode and became more of an 'interested participant', the children appeared to really enjoy this kind of communication and would talk much more openly about mathematics. The children seated nearby would often join in the conversation and these discussions would frequently generate insights into the mathematical 'culture' and peer perspectives of that particular mathematics classroom. This was the most valuable part of the data gathering process and led me to believe that to fully understand a child's world one has, in a sense, to become part of it.

In my observations I particularly noted the ways in which the teacher talked about mathematics and interacted with the children, especially the study child. I wanted to gain a sense of what was valued in the teaching of mathematics, and of the everyday routines that characterised the mathematics programme, such as regular activities, grouping systems and use of tools such as books or equipment. All the while I was attempting to 'see the familiar as strange' as Weigert (1981) urged, so even the most apparently 'ordinary' details were recorded, including, where possible, teachers' exact words when engaging with the children.

On one of the three yearly visits to each of the schools, I took a video camera into the classroom. Consent to do so had been granted in advance by the parents and teachers, but I found that the camera, which I held and used to follow the movements of the

study child, often produced an unsettling effect on the entire class, teacher included. In the first year I filmed complete mathematics sessions but by the second year, I only recorded a few small samples to view later as a backup to the fieldnotes. The videos were extremely helpful in analysing important features of teacher-child interaction such as wait time after questions, and the speed of the *Quick Ten*, or as a visual record of children's mathematics work or seating arrangements. In one instance, the teacher was clearly very anxious about being filmed, so the camera was put away with an assurance that the videos were not crucial to the study.

## Archives

### *Self Assessment Recording Sheets*

I was keen to involve the teachers in some simple data-gathering about the children's attitudes about maths between my visits, so suggested at first that they encourage the children to write about their maths in some way. The Lake School teacher liked the idea of a recording sheet that could be easily completed by the children so we designed one together (Appendix 8) and class sets were copied for all the other teachers in the study. While I encouraged them to try the sheets, I did not insist. Some teachers used them more conscientiously than others and where they were used regularly, they provided a useful record of the topics the class had been studying in maths and the child's response. The sheets later became a helpful talking point when I was interviewing the child and the teacher.

### *Mathematics exercise books*

Mathematics exercise books provided a rich documentation of the ongoing classroom mathematics programme. I was able to learn much about the pedagogy of the classroom and the children's feelings and ideas about mathematics through viewing and discussing the books with the children. The children appeared to be quite eager for me to see their books and during our discussions about the mathematical activities they had been recording, important additional questions and comments would frequently arise. The books were a way of determining what mathematics topics had been taught since my previous visit and I was able to ask about these if they had not already been mentioned. It could be seen how often the books had been used and

therefore how big a part they played in the everyday life of the classroom. I was able to note the frequency of activities such as *Bingo* or *Quick Ten*, identify the source of the activities and gauge to what extent worksheets, textbooks or equipment were being used. They were also an important way of triangulating and verifying teacher and child statements. The books indicated, through the choice of activities, the teachers' comments and use of 'marking' systems, what was valued in the classroom mathematics programme and how mathematics was being taught. With permission from child and teacher, I would sometimes photocopy samples from exercise books.

### *Teachers' records*

These were a valuable source of evidence, not only about the progress of the study children, as perceived by the teacher, but also about the kinds of mathematical skills the teacher was choosing to assess and record, and the ways in which recording was done. They gave an indication of the kinds of assessment practices being used in each classroom and also revealed the limitations of assessment information. As one teacher remarked, when reflecting on her records, 'This assessment doesn't show what she [the study child] can really do. I know she understands it better than this because when we are working in a small group talking about it, she can do it. It's just that when it comes to the test, she can't.'

The teachers frequently showed me examples of children's mathematics work. Some of these constituted part of the assessment records, and others had been completed as a regular part of the programme and had been recently collected for marking or were displayed in the classroom. They proved to be a rich source of information, both as a record of how each study child responded to particular activities, and as evidence of the kinds of activities being regularly chosen by the teacher for the teaching of mathematics, and the kinds of resources being used.

### *School Documents*

School documents such as mathematics policy statements, school data relating to students' mathematics achievement, and school information leaflets, were a source of invaluable information. Where these were available, they provided significant background information about the wider context of the school, the kinds of

mathematical skills the school was identifying as valuable, and those they were choosing to assess and record. They gave an indication of how mathematics was viewed and valued generally as a subject at the school, and the kinds of resource allocation and professional development commitments the schools were making for mathematics. They provided evidence of the kinds of assessment practices expected to be used in each classroom and at different age levels. The ways in which the schools presented their mathematics philosophies to parents were also very revealing.

### **Analysis, interpretation and presentation**

It was originally intended to present the stories of the case study children as ten separate sociomathematical biographies. This seemed not only the most straightforward option but appeared to link most readily to the research questions.

During the ongoing open data coding process following the grounded theory methods described by Strauss and Corbin (1990), the compelling emergence of common themes of classroom observations, participants' accounts, and archival material, dictated the final form of the analysis and presentation of the research. This kind of response to concepts revealed by coding was also experienced by Boaler (1997a). Prus (1996) claims that 'by drawing comparisons and contrasts across settings, we not only arrive at a richer understanding of each setting, but of similar processes across a wide range of settings...indeed, only by being acutely attentive to the ways in which people experience and shape their worlds and drawing parallels across situations can we hope to achieve a theory of action that reflects group life as it is accomplished' (p. 164). Indeed, so distinct were the commonly-found patterns uncovered in this research of everyday taken-as-shared mathematics teaching practice across the thirty-seven classrooms in the thirteen case study schools visited during, and reinforced by home expectations and interactions, that four predominant features of the sociomathematical worlds of the children emerged as demanding particular attention, as outlined in Chapter 1. These four characteristics were:

- speed activities;
- catering for perceived mathematical 'ability';
- establishment of distinctive protocols of 'doing' maths;

- construction of mathematics as a dichotomous subject of 'right' or 'wrong' facts and procedures.

These features have formed the themes of chapters 4, 5, 6, and 7. Within each of these chapters, the individual and unique ways in which each study child responded to these features are discussed, using the key concepts of a symbolic interactionist approach as identified by Woods (1983): context, perspectives, cultures, strategies, negotiation and careers.

The social *contexts* of mathematics learning are described including homes, classrooms, syndicate arrangements, school policy and curriculum directives. The *perspectives* of the participants are examined through their statements, the *cultural features* of the sociomathematical worlds of the children including routines, rituals and symbols are described, participants' *strategies* and *negotiation* of meaning between participants is explored, and the mathematical '*careers*' of the children – their learning progress, their evolving mathematical 'identities' and their commitment to learning in mathematics, are documented. These concepts are not examined in isolation, nor in a stringently followed sequence, rather they are presented as intertwined threads that portray the sociomathematical worlds of the children through a gradual process of revelation. This approach to 'sense-making' within the research process is consistent with the symbolic interactionist theory itself. The findings have been reported through detailed description, the powerful testimonies of the participants, the identification of common themes, and analysis which incorporates examination of relevant policy texts and the perspectives of other researchers.

The following chapter introduces the first of the four distinctive and defining features of the sociomathematical worlds of the children – the presence of speed and competition in mathematical learning - and examines this in the light of participants' everyday experiences through which they constructed distinct mathematical identities and meanings about the nature of mathematics.

## CHAPTER 4

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### THE SPEED PANDEMIC: CREATING MATHEMATICAL 'WINNERS' AND 'LOSERS'

This chapter explores the first of the four distinctive features of the sociomathematical worlds of the children outlined in Chapter 1 - competitive *speed activities* in mathematics learning. These highly ritualised everyday events are illustrated by examples from four different mathematics classrooms, spanning Years 3 to 5. The chapter documents the variants of speed activities, their prevalence, and their frequency. The impacts of speed activities are examined, including how the children interpreted these experiences and how speed performance contributed to their developing mathematical identities. The significance of speed and competition in mathematics, linked to other dimensions of the children's social worlds, are discussed. Government policy documents, classroom texts, and oral histories provide clues about the origins of mathematical speed activities, and evidence of their enduring use.

#### Situation 1: Ritual scene-setting

From fieldnotes, Fleur's classroom, Pukeiti School, Mid Year 4.

*The mathematics lesson is about to begin. On the whiteboard at the front of the classroom, Ms Fell has written 'questions' in red marker (Figure 2).*

<u>Checking Up</u>	<u>Maintenance</u>
1. $6 \times 2$	1. 360
2. $\frac{1}{2}$ of 22	+ 25
3. Total value of <u>6</u> 842	—
4. Place value of <u>9</u> 873	
5. \$2.50 - 25c	2. 967
6. Digital time for half past eight	+ 835
7. $\frac{3}{4} + \frac{1}{4}$	—
8. 19 - 6	
9. 20 + 11	
10. $5 \times 6$	

Figure 2: Questions on the board, Pukeiti School, Mid Year 4



- Ms Fell: Get out your maths books everyone. Turn to the back of your books and put up the date. (*Ms Fell now reads through the questions*) [Fleur later tells me Ms Fell does not usually read the questions first, and often calls them out rather than writing them on the board]
- Ms Fell: You have two minutes. Go! (*Children look at the questions on the board and some begin to write answers in their books while Ms Fell moves around the room*).
- Ms Fell: (*To a child who has not yet written the date*) Quick put the date up.
- Ms Fell: (*Looking over a child's shoulder*) We've got some times. Look at the sign carefully. (*After about one minute, looking around*) Do the ones that you can do first, everyone should get eight, nine or ten because they're easy peasy ones we're doing. (*Children continue to work in silence – many seem uncomfortable because they are jiggling in their seats, looking flustered, or frowning. Ms Fell continues to rove, checking on children's progress*)
- Ms Fell: (*Looking at a child's answers*) Good boy ...didn't even need to give you any clues.
- Ms Fell: (*To Fleur, pointing at her book*) What place is that?
- Fleur: Ones?
- Ms Fell: No. (*Waits for a response then prompts.*) Ones ... tens ...and...?
- Fleur: Hundreds?
- Ms Fell: That's right. (*Continues to move around the room while Fleur writes the correct answer.*)
- Ms Fell: (*When the 2 minutes are up*) Pencils down! Let's see how good our memories are. (*Some children raise their hands. Fleur does not.*) Half of twenty-two? Who got that one correct? ... (*Ms Fell selects a different child to answer each question, even if their hand is not up. Fleur only puts her hand up for the place value question. As the answers are being called out, the children 'mark' their own work with ticks or crosses.*)
- Ms Fell: (*When the marking is completed*) Who got ten? ...(*A few children raise their hands while everyone looks around to check whose hands are going up*). Very good. Who got nine? Good. Who got eight? (*Fleur raises her hand. I can see her score was seven.*) We know if you're being honest. OK, who got less than eight? You have to work a bit harder, you people. Now turn to the front of your books. Numbers one and two. (*Two addition calculations in vertical form are written on the board under a heading 'Maintenance'*). You have thirty seconds each. Go! (*...the lesson continues*).

According to Ms Fell, Fleur, and other children questioned in the class, the timing of the ten questions and maintenance were an everyday occurrence in this classroom, used as a starter to the days' mathematics session. Through this daily social activity, the mathematical scene was set: expectations were established and reinforced,

patterns were constructed and repeated, and for the children and the teacher, mathematics was defined. This activity can be seen as a ritual in which the participants assumed socially-assigned roles: the teacher as supervisor and controller, the children as individual workers and in a sense 'contestants', and the less visible players who exerted their presence through the power of expectation: school management, family, curriculum developers, education policy makers, mathematicians past and present, and members of the wider community who contributed to 'popular culture'.

Throughout this activity, children and teacher were engaged in an everyday social act, involving a shared understanding of procedures and roles through the interplay of language, actions, and physical objects. As Lemke (1990) notes 'a lesson is a social activity. It has a pattern of organization, a structure. Events follow one another in a more or less definite order. It has a start and a finish. But like all other kinds of social activities, it is made. It is a human social construction' (p.2). Lemke points out that classroom activities rely upon a common understanding among all the participants of the sequence of events and their purpose. 'All social cooperation is based on participants sharing a common structure of the activity' (p. 4).

In Situation 1, the teacher's use of language was an important part of the taken-as-shared nature of the activity. She used specialised terms understood by all the children, to refer to the activity and objects within it. She named the activities '*Checking Up*,' and '*Maintenance*'. She referred to '*your books*' meaning special exercise books used only for mathematics. Her instructions were sometimes direct, such as, '*turn to the back of your books and put up the date*', '*go!*', '*pencils down*' and sometimes indirect for example '*let's see how good our memories are*' meaning, 'you will now mark your work.' She used evaluative comments to set expectations or encourage the children, for example '*they're easy peasy ones we're doing*'; '*good boy – didn't even have to give you any clues.*' She used questions such as '*what place is that?*' to 'help' children, while, '*who got ten?*' was asked at the end of the activity as a communal evaluation.

The activity followed a pattern of mutually understood actions on the part of teacher and children:

- the teacher supplied the questions, started the activity, roamed the room inspecting children's answers, monitored the time, stopped the activity, selected children to provide the answers, evaluated answers, and judged children's performances;
- the children sat at their desks, took out their books and opened them at the back, took out their pencils and numbered 1 -10 down the left-hand side of one column, looked at the questions on the board and wrote answers in order vertically, worked alone in silence, raised their hands during the answering time, assigned ticks or crosses to their answers, wrote their total as a fraction out of ten, and raised their hands to indicate their scores.

Physical objects played an important role. The 1E5 7mm graph paper exercise books assigned for use in mathematics time contained a specialised rear section with ruled columns where speed tests were recorded. The ten 'questions' were presented in red marker in a vertical list on the whiteboard. Desks delineated the working space for each child. The special symbols – 'ticks' and 'crosses', indicated right and wrong answers and provided a permanent graphic record of daily achievement.

To the uninitiated, the activity may have appeared puzzling, but not to the children who were so familiar with these symbolic actions that little explanation was needed. When the teacher said '*turn to the back of your books,*' they took out and opened their books at the prepared section. When the teacher said '*Go!*' the children knew that they had to 'answer' the questions on the board, and that these answers were to be written, beside the appropriate numbers, in a vertical list in one of the ruled columns. When the teacher said, '*put up the date,*' the children knew that they were to write the date in its short form with lines between the numbers for day, month and year, and when she said '*let's see how good your memories are*' they knew it was time to provide the teacher with answers and mark their own. Some children raised their hands immediately. For the participants, this performance was a familiar taken-as-shared social ritual.

Implicit in teacher's use of language were beliefs and expectations about mathematics. For example, by referring to '*clues*', the teacher hinted at mathematics as something to be solved, and mysterious to the uninitiated or inept (Bishop, 1991).

'*These are easy peasy*' suggested that while in this situation, difficulty had been reduced to ensure accessibility for all, at other times it might be increased. '*Who got less than eight?*' implied that anything less failed to make the grade. '*How good our memories are*' signalled that memorising rather than understanding, was what the activity required and measured. Those children who failed to get the eight, nine, or ten that she has told them they all '*should*' because they are '*easy peasy*', might have assumed that they had bad memories or some other kind of mathematical deficiency.

The language, actions and objects that were *not* used in Ms Fell's classroom, were as significant as those that were, for example, there was nothing in the activity to suggest that children's thought processes were considered important or even relevant. Ms Fell did not provide children with equipment or encourage its use. Children were not expected to work out their answers by using diagrams or other written strategies, nor was there an expectation of estimating and checking answers for reasonableness. Ms Fell did not ask children about their difficulties or attempt to analyse their errors. Children were not invited to collaborate or share strategies.

During this seemingly innocuous everyday practice, the children were subjected to considerable pressure. In spite of the energy devoted to the activity, particularly on the part of the children, no discernible teaching or learning of mathematics took place.

Speed activities were a significant feature of the daily presentation of mathematics in Ms Fell's classroom, as seen from her description of what usually happened at 'maths time':

- Ms Fell: Normally we start off the day, there's ten questions on the board.  
 (Later) We have a few maths games that we play, they really love. I don't really like *Around the World*, we don't really play that, but we play this one called *Knock Down* that they absolutely love.
- Researcher: How do you play that one?
- Ms Fell: There's sort of five children and it's quick-fire questions. With those two are playing and we might say, 'Three plus four!' and whoever says 'Seven!' first, they stay up and the other one sits down and then it's those two, until you get down to one person and then they go out and you have a final. It's really quick but the kids love it. And then we have *Pipped at the Post* final and they really like that, and we have another game [*Loopy*] where there's a whole lot of cards and we have a stopwatch and it's all

addition, so it's like there's a start card and a finish card so someone's start card might say 'Ten plus five' and someone's got a card that says 'Fifteen' and then the next problem...and someone has the stop card. And we time it and we're trying to beat our time and that's a really good Term 1 game, they really get into that.

Researcher: How does Fleur seem to enjoy those games? Is she an eager participant?

Ms Fell: No. I think she's a bit ...a little bit worried about, you know, about looking silly.

For Fleur, the daily routine of answering questions at speed, was one of the key factors in her growing mathematical identity, that is, how competent she believed herself to be, which bits of mathematics she was best at, and why (self-efficacy). Because the teacher valued speed, and because others were faster, she had developed an understanding of her mathematical 'self' based on her perceptions of her performance within this social setting.

Fleur I'm a bit of a slow learner. They're quick [other children]. There's like two seconds to know them [the speed questions].

Fleur (later) Sometimes she [Ms Fell] goes too fast and I get a little bit sad. She usually goes, (*speaking quickly in imitation of the teacher*) 'Three times two! Three times four!' and stuff like that, and it's a bit too fast.

Researcher How many can you usually get out of 10?

Fleur Well, if she goes a little bit slower I can usually get 10 out of 10. [identifies excessive speed, rather than lack of knowledge, as the barrier to her success] (Mid Year 4)

During the same interview, Fleur rated herself 0 out of 10 for enjoyment of maths and between 4 and 5 for competence. For Fleur, the regular morning Checkup and other speed activities indicated to her that she was 'a bit of a slow learner' because she was not as fast at producing answers as some others. Not only did the daily speed-question ritual define mathematics for Fleur, it was a potent agent in the construction of her mathematical self-image.

## **Situation 2: The silent trial**

From fieldnotes, Georgina's classroom, Island School, Early Year 3

*The class is about to undergo the monthly speed test. It is school policy that all Year 3 to Year 8 children are tested on their basic facts every month. The Year 3s have 6 minutes to complete the questions.*

- Mr Solomon: Today we are going to do basic facts. All you'll need is a reading book and a pencil. *(Children go to reading corner and take a book back to their desks. They prop the books open on their desks to screen their work from other children. They take out their pencils and wait.)*
- Mr Solomon: You need to work absolutely silently to give everyone a chance. *(The teacher walks around and hands out the Basic Facts Speed Test<sup>1</sup> papers, one for the Year 3s and one for Year 4s. The teacher asks several children to move to other desks, apparently to minimise the possibility of cheating.)*
- Mr Solomon: Put your name and date then turn it over. *(When all the children are ready, looking at his watch) You may start now. (Children turn over their papers and begin to write answers. They work in complete silence, some including Georgina, using fingers. Many, including Georgina, appear strained and uncomfortable. The teacher has written the time for the test in half minutes on the board. He crosses off each half minute to indicate to the children how much time has elapsed. After three minutes, she has completed nine of the sixty questions. She looks up to see how much time is left and appears anxious. She bends over her paper, frowning. I can see she is selecting the questions she is able to answer most easily, bypassing the others.)*
- Mr Solomon *(After six minutes) Year 3, turn your sheets over please. (Georgina has now completed twenty-three questions. She has avoided all the multiplication and division questions but completed all the questions involving addition or subtraction of zero. She turns her paper over with a frustrated look, flicking the pages of her reading book and glancing around at others to see what they are doing. After two more minutes, Mr Solomon stops the Year 4s.)* OK everyone, when I call your name, bring your sheets up here and put them on my chair face down please. *(He proceeds to call out the children's names one by one. They walk up to his chair, deposit their papers, which will he will mark, and return to their seats.)*

As in Situation 1, this monthly rite was rich in significant language. Both the teacher and the test paper referred to 'basic facts'<sup>2</sup> on the assumption that the children understood the term. *Basic* refers to the 'fundamental' nature of these facts, but in everyday idiom also means 'dead easy'. Such an interpretation would certainly be disheartening for children who found learning these facts anything but. The teacher used a number of linguistic cues to reinforce the test procedure. 'You will need...'; 'You need to work absolutely silently'; 'Put your name and date' 'turn it over'; The reason for working absolutely silently was supposedly 'to give everyone a chance.'

<sup>1</sup> Basic Facts Speed Test # 3, +, -, x, ÷ from Pinder & Adams, 1996. See sample, Appendix 9.

<sup>2</sup> 'Basic facts' are defined in the glossary of *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) as 'the addition facts up to 9 + 9 and the multiplication facts up to 10 x 10' (p. 210).

In controller role, the teacher adopted a solemn tone as he issued instructions. He walked silently around the room handing out the papers. Stationed at the blackboard he emphasised the time limitations by striking out times as they expired. In worker roles, the children prepared their screens and papers, began as instructed, rapidly and silently entered answers in the appropriate gaps, and finished when told to do so. A videotape of a similar event in this classroom captured the tense and anxious expressions on the children's faces during this activity.

Everyday objects such as screens and pencils became laden with meaning within this ritual. Other well-understood symbols were the test times written in descending order on the blackboard, and the test paper itself. Entitled *Basic Facts Speed Test*, this sheet included a space for name and date, columns of questions, empty boxes for answers, a place for the total correct, and the year group time limits.

The language, actions and objects involved in this activity combined to create a 'test' atmosphere easily distinguishable from any other classroom activity. Regular mathematics tests, introduced early in Georgina's third year of schooling and adopted as part of the routine school mathematics programme, had initiated the children in her class into an educational culture in which tests and examinations would play an increasing part. The heavy emphasis on silence, working alone, shielding answers from others, starting and finishing strictly in accordance with the set times, returning papers face-down, and teacher control throughout, was imposed by the teacher and accepted by the children as the way things *are* – a taken-for-granted, necessary and natural part of learning mathematics at school.

In Georgina's class, mini 'tests' in the form of ten quick questions occurred daily at the beginning of each mathematics session, with the longer written tests interspersed at monthly intervals. Before our first interview Georgina volunteered:

'I hate maths. 'Cause I hate it when we do tests. I only get three or four or five or something, 'cause it's really hard.' (Early Year 3)



While she felt bad about failing, Georgina attributed her failure to the difficulty of the questions. Through the deeply alienating experience of these tests, Georgina had developed an aversion not only to the tests, but also to mathematics itself.

### Situation 3: The solo performance

From videotape, Jared's classroom, Spring School, Mid Year 3

*The mathematics session is about to begin. The teacher sits on a chair at the front of the classroom a digital watch in hand. The children are seated on the mat at her feet. Jared sits hunched at the back of the group. Attached to the front wall of the classroom is a cardboard 'clock face' with the numerals one to ten placed randomly around its rim. In the centre is the digit '6'.*

Ms Flower: OK, six times table. Hands up for their second turn. These people ...*(reads from the assessment book on her knee)* Briar, Tim, Henry...Henry. OK, Henry, you got fifty seconds last time. See if you can go faster. *(Henry walks over to the clock face and looking at it, waiting expectantly)*. OK ...*( looks at her watch)* Go!

Henry: *(Looking up at the clock face and speaking so rapidly that he is barely comprehensible)* Six twos are twelve, six sevens are forty-two, six nines are fifty-four, six threes are eighteen, six fives are thirty, six elevens are sixty-six, six fours are twenty-four, six sixes are thirty-six, six tens are sixty, six twelves are seventy-two, six eights are forty-eight! *(Looks expectantly at the teacher and beams with pleasure at his obvious success.)*

Ms Flower: Woo hoo! Thirteen seconds! *(Children in the class gasp and talk excitedly. Henry sits back down on the mat where some boys congratulate him.)* Well done! *(Records the time in her book)* That wasted your last score. OK, who's next? Ah ... Amy. *(Amy walks over to the clock face and stands in front of it, waiting as Henry did)* And...*(teacher pauses while looking at her watch)* ...go!

Amy: *(Quite slowly compared to Henry)* Six times two is twelve, six times seven is ...*(pauses for at least ten seconds, looking helplessly at Ms Flower who does not respond, just looks at the clock face with an expectant expression – some of the children begin to murmur and fidget.)* Twenty? *(Looking at Ms Flower for confirmation. Ms Flower does not respond. Amy continues.)* Six times nine is ...*(pause)* six times three is eighteen, six times five is thirty, six times eleven is sixty-six, six times four is ...six times six is thirty-six, six times eight is forty eight.

Ms Flower: Good girl. Well done. (*She records the time in her book, but does not announce it*) I can see you've been practising this time. And that's your first go. I'm sure you can improve on that. OK anyone for their second go? (*Looks around*) No one? We've had Briar, Tim, Henry, ...(*reading from the class list in her recording book*)

Amy's answer of *twenty* for *six times seven* indicates that she had little sense of the meaning of six times seven. Given time to do so, Amy could not access forgotten facts in any way. The teacher failed to correct her, to revisit the facts that were unknown or to ask her whether she could work them out. In this situation, the pressure was intense. Jared later explained that everyone had to have a turn.

Researcher: What about that game I saw you playing? People would stand up in front of the times tables clock and they would have to go as fast ...

Jared: (*Interjecting possibly because I had erroneously called it a 'game'*) They had to learn their times tables.

Researcher: Okay, how did you like that?

Jared: It was hard.

Researcher: How did you feel when Ms Flower said it was your turn?

Jared: I hated it!

Researcher: Did you?

Jared: I know the ten times table, that's all.

The patterned ritual interactions of this activity were full of meaning: '*the six times table*'; having a '*turn*' and a '*go*'; '*see if you can go faster*'; '*well done!*' '*you wasted your last score*'; '*you can improve*', and '*you've been practising*', were ways in which the teacher communicated her expectations and defined mathematics, mathematical learning, and mathematical competence. The teacher in the 'controller's chair', the teacher's watch, the yellow cardboard clock face on the wall, and the teacher's recording book were significant objects within this social 'performance'. All the children knew without explanation, the meaning of this activity, for example 'having a turn.' They understood the significance of a 'good' score. There was an air of expectation and tension as the teacher decided whom to choose next.

According to Jared and the teacher, this activity had been a regular lesson starter for most of the term. As in Situations 1 and 2 no discernible teaching or learning took place during this activity. For Henry who was seen as good at maths, there was no

indication of his understanding of the facts he was able to recall with such impressive alacrity. For Amy and Jared, the multiplication 'solo performance' was inappropriate since they were clearly still learning the facts. Jared began his third year at school as quite positive and confident about doing mathematics, rating himself a 10 on the enjoyment scale. His feelings had changed dramatically by the end of the year (1 on the enjoyment scale) and seemed to be directly linked to the speed activities that were a regular feature of his classroom. Based on his lack of success, Jared was identified as one of two children in the class with special needs in mathematics and was placed in a separate catch-up class the following term.

The research showed that it was during their third year of primary schooling that the children were introduced to 'the times tables' which they were then expected to 'learn' gradually over the following two years. Methods of introducing multiplication and 'times tables' and expectations of children's recall varied considerably between the schools in the study. *Mathematics in the New Zealand Curriculum* (1992) does not suggest that children should be able to recall the basic multiplication facts at Year 3, as Jared's teacher was expecting. This does not become a requirement until children are working at Level Three (about Years 5 and 6). Little direct teaching of the multiplication facts appeared to be happening in Jared's or any of the other research classrooms. Instead, the children were given lists to chant, to write down, and to take home and 'learn'. However, there was much evidence of testing of children's automatic recall of these facts, creating great pressure to learn them.

#### **Situation 4: Mathematical combat**

From fieldnotes and photographs, Jessica's classroom, Roto School, Late Year 5

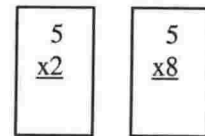
*It is a Monday. A new mathematics unit has just begun. The lesson is nearing its end. It has consisted of a starter of a list of commercially-prepared basic facts questions timed with a stopwatch, followed by checking of the homework task, then the measurement unit starter activity in which the children worked in groups, then on their own from a worksheet.*

*Ms Washbourne:* We might have a quiz to finish. When you're finished, close your books and put it away. Be really quick. *(Children are finishing marking their worksheets)* In a

moment you're going to pack up and when you have done that, get a partner who was in the same group as you last time [ability groups for the previous unit]. I want the groups to be even. [Intensifying competition in the guise of 'fairness']

*Ms Washbourne stands on a chair in front of the whiteboard, holding some cards in her hand. The children arrange themselves into two lines of pairs. The first pair is standing directly in front of the teacher. Ms Washbourne flicks down two cards at once, one in front of each child. On each card, written vertically is a multiplication question (Figure 3). The child who first correctly answers the card in front of them, gets to keep that card, and the pair goes to the back of the line.*

Ms Washbourne: *(To a child who was whispering the answer to another child.) Don't forget you lose a card if you say. (The pairs file up to the front for their turn and Ms Washbourne continues showing cards until all the pairs have had a turn)*



**Figure 3: Cards for speed game**

Ms Washbourne: OK, next round... *(Children are now shouting answers in excitement. To a child in front of her) Calm down, dear... (To a child who 'lost' because she didn't say the answer quickly enough) You know that!*

Ms Washbourne: *(When the cards have all been won) OK, let's see who the winner is this time. (Collects the cards from each line and counts them) Fourteen, fourteen. It's a draw. (A mixture of groans and cheers from the children.)*

Jessica, who 'lost' in both rounds, was later asked about the game.

- Researcher: Do you usually finish with a game like that – a quiz or something?
- Jessica: In Ms Washbourne's we do, we always finish with that.
- Researcher (later): How do you feel about games like the one you had today? What is that one called?
- Jessica: Ah, it doesn't have a name but we're put into teams.
- Researcher: I saw that. How did you feel?
- Jessica: Not that great.
- Researcher: Why not? Some people seemed to be enjoying it didn't they?
- Jessica: I was with this girl called Angela because she was in the same maths group as me. And, um, I'm not sure if she's that good at her times tables, I didn't think that she was, anyway I was quite glad that I was with her and not one of my friends, because then they would find out how bad I am.
- Researcher: So there are still some times tables you don't know yet?
- Jessica: Yeah, like, I know my eights, I'm kind of struggling on my nines and kind of struggling on my sixes, everything under five, and I'm sort of OK on my sevens, sort of on my eights, um, it takes me a little while to work out the nine and ten.
- Researcher: OK. What helps you learn your tables the best?

- Jessica: Look at them then go, OK, whoever says them is going to do them in a mix so it's important not to just add on from the next, so what I do is I may start from five and go to four then go to seven then skip one every time then just go up and down.
- Researcher: The list? Testing yourself?
- Jessica: Yeah. Really mix them up as much as I can.

Teacher language including '*rounds*' and '*winners*', actions including the teacher's controlling role on the chair, and children's lining up in pairs for their turns, and objects such as the question cards, were mutually recognised and accepted symbols of this everyday ritual. In this activity, not only were the children expected to recall their basic multiplication facts at speed in a very public way, but were also pitted one against the other in competition. Helping one another by '*saying*' was banned under the rules of the game.

Jessica practised the times tables with this kind of randomised testing in mind. Given the nature of the everyday basic facts activities she experienced, she had developed a strategy for learning based on her need to be able to recall discrete and jumbled facts. This was not altogether successful, as Jessica knew. She was anxious about her perceived shortcomings and worried that her friends would find out. Earlier in the interview she said that she found maths boring and rated herself between 4 and 5 out of 10 for her enjoyment of the subject. Contrary to their apparent intended purpose as a 'fun' way to practice the basic facts, speed games were a worrying and therefore alienating experience for many children such as Jessica. Educational justification for their use was highly dubious since they not only failed to help children make mathematical sense of the basic facts, but also failed to directly contribute to children's memorisation of them.

### **Speed activities**

These four situations have been chosen to illustrate a genus of activities, variants of which were observed during the majority of the classroom visits. They have been grouped together because they shared a number of features, the most distinctive of which was the emphasis on speed. For this reason they have been collectively named *speed activities*. Had they been isolated or rare events, there would have been little

reason to describe them. On the contrary, they were found to be so usual within the study classrooms that they demanded special investigation and analysis. Evidence was gathered to show that they formed a significant part of classroom routines.

Speed activities were of four main types:

- daily *quick questions* e.g. *Quick Ten* (Situation 1);
- formal written basic facts *tests* (Situation 2);
- oral basic facts *performances* (Situation 3);
- question-and-answer competitive *games* (Situation 4).

### ***Quick Ten* and variants: the daily speed test**

Various called *Quick Ten*, *Quick Ones*, *Quick Questions*, *Daily Twenty*, or *Checking Up*, and with minor variations from class to class, the basic structure of the *Quick Ten* as these activities will collectively be called, were much the same and looked something like this:

Students were asked to ready themselves by sitting at their tables, taking out their maths books, turning to the back of their books and numbering the margin. The 'start' was announced by the teacher and the race against the clock began. The questions were almost invariably closed, that is, there was only one right answer. No talking, concrete materials or written working methods were allowed. When the time was up, the teacher commanded the children to stop. The children marked the answers. The teacher selected children from around the class to provide the answers. The score was written beside the marked questions. The results were publicised by the teacher's asking, 'Who got 10, who got nine ...who got less than 5?' or similar.

A range of variations on this theme were observed:

- *Questions*: Sometimes teachers wrote the questions on the board and while children could work at their own pace they were still expected to finish within a given time period. One teacher concealed the questions until the children were

told to 'go'. At other times, teachers would ask the questions aloud in quick-fire fashion, recording them on paper as they went. One teacher called the questions orally and then wrote them on the board. Sometimes the questions were purely basic facts, at other times related to the current topic;

- *Recording:* Sometimes teachers expected the answers to be recorded in the front or main part of the mathematics exercise book, but mostly the back of their books was reserved for this purpose;
- *Marking:* In a number of the classrooms, the teachers asked children to swap books for 'marking', otherwise they marked their own. Sometimes red pens were required for marking. Some teachers would randomly select any child to provide an answer, while others asked for hands up and selected only from volunteers;
- *Results:* Some teachers asked the children to stand or raise their hands for scores. Others asked the children to announce their scores, which were then recorded. One teacher had the children record their scores with coloured pencils on an ongoing graph in the back of their books. Given that the tests differed daily, the validity of the graph was questionable but the idea that each child was able to track personal progress in visual form had merits.

### **Basic facts written speed tests**

These were less frequent but regular e.g. weekly, monthly, or once a term. Frequency increased with the age of the children, so that by Year 5, such basic facts tests were a significant feature of all mathematics programmes in the study (Table 2, p. 111).

In all cases, the children were given test sheets containing the questions. Over the three years of the research, five of the schools were observed to be using the commercially produced Pinder & Adams (1996) basic facts speed tests, (Appendix 9) as in Situation 2. Other schools had devised their own tests. The number of questions varied from twenty to one hundred and ten, increasing in number with the increasing age of the children. The questions were invariably mixed and of the 'result unknown' problem type e.g.  $3 \times 4 =$  (Carpenter et al, 1999). By limiting the type of question, teachers were also limiting children's use of strategies and so too their understanding of the nature of basic facts and the connections between number operations. The tests



were always timed and completed in silence by each child. Test papers were collected and marked by the teacher. Results were documented and often became part of each child's permanent assessment records.

### **Basic facts oral speed performances**

Within the small sample group of ten children, three were found to have experienced these activities. They both took a similar form - with a 'clock face' on the wall made of randomised multiplicands around the outer edge and a multiplier in the centre. Children were called upon one by one to stand in front of the clock and perform the multiplications orally as fast as possible in a clockwise direction around the clock. In one classroom, individual children were simply asked to recite a particular times table on their own in front of the rest of the class. While apparently less widespread than the written tests, it is not unreasonable to assume that a significant proportion of New Zealand teachers are regularly using this kind of assessment in their mathematics classrooms.

### **Mathematical speed games**

A number of different competitive basic facts games were observed or described by the children. The most common of these was a game called *Around the World*, in which one child was selected to stand by another. The teacher asked a basic fact question and the child who most quickly responded with the correct answer, stayed 'in'. The loser remained seated while the winner moved to stand beside another child for the next question and so on. The object of the game was to stay 'in' for as long as possible and the ultimate challenge was to go 'around the world', that is, to beat every other child in the class. In the four games that I observed in four different classrooms, it was obvious that some children were never 'in' and it seemed that they simply gave up trying. Dominic described a similar game regularly played in his classroom called *Shoot Out*, where contestants were eliminated for slow or wrong answers. He also described a game played regularly as a starter activity in Year 4:

- Dominic: *Maths Challenge* it's called.
- Researcher: Do you have to answer basic facts questions?
- Dominic: Yeah, you get a winner. 'Cause there are four people standing up and the rest of the class are sitting down. They challenge the people that are up there and the people that sat up there all the time [because they get the answers right].

The pairs game of Situation 4 was also fairly popular. Three of the teachers were regularly using variants of this game. In Jessica's Year 5 class, cards were used and kept by the winner and a number of successive rounds were played, while in Jared's Year 3 class, the teacher called the questions for the front pair, and the winner stayed at the front while the loser went to the back of the line and sat down, having been eliminated. This continued until a final winner was found. Jared describes how the game was played in his class:

We play *Shoot*, it's a maths test and two people are up there [in front of the class at the whiteboard] and Mr Waters writes something [on the board] and you're looking at the people [facing the class with backs to the board] and Mr Waters says 'Shoot!' and they've got to turn around and get the answer right, and then the other person goes out if they get it wrong.  
(Early Year 5)

Note that Jared has called this activity a 'test', whereas the teacher implied it was a game when he said 'we play *Shoot* on the blackboard.'

*Buzz* was another commonly found speed game. With the class arranged in a circle, the teacher would announce the numbers that were to be substituted by the word 'Buzz' e.g. multiples of five, and would then select a child to start. Taking turns around the circle, the children would rapidly recite a number sequence, either forwards or backwards while the teacher and children would keep track to make sure that the correct words were being said, either the next number in the sequence or 'Buzz', as appropriate. Children who made an error on their turn were eliminated and sat down, while the others continued. Helping others was forbidden, and wrong answers were very public.

*Loopy*, another speed game, was observed in Peter's Year 5 classroom. Fleur's teacher Ms Fell also used a version of this game as described on page 6. While there

was no winner of the game, the aim was to produce the fastest class time for loop completion. Those children who were slow to answer were seen as weak links in the loop, thus preventing the class from achieving a record time.

Other common speed games observed were versions of *Lotto* or *Bingo*, where the children would select numbers to write in their books. The teacher would ask basic facts questions in quick succession and the children would cross off corresponding answers from their number selection. Since winning was a matter of chance, there was less emphasis on competition in these games, and the practice of calling out of answers seemed to be more accepted by teachers than it was in the very competitive games. The children valued winning. They would become visibly disturbed if they missed their numbers through not having been able to keep up with the questioning. Liam describes *Cross Out*, the variation of this game played in his classroom.

Researcher: What's *Cross Out*?

Liam: You put down the [selected] numbers [between] one to twenty and then she [teacher] calls the numbers out and does them quite fast so that some people can't hear them and she calls them out about twice and the first one to get all the numbers, they win and they put their hand up and, um, the teacher normally puts their name up on the board for Star Student. (Early Year 5)

### **Speed activities as part of everyday life**

In order to determine what usually happened at maths time, teachers were asked to describe the format of a typical mathematics lesson, children were asked what usually happened at maths time, and records of the children's work such as their mathematics exercise books, were examined. Speed activities featured prominently.

Researcher: What would a typical maths lesson look like in your class?

Ms Flower: To start, a game, we mostly do a game ...they really, really like the game that we played [the competitive game I saw while observing the class] so I play it heaps – yeah, basic facts all through, pretty well. (Spring School, Early Year 3)

- Mrs Joiner: The structure is always the same – some sort of whole class activity to start with like *Basic Facts Speed Tests*, *Bingo*, *Buzz*, chanting tables, looking at factors, addends, patterning ... (Bridge School, Early Year 3)
- Ms Fell: Normally we start off the day, there's ten questions on the board. (Pukeiti School, Mid Year 4)
- Ms Seager: We start off with some, um, basic facts recall. This morning I just did the *Quick Ten*, other mornings I do problems on the board, word problems and things like that, so some sort of recall of basic facts. (Roto School, Early Year 4)
- Mrs Waverly: We come and we have *Quick Ten* which is so basic and easy, but that's really just to settle down. (*imitating herself*) 'Get your books out, turn to the back, do those'. (Beach School, Early Year 4)
- Mrs Ponting: It's a very busy time. It's heads down and we work really hard. We have our *Daily Twenty*, then *Computation*. You're really working hard to get them into their basic facts of addition and subtraction, from memory, recall. I think that's important. They've got to know it. So many with their fingers! (Bridge School, Early Year 4)
- Ms Linkwater: I tend to usually run, um, fairly structured programmes, with basic facts tables at the beginning, might be a maths game, um, some computation, then new learning....(Later) Basic facts tests, this one I'll often throw at them. I'll give them this one once a month. (Bridge School, Early Year 5)
- Ms Matagi: We usually start with maintenance of our basic facts. We do our daily drills, they say them, just chant them,... then those children I know who feel confident will do it on their own. (Mountain School, Early Year 5)
- Mrs Iles: I have 20 quick-fire questions and that's the tables practice, yeah, and the basic facts. (Motu School, Early Year 5)
- Mr Ford: Basically it's run the same system most of the days. We start off, we have, Daily Drill, Quick Twenty kind of thing, just basic facts. Whip that through.
- Researcher: Is that on the board or do you call it out?
- Mr Ford: I put a little grid, they write it down, I put up the time, not so much as a competitive thing, but I've explained ... I give it to them and I've explained that it's there for them to monitor how they're going. Yeah, and it's also for those ...'cause it's hard to make it something ...a motivating kind of thing but if you sell it as something like

'Hey, what's your time? See if it goes down a bit over the term,' that sort of drives them along a little bit. (River School, Early Year 5)

Mr Waters: In the first term ... maths was your *Quick Ten*, you got it in, you got it done, a concept, then maybe a worksheet that we use then straight onto the English. (Spring School, Early Year 5)

Researcher: What do you usually do first at maths time?

Fleur: We usually start with times tables or take aways. She says, like, 'Six take away seven' [sic] and we write them down in the back of the book and sometimes she mixes them all up... She calls it *Quick Ones*. (Late Year 3)

Fleur: We usually do one of those tests (*points to yellow times tables achievement chart on classroom wall*) or else just questions like five take away five and stuff like that. (Mid Year 4)

Fleur: We usually do *Quick Questions*. On Friday we do *Fifty Quick Questions*. And there's like two seconds to know them. (Mid Year 5)

Georgina: Mr Solomon claps his hands and says, 'Get out your maths books and set up one to ten.' (Early Year 3)

Georgina: Get out our maths books and do our maths. Twenty basic facts in two minutes. And ten for one minute. (Early Year 4)

Georgina: On Mondays we have to do this thing. It's got eighty questions and it's got, like, eight times nine and stuff like that. And you have to start doing that and you have to do it under seven minutes. (Early Year 5)

Toby: Usually he says, 'Get your maths ready in your speedy maths books and do it as fast as you can.' (Mid Year 5)

Although speed activities may not have been observed on every visit, there was ample evidence in the children's mathematics exercise books that such activities were a regular feature of the classroom programme. In such cases, the children's exercise books typically appeared as in Figure 4. This confirmed the everyday nature and extent of activities of this type.

Given the broad spread of school types and classrooms included in the study, it is not unreasonable to consider speed activities as one of the ‘universals’ of mathematics teaching practice as shown in Table 2. The table also shows that speed activities increased with the age of the children: by Year 5, all children were experiencing daily speed activities, and regular speed tests. Eight of the ten were also experiencing frequent speed games. This provides strong evidence for regarding speed as pandemic within mathematics teaching practice.

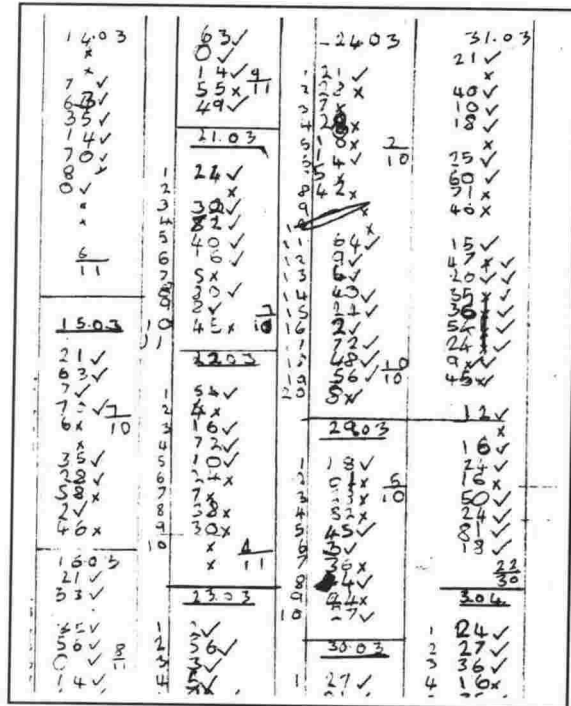


Figure 4: A page from the back of Fleur’s maths book, Pukeiti School, Early Year 5

Table 2: Speed activities experienced by the study children over Years 3, 4 and 5.

Key: **D**= Daily, **W** = Weekly, **F** = Frequently, **M** = Monthly, **O** = Occasionally.

= quick-fire questions written or oral, = written speed test sheet, = speed games

Child	Year 3	Year 4	Year 5
Fleur	D	D  F	D  W  F
Georgina	D  M	D  M	D  W  O
Jessica	O	D   1x a term	D  F   2x a term
Rochelle	D   O	D   2x a year	D  F  F
Dominic	D	D  W	D  W  F
Jared	D	O	D  F
Liam	D	D	D  O   1x a term
Mitchell	O	D	D  F
Peter	D	D  O	D  W  F
Toby	W	D  F	D  F

### 'Making the familiar strange'

Ernest (1998) describes school mathematics as 'a set of artificially contrived symbolic practices, a significant part of the meaning of which is not already given but deferred to the future' (p. 22). To lay bare the underlying explicit or implicit social 'rules' or conventions that govern such 'contrived symbolic practices', careful and detailed description is needed. While subtle variations were found in the ways in which speed activities were implemented within the group of study classrooms, the following list identifies the common *rules*, *conventions* and *norms* and provides a range of examples of teacher reinforcement of these norms.

Rules/Norms for Speed Activities	Examples of Teacher Reinforcement of Rules/Norms
<ul style="list-style-type: none"> <li>Participation is compulsory for all children in the class</li> </ul>	<ul style="list-style-type: none"> <li>'OK everyone, get out your maths books, turn to the back and put up the date.' (<i>Checking Up</i>, Miss Fell, Pukeiti School, Mid Year 4)</li> <li>'Can you turn to the back of your books for numeracy?' (Numeracy sheet, Mrs Mere, Roto School, Late Year 5)</li> </ul>
<ul style="list-style-type: none"> <li>Each child must engage in the activity without help from others</li> </ul>	<ul style="list-style-type: none"> <li>'There shouldn't be any discussion. People are thinking. Do it on your own.' (<i>Quick Ten</i>, Miss Matagi, Mountain School Mid Year 5)</li> <li>'Don't help him! You know the rules!' (<i>Buzz game</i>, Mrs Joiner, Bridge School, Mid Year 3)</li> </ul>
<ul style="list-style-type: none"> <li>'Answers' only must be produced in response to 'questions'</li> </ul>	<ul style="list-style-type: none"> <li>'Write the answer down.' (to Peter during <i>Quick Ten</i>, Ms Summers, Beach School, Late Year 3)</li> </ul>
<ul style="list-style-type: none"> <li>The same questions are used for all the children in the class - 'one size fits all'</li> </ul>	<ul style="list-style-type: none"> <li>'Everyone should get eight, nine and ten because they're easy peasy ones we're doing.' (<i>Checking Up</i>, Miss Fell, Pukeiti School, Mid Year 4)</li> </ul>
<ul style="list-style-type: none"> <li>The teacher writes or selects the questions</li> </ul>	<ul style="list-style-type: none"> <li>'Right, we're doing number forty-one this morning.' (Numeracy sheet, Mrs Mere, Roto School, Late Year 5)</li> </ul>
<ul style="list-style-type: none"> <li>The questions are quiz-like - they are usually disconnected from one another and presented at random.</li> </ul>	<ul style="list-style-type: none"> <li>'Eight plus nine. (<i>Short pause</i>) How many corners does a square have? (<i>Short pause</i>)... (Ten quick oral questions, Mr Solomon, Island School, Late Year 3)</li> <li>'Number one. Four plus seven. (<i>Short pause</i>) Number Two. How many legs on three cats? (<i>Short pause</i>)... (Ten quick oral questions, Ms Torrance, River School, Mid Year 3)</li> </ul>
<ul style="list-style-type: none"> <li>'Questions' have to be 'answered' within a very limited time.</li> </ul>	<ul style="list-style-type: none"> <li>'Go faster! Flash cards are supposed to be fast!' (<i>Daily Twenty</i>, Mrs Ponting, Bridge School, Late Year 4)</li> <li>'See if we can do it in double quick time. See if we can do it in five minutes then we'll stop and mark' (<i>Quick Ten</i> on whiteboard, Ms Summers, Beach School, Early Year 3)</li> <li>'You have to be quick to beat Nigel' (<i>Around the World game</i>, Miss Peake, Mountain School, Early Year 3)</li> <li>'Excellent, that was much faster today.' (<i>Buzz game</i>, Mrs Joiner, Bay School, Early Year 3)</li> </ul>



- The time limit is imposed and controlled by the teacher
  - 'Don't start yet because we want to see if we can do better than last time.' (Basic facts test, Mrs Heath, Hill School, Early Year 4)
  - 'Ten seconds until we start...*(pause)*...Go!' (Ten Questions on the board, Ms Palliser, Pari School, Late Year 4)
  - 'I think you only need two minutes today. I'll be generous and give you one minute more.' (Multiplication array, Mrs Kyle, Bay School, Late Year 4)
- The pace of answering is controlled by the teacher
  - Child: 'I missed out a couple because you said it too fast.'
  - Ms Matagi: 'That's how it is.' (*Quick Ten*, Mountain School, Mid Year 5)
- The children must work in silence and concrete aids or basic facts charts are neither provided nor considered appropriate
  - 'Please work it out in your heads otherwise it's very distracting for the people around you. Alice, I can hear you all the time. Be quiet.' (Numeracy questions, Ms Mere, Roto School, Late Year 5)
  - 'You don't need to use your fingers!' (*Twenty Questions*, Mrs Ponting, Bridge School, Mid Year 4)
  - 'Try to work it out yourself, Georgina please.' (*When Georgina looks at the chart on the wall*) *Quick Ten*, Mr Solomon, Island School, Mid Year 3)
- The answers are judged correct or incorrect by the teacher
  - 'Not quite' (*teacher selects another child to answer*). (*Quick Ten*, Ms Seager, Roto School, Early Year 4)
  - 'Not quite. Who can help her?' (*teacher selects another child*). (Multiplication array, Mrs Kyle, Bay School, Early Year 4)
  - 'No. Someone else?' (*Quick Ten*, Mr Solomon, Island School, Mid Year 3)
  - 'Are you sure?' (*to child who is wrong*) (*Buzz game*, Mr Cove, Whanga School, Mid Year 5)

The achievement of individual children is exposed to others in the class in at least one of the following ways:

- Compulsory swapping of books for 'marking'
  - 'Okay, swap books children please'. (*Quick Ten*, Ms Seager, Roto School, Early Year 4)
  - 'Right swap books, red pens out, quick!' (*Quick Twenty*, Miss Meadows, Pukeiti School, Late Year 5)
- The teacher picks out willing or unwilling individual children to provide answers orally so that the whole class will hear
  - 'Anna, first three answers please.' (Numeracy sheet, Mrs Mere, Roto School, Late Year 5)
  - 'Twelve take away three is ... Andrew?' (*Andrew did not have his hand up*). Andrew: Um ... eleven? (*Quick Ten*, Mr Solomon, Island School, Mid Year 3)
- The teacher requires individual children to publicise their attainment by standing, raising hands or calling out the total of correct answers.
  - 'Stand up who got 5. Stand up 6. Stand up 10. Well done you three.' (*Quick Ten*, Mr Solomon, Island School, Mid Year 3)
  - 'Who got all right? Great, I'll give you a sticker.' (*Twenty Questions*, Mrs Cayo, Island School, Mid Year 4)
  - 'Now I want to check how you went. Hands up who got them all correct?' (*Quick Ten*, Ms Summers, Beach School, Late Year 3)
- The teacher comments on individual children's progress or results so that the whole class may hear
  - 'You're only up to number one. I don't want to be bothered with boys who don't try.' (*Daily Twenty*, Mrs Ponting, Bridge School, Late Year 4).

### The children's responses

The children reacted in diverse ways to the speed activities they experienced in their classrooms. The processes by which each child constructed meaning from these activities were individual and complex, yet common responses can be found within the sample group.

Rochelle, Dominic, Liam and Toby reported that they enjoyed the speed activities. They usually finished within the given time, scored highly, or succeeded in the games. Besides enjoying mathematics, Dominic, Liam and Toby were also very keen on competitive sports and their parents and teachers remarked on their enthusiasm and success in sporting activities. The competitive aspect of the *Quick Ten* seemed to appeal to them as these conversations reveal, and inspired their learning of the facts.

Researcher: Have you ever won it? (*Dominic has just described how to play Shoot Out*)

Dominic: No I haven't, but I've nearly won it.

Researcher: Do you like that game?

Dominic: Yep! (*very enthusiastically*)

Researcher: What do you like about it, Dominic?

Dominic: 'Cause it helps me to learn. (Mid Year 4)

Researcher: How do you feel about those [the weekly basic facts tests]?

Dominic: I feel real good when we have those. Because I always get stickers and stuff.  
(Late Year 4)

Dominic: We get eight minutes to do the whole thing [weekly basic facts test]. I actually do it in seven and a half minutes. (Mid Year 5)

Researcher: How do you feel about the *Quick Twenty*?

Liam: Good.

Researcher: OK. Why do you feel that?

Liam: 'Cause it's a competitive thing and I like to compete.

Researcher: What about *Buzz*? You said earlier that you didn't like that as much.

Liam: No, I can't compete with it. (Late Year 4)

Toby: We play this thing called *Around the World*. I like it, like when I beat somebody when they come to my desk. (Late Year 4)

Toby: Speed maths that's my favourite part. I like being ahead of other people. (Mid Year 5)

Researcher: What tells you you've got better at maths, Toby?

Toby: Well, I'm much faster.

(Later)

Researcher: Are there any kids who are better than you at maths?

Toby: Yes.

Researcher: How do you know?

Toby: Because they're much, much faster. (Late Year 5)

Rochelle, who also enjoyed the '*Daily Twenty*', seemed to regard it as a reassuring and satisfying measure of personal success and as a pleasing indication of compliance with teacher expectations. Her teachers remarked on how Rochelle was 'eager to please' and for Rochelle, 'getting answers right' appeared to fall into this category.

Researcher: What do you like most about maths, Rochelle?

Rochelle: *Daily Twenty*.

Researcher: What is it about *Daily Twenty* that you like?

Rochelle: You get to answer questions.

(Later)

Researcher: Why do you think you're pretty good at maths, Rochelle?

Rochelle: 'Cause in the *Daily Twenty*, I can get nearly all the questions right.

(Later)

Researcher: What's the most important thing you do in maths time do you think?

Rochelle: *Daily Twenty*.

Researcher: Why do you think that's the most important, Rochelle?

Rochelle: Well, you answer questions. (MidYear 4)

Rochelle also enjoyed competition.

Researcher: What about that game *Around the World* that I saw you playing?

Rochelle: It's fun.

Researcher: What do you like about it?

Rochelle: Because you get to beat people at saying the answers.

Researcher: Do you usually beat people?

Rochelle: Sometimes. I nearly got around the world, but [for] Tania, she's quite good at maths.  
(Early Year 5)

Fleur, Georgina, Jessica, Jared, Mitchell and Peter were less positive about speed activities. Fleur was on holiday with her parents when the class began to learn the

'times tables'. She returned to find herself behind the others and for her, catching up was difficult. The teacher told me that Fleur had become extremely agitated when she could not do what the other children could.

- Researcher: Are there any people in the class who are better than you at maths?  
 Fleur: Ella.  
 Researcher: How do you know Ella's good at maths?  
 Fleur: 'Cause she can get to finish all her times tables and take aways and pluses all right most of the time. She's a lot faster than me too.  
 Researcher: How many do you do?  
 Fleur: We get a hundred, in fifteen minutes. (Late Year 3)

Fleur felt pressured by the speed of basic facts activities. The Year 4 teacher described how Fleur would burst into tears on occasions during mathematics tests.

- Researcher: Do you usually start with ten questions in the back of your book? [*as just observed*]  
 Fleur: We usually do one of those tests (*Points to yellow times tables achievement chart on classroom wall*) or else just questions like five take away five and stuff like that.  
 Researcher: I see, and does the teacher call those out?  
 Fleur: Yes.  
 Researcher: How do you feel about that?  
 Fleur: Sometimes she goes too fast and I get a little bit sad... If she goes a bit slower, I can usually get ten out of ten. (Mid Year 4)

By Year 5, Fleur was experiencing daily speed activities on a regular basis, with ten quick questions to begin each regular lesson, and fifty questions on Fridays. At the end of their Quick Ten test Mrs Meadows required the children to exchange books for marking and when completed, asked the children to call out their scores which she recorded in her assessment register. After one such event in which Fleur had scored seven out of ten, the following discussion took place within Fleur's desk group.

- Joshua: Maths? I hate it.  
 Zac: Maths is my favourite subject.  
 Researcher: What do you hate about it, Joshua?  
 Joshua: It's hard. The times tables. The eights.  
 Researcher: You can do those can't you, Fleur? (*Fleur nods*)  
 Zac: I know them all.  
 Researcher: How did you get so good at them?

- Zac: I practise them at home  
 Researcher: Did you have to practise alot?  
 Zac: Yeah.  
 Joshua: I only do them at school.

Fleur had already told me that she practised the tables at home, and she showed me her homework notebook with lists of the times tables written in the back. In Fleur's class, it was expected that the tables be learned at home rather than at school and Fleur and Zac had been able to do so while Joshua had not. Teachers' relinquishment of responsibility for teaching the times tables was clearly resulting in problems for those children for whom learning at home was difficult.

Georgina had also experienced problems learning her basic facts at home. Daily speed tests had become a real problem for her and the three years' observation showed that progress was minimal. The rear section of her mathematics exercise book told the tale: she regularly achieved less than 50% of correct answers. Day after day, *Quick Tens* served only to reinforce her sense of failure. Consistently over the three years of the study, Georgina expressed her dislike of, and lack of confidence in, mathematics.

- Researcher: When the teacher says 'Ok, it's time for maths now, how do you feel?'  
 Georgina: Ugh! (*Grimaces*). We have to do this 20 or 10 question thing and Mrs Cayo calls out the questions and you have to write the answer and she goes really fast now and I can't do it. (Mid Year 4)

Given concrete materials, opportunities to talk about her mathematics, and above all more time, Georgina demonstrated understanding and creativity in the subject. With visible excitement and obvious understanding, she described in both words and diagrams, two activities she had really enjoyed: using a three-bar abacus, and drawing pictures using paired co-ordinates, and yet Georgina felt that she was no good at maths, pinpointing basic facts speed activities as the chief indicator. As her Year 4 teacher said: 'Georgina has more ability than the assessments show.' (Mid Year 4).

- Researcher: What subjects that you learn at school do you think you're the worst at?  
 Georgina: Maths.  
 Researcher: What makes you think that?

Georgina: Because I never get my basic facts right.

(Later)

Researcher: What makes someone good at maths do you think?

Georgina: They learn all their times tables and learn all the, um, take aways and, um, pluses.  
(Late year 4)

Georgina: I put my goal to get faster at it. (*She shows me the basic facts self test sheet on which she has written: 'My score this week was 28, my goal for next week is to get faster.'*)

(Later)

Georgina: Bradon, the person next to me, he's pretty good at it [maths].

Researcher: How do you know?

Georgina: 'Cause he just goes like this on the test (*mimes writing very quickly*). And it's two minutes [to finish]. (Early Year 5)

Georgina was also expected to learn the basic facts at home. Her teacher expressed this belief in the comment below, and in noting that this did not always happen, acknowledged that more time should be spent on learning them at school. This lack of school support seriously disadvantaged Georgina.

Mrs Eyles: But it is, um, I mean, part of the onus is on them too, to learn their tables at home and then come back, and I've said, 'Well you know, you should, up to now, know up to your six times tables, and if you're not feeling confident then you should spend time learning them'. But there probably should be more time allowed [at school] for those children that, ah, at home, they're not always done. (Early Year 5)

Jessica, too, believed she was not very good at mathematics. As we have already seen, speed activities helped to create and reinforce her feelings of inferiority.

Researcher: Do you think there are people in the class who are better than you at maths?

Jessica: Way better!

Researcher: Okay, how do you know they're better?

Jessica: Well, because we do *Around the World*, things like times tables, adding and dividing and there's this boy, he goes around and he's, like, really, really good. He's made it, like, three-quarters of the way around.

Researcher: How do you feel when he comes around to stand by you?

Jessica: Not Good. I feel kinda nervous. Because there's the whole class there and stuff.  
(Late Year 3)

Basic facts speed activities compounded her feelings of failure. Based largely on the basis of their basic facts test scores, the children in Jessica's class had established a hierarchy for mathematics.

- Researcher: How do you feel about the tests? [basic facts speed tests]
- Jessica: I feel nervous, ... 'cause if you don't get very many, we've got these graphs [of their basic facts scores] and mine starts up there and then it goes down, up a bit, but down and up... So somebody could tell as soon as they saw it, so they can tell you got a low score.
- Researcher: Does anyone else see your graph?
- Jessica: Some people do but they're not really supposed to look at it.
- Researcher: Do people know each other's scores or is it private to you?
- Jessica: It's meant to be private but some people go, (*using a wheedling voice*) 'What did you get?' (Late Year 4)

Jessica: Sometimes we do these challenges. It's the *Times Challenge* and we've got a clock, with, like, in the middle there's times, whatever the times table we're doing, and then around the inside, the circle, Ms Washbourne puts numbers, and you have to go 'Twelve times six is...twelve times eight is ...whatever the answer on the outside of the circle is and I've never tried that but I don't want to because we're up to the six times tables in twenty-five seconds.

(Later)

- Researcher: How do you think you're getting on with your tables these days?
- Jessica: Not very well, but I've had a few days off, so I'm still on my four times tables. (Early Year 5)
- Jessica: Well when we do this numeracy skills mastery programme, some people are doing a different sheet because they're not as up to the others.
- Researcher: Why is that?
- Jessica: Maybe they're not as fast as us. (Mid Year 5)

Mitchell experienced learning difficulties in all areas of his schooling and tried to make sense of the activities in which he was expected to participate at mathematics time. It was the daily basic facts tests that most stood out for him.

- Mitchell: We have to do a sheet [of basic facts questions] and Miss Palliser times us and you have to put your hand up and it's really short, like for three minutes. (Mid Year 4)
- Researcher: Do you ever play games like *Buzz* or *Around the World*?
- Mitchell: *Around the World*.



- Researcher: How do you like that game?  
 Mitchell: Bad.  
 Researcher: Bad? What happens for you when you play that game?  
 Mitchell: I always lose, 'cause the other kids know and I don't. (Early Year 5)

In spite of having had plenty of help and encouragement at home, *Quick Ten* from the whiteboard every morning for two years, and a basic facts speed test every Friday for most of Year 4, Peter did not feel entirely happy with his own progress. He was a child who worked methodically, carefully, and accurately, but slowly!

- Researcher: Is there anything you've done in maths that you really haven't liked much?  
 Peter: Um, times tables.  
 Researcher: What makes the times tables not so good for you?  
 Peter: Because they're hard. (Late Year 4)

All the children in the study group nominated *Quick Ten* and similar kinds of basic facts practice and testing as the most important part of their mathematics learning. This is consistent with the research of Flockton and Crooks (1998) who found that in response to a question where children were asked to name three things that a person needs to learn to do or be good at maths, 100% of 1440 Year 4 students in their sample group, gave basic facts and tables as one of their choices. The next most frequently-chosen items were work and study skills (26%) and classroom behaviours (26%) such as seeking help and paying attention, well down compared with basic facts. By Year 8, basic facts was still the most frequent response (67%) compared with personal attributes (26%) such as good attitudes and concentration, the next most popular choice. These findings are revealing. It seems that the daily emphasis basic facts receive through frequent speed activities in the middle primary school, leads children to believe that basic facts are the heart of mathematics, that 'knowing' the basic facts is indicated by rapid recall, and that those who are able to recall all the basic facts at speed, are good at mathematics.

### **Speed activities and pressure**

- Researcher: Do you feel comfortable when you're doing maths, or uncomfortable?  
 Toby: Well it depends like if we have to get this, like something done by a certain time, I

don't feel too comfortable, I have to hurry up, and well, if it doesn't matter how long it takes, I feel comfortable. (Late Year 4)

Toby expressed feeling pressured by time constraints as did many of the other study children. Speed activities exerted this pressure by:

- forcing the child to participate whether s/he wanted to or not (compulsion/coercion);
- pushing individual children to perform beyond their natural and comfortable pace (too fast);
- pushing individual children to perform beyond their current state of skill and knowledge (cognitive inaccessibility);
- calling for only one 'correct' response (creating a high risk situation through the chance of being 'wrong');
- exposing the child to public (in this case class mates', teacher's and sometimes parents') scrutiny with subsequent 'failure' or 'success' labelling by teacher or classmates or self or any combination of these (creating the chance of being shown up and/or ridiculed);
- banning or discouraging the use of materials, peer discussion, and checking of answers (reducing support and increasing likelihood of error).

The four situations described in this chapter demonstrate that these six forms of pressure often occurred simultaneously. It was the multiple nature of pressure that served to intensify the impact of each aspect upon the others, for example, if an activity called for 'right' answers to be produced faster than was comfortable or even possible for the child, then the pressure was far greater than if the expectation were for accurate answers alone. Pressure appeared for some children, to contribute to their dislike and fear of these activities in particular, and mathematics in general. As we have seen, activities based on pressure appeared not only to contribute significantly to the way children come to view mathematics, what it was, how it was learned and how they felt about it, but also to the way they constructed a view of themselves as competent or otherwise.

Pressure was found to be deeply embedded in mathematics pedagogical practices in the classrooms observed, and formed either an explicit or implicit part of the everyday routines. These practices were inherent in teacher beliefs about mathematics teaching and learning passed on unchallenged through successive generations of teachers. This research would suggest that these practices appear to be remarkably stable over time and deeply resistant to curriculum change.

Harries and Spooner (2000) acknowledge the pressure of speed activities as reported by students and teachers relating personal experiences of learning mathematics:

Being asked to respond at speed is identified as a source of great anxiety. The nightmare of the 'runaway test' is a common reminiscence. A victim of this experience recalls listening to question 3 in a mental arithmetic test, starting to work it out but before managing to write the answer hearing the teacher say 'Question 4'. At this point they start to debate with themselves as to whether to finish question 3 or listen to question 4, only to find that debate truncated by the teacher saying 'Question 5'. The other excruciating image is that of the opportunities mathematics seems to provide for public humiliation. These situations are usually related to being asked a question which under different circumstances might be fielded without difficulty but is rendered impossible by the pressure created by a mixture of the need to respond at speed and the presence of an audience that one suspects might derive more pleasure from any response other than the correct one.' (p.36)

Fleur, Georgina, Jessica, Jared and Mitchell suffered most from the pressure of speed activities since they took longer than their peers to learn the basic facts. There are many reasons why this may be so, including lack of appropriate learning activities for these children at school, lack of time spent on learning the basic facts during class time for these children, lack of appropriate support at home, lack of fundamental understandings, lack of interest, or lack of confidence on the part of the child. These children were compelled to engage in daily whole-class speed activities which doomed them to failure, while contributing little to their learning. Peter voiced his frustration and his needs in this regard:

Researcher: Is there anything that we could do to make maths better for you?

Peter: Um, learn more times tables and learn them all.

Researcher: What would help you to learn them all do you think?

Peter: Just getting a piece of paper and writing them all down then copy the answers and just looking at them for ten minutes or something.

Researcher: Yes? Do you get enough time to do that in class do you think?

Peter: No. (Late Year 4).

The children had developed strategies to contend with the daily ordeal. Fleur was sometimes reduced to 'helplessness' such as crying. She raised her hand only when she was sure of answers. She gave a false score on the *Checking Up* questions to appear less incompetent. Unlike Fleur, Georgina, often raised her hand to give the impression of competence. For the same reason, she finished tests early rather than taking the time allotted for her ability group. She looked at the answers of those who were known to be good at maths and copied them. If she could get away with it, she used the times tables chart on the wall or her fingers to access unknown facts. She explained her lack of success as external - a problem with the teacher's going too fast and mathematics' being too hard. Jessica adopted a philosophical face-saving approach. She explained her lack of success as natural - not all people can be good at everything. She looked at others' work for answers and like Fleur, raised her hand only for known answers. Jared devoted great energy to rapid completion of tasks at all costs. He rarely checked the sense of his answers and avoided reflecting on his lack of success. Mitchell tried to avoid activities that made no sense to him. This behaviour was interpreted as deviant. Peter kept an extremely low protective profile and almost never raised his hand for answers. He checked others' books to confirm his answers and worked very quietly and methodically without attracting attention. He explained lack of success as insufficient learning time or provision of support for learning facts.

The evidence of these children suggests that the process of alienation and marginalisation created through the pressure of speed activities is commonplace and likely to affect a significant proportion of primary school children learning mathematics.

### **The teachers' perspectives**

Teachers seldom directly stated their reasons for using speed activities. Nor did they question the effectiveness of these practices in terms of enhancing children's *learning*. Some indication of their assumptions and beliefs can be gained through the statements they made about their use of these activities in particular and about their mathematics programmes in general.

- Miss Puna: That's why I'm having a blitz on the times tables. They've just got to know it, they've got to learn it by rote – memorise it – there's no other way. (Late Year 3)
- Mrs Sierra: We have daily drills, either basic facts or whatever, a game or a maths icebreaker. (Early Year 4)
- Mrs Waverley: We come and we have *Quick Ten*, which is so basic and easy, but that's really just to settle down. (Early Year 4)
- Mrs Kyle: I often think the basic facts are crucial to a lot of what they're doing and once they've cottoned on to those they seem to find maths a lot easier, you know, especially if they can do them quickly. (Early Year 4)
- Ms Fell: Normally we start off the day, there's ten questions on the board....it's reminding them of things we've done. (Mid Year 4)
- Ms Linkwater: Yes, the thing that has been my concern is the lack of recall of basic facts and tables which I feel are just so critical. (Early Year 5)
- Mrs Isles: It's always the basic facts. It always comes down ... because so much hinges on that...I just call them out because the idea is that they give a quick response and they can get them down, because when we were doing partner testing, when they wanted to come to me and say 'Look I think I know my seven times table now. Will you test me?' I just quickly fired, and they knew that that was what was going to happen. That they've got to be able to give a quick response. ... (Early Year 5)
- Mr Waters: (*Talking of Jared*) His recall, his speed of recall is improving, so the times tables are something you shouldn't have to think about...I missed out on that. My tables are shocking, they really are so I'm learning. I don't dare let them know, so I'm learning with them and that's something that I really want to work on because that's a major thing. (Early Year 5)
- Mr Cove: I'm a bit old-fashioned. I believe in the times tables. Where would they be without them? (*Mimes using his fingers to work them out*) I go quite fast because I believe they either know them, or they don't. (Mid Year 5)
- Ms Washbourne: We generally start off with our mental arithmetic...I call it, and they write. I like to do a multiplication array so the children challenge themselves to do those. I know some teachers don't like them but I do. They're really good for mental agility. So that'd be most days. Friday's basic facts testing which is tables and once a month is

the basic facts test which is out of a hundred and they have six and a half minutes to complete it. And so they work their way down in time and up in accuracy. Here they are [*Pinada Publications* resource] And each month it's a different one. (*Looking at her assessment records*) We haven't had anybody who's beaten the 6.30, [time expectation] but if they do beat the 6.30, that's recorded as well. (Mid Year 5)

Fleur: (*Talking of her teacher*) Mrs Meadows is always telling us, 'You don't know maths if you don't know your times tables.' (Late Year 5)

These comments show that the use of speed activities, almost always linked to basic facts, was rationalised by teachers in a range of ways. Where they were used to start a lesson, classroom management ('to settle down') or setting the scene for the lesson ('ice breaker') appeared to be the main purpose. Some teachers seemed to be suggesting that the activities were providing necessary practice for the children. ('I'm having a blitz', 'daily drills', 'reminding them of what we've done'). Some apparently viewed the activities as a stimulating kind of exercise for the brain ('good for mental agility') while others perhaps regarded their use as a means of making transparent those children that knew and those that didn't ('they either know them or they don't'). For some, automation of recall was the chief aim ('the times tables are something you shouldn't have to think about'). Bibby (2002) in her study of the epistemological beliefs of primary teachers found that efficiency was most often mentioned as something they viewed as important in mathematics. She notes that the notion of efficiency implies speed, and the danger that 'this belief would militate against developing perseverance and may be encouraged by an over-emphasis on the testing of rapid recall. With an over-emphasis on speed, accuracy is forfeit' (p. 168).

### **School Policy**

Schools had often developed specific policies regarding the learning of basic facts. While these differed from school to school, they often appeared to be driven by school and parental expectations rather than the learning needs of particular children or requirements of the curriculum, as the following example shows:

'Our survey revealed a level of concern from some parents regarding aspects of our programmes in maths and particularly at the Year 3 level, with regard to transition to maths education in the senior school. In response to this, and as a normal part of our ongoing review programme we will review the emphasis currently placed on basic facts and basic operations in our Year 3 maths programme, continue with the basic facts emphasis started in 1999 for the Year 4, 5 and 6 classes and reaffirm the goal of children knowing all addition/subtraction facts by the end of year 4, and all multiplication and division facts by the end of Year 5.' (from Development Plan 2000, Pukeiti School)

### **The Home Dimension**

Having also grown up in a mathematics culture where speed pressure was an accepted way to learn, parents often communicated their beliefs about mathematics to their children. Through a desire to help them learn what they believed to be 'the basics', parents often provided their children with extra practice at home through informal 'testing'. This often served to reinforce in the children's minds, the overriding importance of speedy recall of basic facts over most other mathematical skills.

In the first interview early in Year 3, the parents were asked about what they thought was the most important part of mathematics for their children to learn. Although speed was never mentioned, 'the basics' featured most highly, as did the need for mathematics in everyday life.

- Researcher: What is the most important part of maths do you think?
- Georgina's mother: Addition, multiplication, fractions. Because it's an everyday occurrence. You use a mathematical equation every day. (Early Year 3)
- Researcher: What do you think is the most important part of maths that Rochelle needs to know?
- Rochelle's mother: Just the basics, basic facts, yeah, I think would get her going in the right direction. Just the very basic stuff. If they've missed out on it, they've missed out on everything, they just won't pick it up...Just being able to add and subtract and, um, just that type of thing...deal with money...They need to understand what they're doing. They need to just understand the basic concept of maths and then they should be right all the way through. Yeah, it's the basics I missed out on. (Early Year 3)



- Jared's mother: Basically number concepts of how to add and subtract. They get too reliant on a calculator. I just learnt it no arguments. No calculators, nothing! You just learnt it, end of story. (Mid Year 3)
- Liam's mother: I still believe in the three R's, reading, writing and arithmetic...
- Researcher: So what do you think in maths is most important? You talked about arithmetic.
- Liam's mother: It sort of means everything, multiplication, your take aways, division. Everything you've got to have a grasp of these days. I find it's not one thing really, as long as you've got a grasp of the basics of all of them, you're going to be OK. (Early Year 3)
- Peter's mother: We think maths is the most important subject at school.
- Researcher: What is the most important part of maths to learn, do you think?
- Peter's mother: Probably the practical maths questions that have to occur. I mean I can't imagine not being able to compute things. (Early Year 3)

Sibling rivalry in mathematics and parents' testing children on their basic facts were common features of family life which served to strengthen the children's beliefs that answering basic facts questions at speed was what mathematics was all about.

- Researcher: So what sort of maths do you do at home?
- Georgina: Sometimes my Dad tells me, 'What's twelve plus twelve?' and I try thinking. (Late Year 3)
- Rochelle: Sometimes I do it [maths] with my Mum. She asks some questions when she's doing the ironing. She asks us the times tables and me and Cheyenne [older sister], we do the quickest who can answer them. (Mid Year 4)
- Dominic: I would get Mum to test me, like while we're in the car coming to school and things like that. (Late Year 4)
- Liam: Me and my sister have competitions. I ask Mum if I can have some questions and she [older sister] goes 'I'm better than you' and I go, 'OK, we'll have a competition then'. (Late Year 3)
- Toby: Mum, she made up a big sheet of multiplication questions and I had to do every single one of them in the fastest time I could. (Late Year 5)

In these examples, the questioning activities had sometimes been initiated by the children, and sometimes by the parents. The development and perpetuation of beliefs about the importance and desirability of a child's being able to speedily answer mathematics questions provided at random by an adult can be seen as a two-way interaction between school and home.

### **Origins of the speed pandemic**

The prevalence of speed activities indicates that faith in them, as enshrined in everyday life, is widespread. While it is difficult to pinpoint the precise origin of any social practice, historical sources such as teaching manuals, syllabi, curricula and oral histories provide clues. For many, the four situations described earlier in the chapter will resonate with great familiarity. There is evidence that speed activities have been practised for decades in various guises. Atkinson (1996) for example, reflects on her schooling in Britain: 'For many of us, we realise that we were brought up on a rather monotonous diet of mental arithmetic every morning (where we wrote down the answer and had them marked out of twenty), arithmetic from a textbook every day, 'problems' (usually work sums) about twice a week...and arithmetic tests every Friday. Rivetting stuff!' (p. 42 ).

Flournoy (1964) in describing the history of mental arithmetic in US elementary school mathematics, writes:

During the 1920s and early 1930s, the 'connectionist' or 'stimulus and response bond' theory of learning was predominant. (pp. 53)

Mental arithmetic received very little attention in our schools during the first half of the twentieth century. The swing towards little or no emphasis on mental arithmetic about the beginning of the twentieth century seems to have arisen because of an over-emphasis on mental arithmetic during the latter half of the nineteenth century. Within the last decade, more attention has gradually been given to the development of mental arithmetic ability. (p. 60)

In spite, or perhaps because of, the sheer ordinariness of speed activities in mathematical pedagogy, surprisingly few detailed descriptions of them exist in mathematics education literature, although a number of passing references can be

found. Davis (1996) notes the 'Mad Minute' in North American classrooms, where students are expected to complete as many computational questions as possible within sixty seconds.

Given the universality of these sociomathematical norms, it can be assumed that there exists a global belief and tacit agreement among educators that these activities constitute effective mathematical pedagogy. How and why have speed activities been 'contrived' as classroom practice (Ernest 1998), implemented, and perpetuated on such a wide scale?

The history of mental arithmetic including the memorisation of basic facts can be traced to the likes of Thorndike (1922) who believed that children learned through a 'stimulus-response bond' and that regular repetition was essential for habit-formation and connection-making that contributed to memorisation (p. 70). He wrote: 'Learning arithmetic...is in some measure a game whose moves are motivated by the general set of the mind toward victory – winning right answers' (pp. 283-284). Other writers have stressed the need for understanding rather than blind recall. For instance, Wheat (1937) criticised 'the resort to drill' at the expense of comprehension, stating that 'reasonable speed and reasonable accuracy...are the proper goals of drill...the pupil's progress must be measured in terms of what he understands, not in terms of accuracy and speed' (pp. 158-159). It would seem that the Thorndike approach has proved particularly compelling and enduring in mathematics classrooms.

Joan Paske, who became a primary mathematics advisor in the Wellington region in the late 1960's testifies that the *Quick Ten* was alive and well in schools at that time (J. Paske, personal communication, 19 June, 2002). Joan recalls answering ten quick questions, either from the board or dictated by the teacher, as a primary school pupil in England in the 1930's. She says that as an advisor, she worked hard to eliminate the practice, unsuccessfully it appears.

### Promotion of speed activities through commercial texts

Paul Dowling (1998) maintains that all mathematical text books 'constitute in their reading, *voices*, and in particular, authorial and readerly voices' (p. 122). He believes that they serve to construct a hierarchy between the learner (acquirer) in an apprentice position and the teacher/authority (transmitter) in the expert position. In the same way that a teacher assumes authority in the classroom by positioning him/herself in a particular role through using comments, statements, instructions or questions, the textbook uses the voice of written statement, comment, question and instruction to take on a similar authoritative and didactic role. The authoritative 'voice' of the textbook can therefore be seen to echo, support, legitimise and strengthen the authoritative role of the teacher.

Many teaching resources can be found in which speed has been emphasised for many decades, for instance, Burn (1968) provides lists of questions and introductory instructions for teachers - 'The aim is to help children be quick and accurate with calculation' (p. i), and children - 'Work as fast as you can and race yourself every day' (p. 2). The *Macmillan Mathematics Children's Recording Book Level 2b* (Beesey & Davie, 1991), instructs students to 'fill in the grid at two different times during the year. You will need a stop watch, or a clock with a second hand, to record how long it takes you to complete the grid' (pp. 52-53). *New Wave Mental* (Moore, 2000), provides a very recent example of the promotion of daily mathematics speed activities through commercial texts. This teaching resource provides a list of speed questions for every day of the school year, and a test for every Friday.

Many instances were found in the study classrooms where commercial mathematical texts were used that supported speed and competition in the so-called learning and practice of basic facts. The most common of these were the AWS<sup>3</sup> numeracy sheets and the Pinada<sup>4</sup> resources. Neither of these resources provided teachers with methods of helping the children to understand important connections and relationships between the facts they were expected to learn.

<sup>3</sup> *AWS Teacher Resources*, (Stark, 1997).

<sup>4</sup> E.g. *Speed, Skill and Success*, (Pinder and Adams, 1996).

### Speed in the official mathematics curriculum

At the time of its development and publication, *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) represented a significant change in mathematics education philosophy in New Zealand. In its introductory pages, the curriculum advocates that teachers take a problem solving approach to the teaching and learning of mathematics and states that 'students need frequent opportunities to work with open-ended problems.... Such problems encourage thinking rather than mere recall' (p.11). This idea is further strengthened by the inclusion of the *Mathematical Processes* strand. There is no suggestion in the introductory section of the document, that speed pressure in any of the forms described in this chapter, is desirable in teaching mathematics. On the contrary, concern is expressed that girls and Maori are underachieving and/or losing interest in mathematics and in the case of Maori, there is special mention made of the need for a wider range of assessment techniques than 'traditional time-constrained pencil and paper tests' (p. 13).

Alongside the concern about the negative effects of time constraints, the curriculum also states that students should be 'developing *instant recall* of basic addition and subtraction facts through a programme of regular maintenance' (Level 2) and 'demonstrating the *instant recall* of basic multiplication facts' (Level 3) [italics added]. 'Instant' meaning 'at great speed', would suggest that the curriculum writers consider this skill to be important. The curriculum can be seen on the one hand to be advocating a teaching approach that removes the undesirable pressure of traditional methods, while on the other continuing to value speed in the recall of basic facts. Thus the valuing of speed (instant recall) at least for basic facts, is embedded as expectation within the national syllabus.

In a Ministry of Education teachers' handbook *Developing Mathematics Programmes* (1997) practice and maintenance of basic facts is discussed in a special four-paragraph section called *Basic Facts*, under the general heading *Providing for Maintenance*. While the word 'instant' is not used, it is advocated that:

students must have *rapid* recall of the basic addition and multiplication facts. Being able to recall these facts *quickly* and easily through knowing them 'by heart' increases students' confidence, allowing them

more time to concentrate on the higher-order thinking and communication skills they need to solve problems relevant to their level of development. Even when students understand the basis for addition and multiplication facts, they need lots of practice to make their recall *automatic*.' (p.25) [Italics added]

A list of possible strategies for learning and practising these facts is provided with the comment that, 'whatever strategies are used, they must be enjoyable and provide positive and immediate feedback to the students' (p. 25). While there is no suggestion that speed pressure be used as a means to teach or maintain the basic facts, the writers say that 'being able to recall these facts quickly and easily through knowing them 'by heart' increases students' confidence, allowing them more time to concentrate on higher order thinking' (p. 25). Deep structural understanding of the facts, being able to derive unknown facts from known, and appropriate application of the facts in authentic contexts appear to receive far less attention than speed of recall. Issues of understanding and memory are explored in Nuthall (2000) and Anthony and Knight (1999) who both stress that 'learning' and 'recall' are complex cognitive processes, and that meaningless rote memorisation and drill are either ineffective or lead to impoverished learning. Nuthall argues that the quality of the activity itself is significant in the child's development of understanding of the subject matter.

Mathematics is the only school subject where speed of task performance is explicitly valued through the official curriculum. In *English in the New Zealand Curriculum* (Ministry of Education, 1994b) spelling, also characterised by regular testing in primary school, is valued for its sense-making in context, not for the rapidity of its recall (see p. 96). In *Health and Physical Education in the New Zealand Curriculum* (Ministry of Education, 2000a) where physical speed might feature, there is no such suggestion. No one would expect a national curriculum to specify for instance, that by a certain age all students must be able to run 100 metres in at least fifteen seconds.

While the Department of Education School Mathematics Booklets published in the 1980s did not emphasise speed in learning basic facts, more recent government mathematics texts overtly encourage the use of speed activities. *Cycle 12* of the *Beginning School Mathematics* resource (Ministry of Education, 1993) intended for use at Year 3, promotes the use of speed through activities such as 12:3:46 -*Time is*

*Running Out* and 12:3:48 - *Who Gets a Hard-Boiled Egg?*, both of which require an egg-timer to set the time limit for activity completion. In the instructions for *Don't Fall off the Roundabout* (12:3:8) a basic facts game in which children 'fall off the roundabout' if they cannot produce the correct answer, it is suggested that 'the roundabout could speed up as it goes around each time' (p.123). The series *Figure it Out* (Ministry of Education, 2000a) commonly used in many of the classrooms observed, provides, in the activity *Beat yourself Down*, a vivid example of how mathematics, speed, and sporting competition are linked. The children are instructed to:

Choose an addition or subtraction section and write down the answers in your exercise book as fast as you can. Use a stopwatch to time yourself. Your aim is to answer correctly all the equations in one section in the shortest time possible. Try a new section each day. Aim to increase your speed and accuracy each day. (*Figure it Out, Level 3, Basic Facts*, pp. 2-3)

Lists of questions are set on a background featuring photographs of a boy in sports gear with stopwatch in hand and six pairs of children's sports shoes placed around an athletics running track.

In light of the stress that the New Zealand Ministry of Education currently places upon rapid recall of basic mathematics facts, important questions arise. Why is rapidity of recall regarded as the greatest indicator of 'knowing' mathematical 'facts'? How fast is *rapid*, *instant*, or, *automatic*? Is a child who takes two seconds to recall a basic fact less 'able' than a child who takes one? At no point is rapid or automatic basic facts recall either clearly defined or convincingly justified in any of these government publications.

### **International perspectives on speed activities**

New Zealand is not alone in valuing speed in mathematics teaching and learning. In England's *National Numeracy Strategy* (Department for Education and Employment, 1999) implemented to raise standards, a clear directive is given for the deliberate use of speed in everyday mathematics pedagogy. Rapid recall is frequently stipulated from Year 1 onwards, eg: 'As outcomes, Year 1 pupils should for example: Respond rapidly to oral questions phrased in a variety of ways' (p. 30). Pace is emphasised in



the lesson itself where it is suggested that 'in the first part of the lesson you need to: get off to a *quick start* and maintain a *brisk pace*; target individuals, pairs or small groups with particular questions' (p.13). While creating time constraints and an atmosphere of 'businesslike' efficiency are advocated as teaching strategies for the whole mathematics lesson including the oral whole class starter activity, *Quick Tens* and competitive games are not explicitly specified as effective teaching practice.

In the USA, the National Council of Teachers of Mathematics standards (2000) carefully avoid mentioning speed, but speed is implied when it is stated that students should develop *efficiency* and *fluency* with basic number combinations (see p. 35).

Speed or 'efficiency' in curriculum documents can be seen as a distinctive feature of current mathematics classrooms in countries characterised by common social and educational heritages. Indications are that speed activities have existed in these countries as everyday classroom fare for many decades in spite of the lack of any explicit official mandate. This suggests that the widespread recurrence of speed activities must be viewed as a genre of practice embodied in and validated by the everyday life of school, supported by commonly used mathematical texts, reinforced in the home, implicitly endorsed by curriculum documentation, and reproduced by successive generations of adults who sustain this pedagogical icon.

### **Exposing the taken-for-granted**

Concealed within the statements of teachers, parents, curriculum documents and mathematics textbooks, and made more visible through classroom ritual, were a range of commonly-held beliefs about the place and purpose of the recall of basic numerical facts in the learning of mathematics:

- 'knowing' basic facts is essential for all other learning in mathematics;
- rapid recall indicates 'knowing';
- 'knowing' facts is the responsibility of the individual child;
- there is only ever one right answer;

- right answers are more important than the methods of finding them;
- a child's ability to perform in basic facts speed tests is a good indicator of her/his overall mathematics ability;
- children are all 'ready' for this kind of activity at a certain age;
- frequency of speed testing will enhance learning for all children;
- it is the teacher's role to choose the questions and the teacher's role to determine the pace of questioning;
- sharing scores with the rest of the class is good for children because a degree of pressure through public exposure motivates them to learn.

Competitive games and written tests provided an immediate, tangible and quantifiable indication of a child's success or failure. Evidence showed that the study teachers were apt to focus on the easily measurable since basic facts test results were seen to be the most recorded of teachers' assessment practices in mathematics. Teachers rarely expressed an awareness of, or concern about, the pressure they were creating for children through their routine basic facts testing and game-playing. Because some children in the class displayed enthusiasm for competitive mathematics games, teachers wrongly assumed that *all* the children loved such games.

### **Speed activities, power and control**

In his analysis of mathematics education as a 'culture', Bishop (1991) has isolated what he believes are the key underlying values of this culture. One of these he calls *control* and says 'there is no doubting the fact that when mathematics is understood and mastered it develops strong feelings of control, security and even mastery in the adept' (p. 71). Conversely, it is likely that lack of mastery will develop strong feelings of powerlessness, insecurity, bewilderment, and failure in the inept (Mayo, 1994).

Some have suggested that pressure in mathematics teaching is linked to wider cultural practices surrounding power, control, security and mastery. Appelbaum (1995) for example has investigated the links between popular culture and mathematics education and argues that 'popular culture is an important force in shaping how students, teachers and others view themselves and various forms of teaching and learning' (p. 108). He provides the example of televised quiz shows that he believes

echo the ritual functions of the education system generally, and in particular, mathematics education. Many of the teachers conducted speed activities in a remarkably similar way to the question answer format of the quiz show in which children as contestants, became winners or losers.

Parallels can also be drawn between the ways in which children were expected to perform during speed activities in the classroom, and popular culture surrounding competitive sport. In New Zealand, as in a growing number of other countries, competitive sport has become a significant part of everyday life and even of national identity (McKay 1991). Competitive sport values performance against the pressure created by time constraints, difficulty of task, and strong opposition. Winning is paramount. Winners are hailed as heroes and seen as powerful and masterful, while losers receive little respect or tolerance. This sentiment is captured in the words of Oliver<sup>5</sup> (2001) who said: 'Winning is everything. If we lose, it's all for nothing'. Significantly, 'Loser!' is currently used as a common term of abuse.

When mathematics activities in the classroom resembled sporting competition, as in games such as *Shoot*, the same values were seen to apply. Arithmetical skills and knowledge in mathematics lend themselves to competition because winners and losers can be created through imposing time limits in which to produce unequivocal correct answers. Teachers who have invented games like *Around the World*, *Shoot* and *Buzz*, have capitalised on the motivational nature of competition to endow learning mathematical 'facts' such as the times tables, with what they believe to be 'worth' and 'purpose' in the eyes of their learners.

A number of children and some teachers in the study referred to attainment in written tests as *scores* rather than the more traditional term *marks*. In New Zealand, *score* is a word that was once associated only with competitive sport. For many decades in the USA, *score* has been used in education to mean the result of a test or examination and it appears that this American usage is beginning to take hold in New Zealand. It is possible that this linguistic change has served to subtly connect, in the children's minds, sporting competition and mathematics.

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<sup>5</sup> Anton Oliver, captain of the New Zealand All Blacks national rugby team, TV1 News interview, 6.00 p.m., 15 June, 2001

The belief that speed pressure was justified, was shown in teachers' statements for example Mr Cove who said, 'I go fast because they either know them or they don't,' indicating not only that speed of recall was the chief criteria for 'knowing', but implying that speed would sort out those who knew them and those who didn't.

By subjecting their pupils to the intense pressure of speed activities camouflaged as benign 'check ups' or 'games', teachers were able to exercise a high degree of control over children in their teaching of mathematics. For those children who succeeded at these activities, such as Rochelle, Dominic, Liam and Toby, their sense of control was increased. However, such control was reserved for a few and won at the expense of others. Teachers in the study were observed to consciously or otherwise, manipulate speed activities in order that only a small group of children could ever be 'winners'. They were able to do this by increasing the pace, decreasing the time limit, or increasing the difficulty of the questions. Success was always unattainable for a significant proportion of the children in the classrooms. By limiting access to success, they were effectively creating and maintaining an elite group. Children were also controlled by being targetted with questions if they appeared to be inattentive or off task as Jessica noted when she said: '...when she's [the teacher] asked someone who isn't paying attention.' (Mid Year 5).

It can be seen that some children, Fleur, Georgina, Jared and Peter for example, processed their thoughts more slowly than others. Peter either knew, or could derive most basic number facts number facts, as evidenced by his accurate though incomplete test papers, but he still 'failed' at these speed tests because he could not think or write as quickly as the teacher demanded. For those children in this study for whom the expected speed of recall was unrealistic, high-pressure speed activities not only increased their feelings of failure and reduced their confidence, but also prevented these children from improving either their knowledge of the facts or their speed of recall.

For those who were 'too slow', speed activities resulted in a deeply disempowering and negative affect, the strength of which cannot be underestimated. Fleur, Georgina, Jessica, Jared, Mitchell and Peter, provide us with disturbing insights into the social worlds of children who fail, literally, to come up to speed in mathematics.

### Speed activities as part of everyday life: resistance to change

In spite of its profound detrimental effects on learners, the speed pandemic in mathematics has proved particularly resistant to change. Ms Fell's self-reflection on her use of speed activities provides important insights into the social mechanisms by which these activities are perpetuated.

Ms Fell: Normally we start off the day, there's ten questions on the board. I went on a maths course where the lady said, 'Don't do it. Don't put questions on the board because if they don't know it, they're going to feel like they're failing, and if they do know it, they don't need to practise it. And I thought 'Oh, that might be a really good point and I came back and thought about it and then I thought, 'But often it's reminding them of things we've done'. (Mid Year 4)

Ms Fell was reluctant to alter her daily routine that began with the ten-question speed activity. While she could appreciate the reasoning of the adviser, rather than accept it and make changes to her regular, familiar programme, she rationalised her current practice as being beneficial for the children because it 'reminded' them of things they had done in previous lessons.

Justification of habitual behaviour in our daily lives, and reluctance or inability to change when challenged, is a commonly observed human phenomenon. Vaughan and Hogg (1998) describe how the theory of *cognitive dissonance* has been used to explain this behaviour. Cognitive dissonance, they say, is 'a state of psychological tension produced by holding two simultaneous and opposing cognitions' (p.173). A person in this state is thus motivated to reduce the tension by discarding or altering one of the cognitions. In the example above, Ms Fell discarded the new and conflicting cognition produced by contact with the adviser.

While the theory of cognitive dissonance may explain how individuals might feel a need to create harmony when faced with incompatible cognitions, symbolic interactionism may go further in explaining why 'new' cognitions are often rejected in favour of the everyday, familiar, and habitual. From this point of view, it is regular, routinised social interactions that enable individuals to make sense of their lives. Ms Fell knew of no alternatives with which to replace her established routines. Without those routines, teaching could, for her, become risky, unmanageable and lacking in

sense. How could she be sure that children were 'reminded' of what they had done without *Checking Up*? How else could she begin mathematics sessions without the familiar 'opening scene'. How else could she control the whole class without the pressure system? How else could she 'see' who was struggling and who wasn't? In reflecting on the meaning of these realities of her everyday life, Ms Fell's pupil Fleur came to the conclusion that she was a slow learner, and less mathematically capable than many of her peers (p. 96).

Commonly accepted use of speed activities in the teaching of primary mathematics is established through complex, interwoven processes of social interaction which appear to include:

- the teacher's own childhood experiences of learning mathematics at school where speed activities may have been an everyday affair;
- the teacher's pre-service training that may have emphasised the need, in a *balanced* mathematics programme for daily *maintenance* of basic concepts and skills such as recall of basic facts;
- the teacher's interaction with other teachers who may pass on ideas about speed activities;
- the school's policy which may stipulate the use of speed activities
- the school's use of precisely timed Progressive Achievement Tests for mathematics and/or involvement in speed-dependent mathematics competitions;
- the parents' own childhood experiences of learning mathematics at school;
- the teacher's interactions with parents who expect speed activities to be used;
- the teacher's interactions with the children who may expect the same routines as used in previous classes or the same teaching methods experienced and discussed by their parents and older siblings;
- the teacher's and/or school's use of textbooks and commercially-produced mathematics programmes that promote the use of speed activities, including providing ready-made lists of speed questions and timed tests;
- the teacher's exposure to aspects of popular culture that validate and reinforce speed and competition.

Ms Fell, a young teacher with eighteen months' classroom experience, had been trained to teach mathematics based on the achievement objectives and constructivist learning philosophy of *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992). Without living models of this changed view of learning, Ms Fell resorted to the known: she reproduced, maintained, and justified entrenched traditional approaches to mathematics teaching, including speed activities. It is only to be expected that pressure for change involving a radical shift in widely-accepted social practice will meet with resistance. Not surprisingly, amongst all the players in the sociomathematical worlds of the children, the research did not reveal evidence of any significant challenge to the entrenched use of speed activities.

### **Conclusion**

Mathematics speed activities were found to be a distinctive and dominant routine within classrooms. Their privileged place in the teaching of mathematics, supported by parents, implicitly sanctioned by *Mathematics in the New Zealand Curriculum*, and explicitly endorsed by widely used teaching guides, was found to have survived decades of curriculum reform. With daily exposure and much kudos for those who were successful (winners), the children concluded that 'knowing your tables', as defined by memorisation and speed of recall, was the most important of all mathematical skills. Correspondingly, they developed the belief that anyone who underperformed in speed activities (losers), was no good at mathematics. These understandings were particularly significant in the children's developing mathematical identities and their views about the nature of mathematics. Children were compulsorily subjected to speed activities, which for some became the prime source of a sense of achievement and confidence in mathematics, but for others produced feelings of deep anxiety and inadequacy. Most teachers appeared to use speed pressure in teaching mathematics with little awareness of, or concern about, its impacts.

The following chapter examines how classroom practices in the teaching of mathematics, including speed activities, contributed to the construction of shared beliefs about mathematical competence and differentiation by socially defined notions of *ability*.



### TOP, MIDDLE AND BOTTOM : THE CONSTRUCTION OF MATHEMATICAL 'ABILITY'

Chapter 4 explored the widespread use of speed activities in the sociomathematical worlds of the children and the ways in which such activities contributed to the children's developing perceptions of what mathematics was, who was good at it, and their own mathematical identities. It was shown how, for some children, speed activities caused disaffection with mathematics. This chapter explores another facet of the children's mathematical worlds. It centres on the ways in which teachers, parents and children viewed mathematical achievement and how teachers classified, separated and labelled children through their everyday teaching of mathematics.

Societal beliefs and values are often reflected in metaphors found in everyday language (Lakoff & Johnson, 1980). Metaphors are commonly used to express and reinforce our beliefs about human differentiation and segregation, for example the biblical story about separating 'sheep' from 'goats'<sup>1</sup> or the saying 'separating the wheat from the chaff'<sup>2</sup>. Recurrent use of such metaphors denotes a popular conviction that there are naturally-occurring, distinct 'types' of people, some better or worse than others, and that they can and should be identified, categorised, separated, and treated differently. This chapter examines beliefs and values that shape perceptions of mathematical achievement and ability, and show how these are reinforced through the metaphorical language and everyday practices of the classroom. 'Mathematics' claim Dorfler and McLone (1986), 'is one of many subjects but it nevertheless is in a unique position, because of its highly differentiating effect. There are the talented students and the underachievers, there is the necessity for remedial teaching, there are minimal competencies and many other features which demonstrate the quite peculiar position of the subject mathematics at school' (p. 71).

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<sup>1</sup> New Testament, St Matthew, 25:31-3.

<sup>2</sup> Psalms, 1:3-4.

Three of the children in this study have been selected to illustrate the ways in which differentiation, separation, and labelling operated within the sociomathematical worlds of the children. The first is Mitchell, a pupil so exceptional that he failed to fit within the parameters of expected mathematical achievement. The second is Jessica, a 'middle' student who survived, rather than flourished, within the environment of school mathematics. The third is Rochelle, a 'top' student, who derived satisfaction and a measure of security from her daily experiences of mathematics at school.

The chapter explores the criteria by which learners were sorted, grouped, and labelled, the prevalence and frequency of grouping systems, and the children's views of ability and achievement in mathematics. It is shown how their developing mathematical identities including self-efficacy beliefs and causal attributions for success or failure, were influenced by the differentiation practices of their social environments. In the light of curriculum documents and classroom texts, the beliefs that appeared to support sorting and labelling practices in school mathematics are examined.

### **Mitchell: 'Behind the eight ball'**

From fieldnotes, Mitchell's classroom, Pari School, Mid Year 3

*Mitchell is seated on the mat with the other children in his class.*

Mrs Hite: Room 6 people go through for maths please.

*(Three of the children, including Mitchell, walk to the classroom next door. They sit on the mat where the teacher is already seated in front of the children. The children are chanting rhymes from a big picture book. Mitchell looks bemused and begins to suck his thumb.)*

Mrs Craig: *(To Mitchell, sharply)* Put your hand on your lap so you can speak out.

*(Mitchell appears more interested in the next chant, Little Puppy Rap and tries to keep up with the chant, but begins to suck his thumb again after a short time, and appears to miss the teacher's instructions when she directs the two groups to their tables. They are working on activities from the Beginning School Mathematics programme. The teacher is working with Mitchell's group today, while a student teacher takes the other group. She takes her group to a table and directs the children to sit around it on their chairs. Mitchell sits covering his nose with his hands.)*

Mrs Craig: I want you to do two things today. I want you to draw on the green paper a picture of your house. Then I'm going to get you to take the pink paper and draw a picture of yourself.

*(While others in the group are excitedly discussing what their house looks like, Mitchell is silent. He watches as the teacher demonstrates by drawing her own house.)*

Mrs Craig: Mitchell, what's special about your house?

Mitchell: I don't know.

Mrs Craig: Well you have a think in your head. *(The teacher asks other children. Some others also have trouble describing their houses).* Close your eyes. *(Mitchell does so)* Now draw your house. *(Mitchell takes a brown felt pen and draws a large square that takes up most of the page. He looks at the teacher's picture and draws his picture like hers. He looks around at the other children's drawings. Some children talk as they are working.)*

Andre: My house is green. That's why I'm doing it green.

*(Mitchell is silent throughout. Some children begin discussing where they live, pointing in the direction of their houses. Mitchell does not join in. He draws a tree beside his house, then sucks his thumb and looks out of the window. He then arranges the felt pens in the tray.)*

Mrs Craig: *(Looking at Mitchell's picture)* Tell me about your house, Mitchell. What's this?

Mitchell: *(Very quietly)* The shed.

Mrs Craig: *Inside* the house? What's this?

Mitchell: *(Shrugs)* I don't know.

*(The teacher now instructs the children to draw a face on a yellow circle. Mitchell draws eyes.)*

Mitchell: What colour's my hair?

Mrs Craig: What colour's your hair? *(Mitchell does not reply)* Go and look in the mirror.

*(Mitchell goes to look in the mirror then comes back. He draws a mouth. He goes back to the mirror, returns and works on the cheeks, goes back to the mirror once more and pulls faces at himself. He looks back at Mrs Craig to see whether she has noticed.)*

Mrs Craig: I want you to write your name under your face. *(Looks around and sees Mitchell at the mirror.)* Finished looking at yourself, Mitchell? *(He returns to the table.)* You've got a nice cheery smile on your face, Mitchell. *(looking at Mitchell's drawing. Mitchell returns to the mirror twice more to look at himself, oblivious of the other children who are now gluing their houses, faces and pink connecting arrows onto a chart.)*

Mrs Craig: Come on Mitchell. *(Mitchell doesn't move from the mirror. The teacher goes over and puts her arm around Mitchell's waist, guiding him back to the table to wait his turn for gluing.)*

*Mitchell looks restless.) You could put the felts away. (Mitchell returns to the mirror then plays with some metre rulers nearby.) Mitchell, come here. I've got a space for you now.*

*(Other children who have finished have gone off to the mat and are engaged in mathematics games and independent activities from their BSM box. Mitchell glues his house onto the chart. The teacher gets him to finish another piece of work from the day before, involving cutting and gluing some objects onto paper. The teacher sits beside him. The lesson finishes with the teacher asking the children to pack up, and sending the Room Seven children back to their classroom.)*

Mitchell appeared to be far less engaged in the activity than the other children were. His attention wandered after a short time, and there was little verbal communication between Mitchell and the others. The teacher did not introduce the activity by explaining its purpose nor finish the activity by talking about the chart, so it was difficult to tell whether any of the children, Mitchell included, understood why they were doing what the teacher had asked, and why such an activity might be helpful for their mathematical learning. It seemed that Mitchell had tried to draw the shed in front of his house, but it looked as though it were inside. His lack of verbal and social skills prevented him from explaining this to the teacher, or from engaging in mathematical talk such as the direction of his house, with the other children. Donlan and Hutt (1991) link many problems in mathematical learning to language. Children who are not fluent in the specialised language of the classroom are therefore at a great disadvantage. Zevenbergen and Lerman (2001) and Zevenbergen (2001) note that children from economically underprivileged backgrounds are also disadvantaged at school because for them, the languages of home and school are more likely to be incongruent. The two schools Mitchell attended were in the poorest housing areas of the city, and a parent interview confirmed that his one-parent family was financially deprived. It is likely that social circumstances were a significant contributor to Mitchell's difficulties at school.

### **The teachers' responses to Mitchell**

Mrs Craig, Mitchell's teacher for mathematics, was perplexed and somewhat frustrated by this child.

Mrs Craig: He is by far the oldest in the class and I think he's quite capable of working at a higher level but he hasn't got the independence. He's very easily distracted and goes off task and sometimes he will not cooperate and if he is working one to one, eventually he will probably give the answer that you want. (Mid Year 3)

The teachers were concerned that Mitchell was not conforming to the classroom routines in the same way as the other children. Although they had sought help from appropriate agencies, this had been limited. The following conversation reveals the ways in which they strove to understand Mitchell in light of their expectations.

Mrs Hite: Of late I have noticed, like this term, he has begun to show signs of maturity, not leaps and bounds but he will sit down and he will listen and he will actually follow instructions but he still needs very close monitoring to make sure he actually follows the task through. *(Later)* I think Mitchell will always be a little behind the eight ball in terms of maturity. I mean, like, there'll probably come a stage when he does actually catch up so it's not quite so noticeable in terms of his peer grouping, but he will always be an individual, he will always be, um, a special child, you know, in terms of his wants and needs, and he'll always be a little bit different but that's fine.

Mrs Craig: He's a very interesting little boy...He's on a different plane altogether...the eye contact or the absorption, it's not there...fiddles ...uncooperative in responding...bewildered and perplexed...his writing is still very untidy compared to the rest... He'll sit down and not be so obtrusive, but he's still not necessarily listening... He's a special needs boy. (Mid Year 3)

They also recognised some of his mathematical capabilities, particularly apparent when he was provided with concrete materials.

Mrs Craig: I was saying to him, 'Are there more or less?' and he was quite lost, so the next day I did an activity with everyone counting out some little BSM toys and he had no problem telling me when he could see them...He's OK when he's got the apparatus there.

(Later)

Mrs Craig: The other day we were doing this measuring one, with things we had from the developmental room, they all had a small ruler, and he marked the end of each one *(She shows his work, where objects had been traced around with their left hand ends aligned and whose lengths they were comparing by looking at the right hand ends. Mitchell had drawn little vertical lines from the ends of the objects to help him*

*compare their lengths. One object, a comb, had a curved end so his technique was particularly useful there in finding the longest point of the comb.*) I hadn't said that to the children, it wasn't part of my instructions, but he had actually done that ...I thought that was particularly clever of him, and I was standing by him when he got down to the toothbrush at the end and he says, 'Oh I think this one's bigger, this is bigger,' and he sat and did that without any of the nonsense that we would have had earlier in the year.

(Later)

Mrs Craig: I was distracted for a moment [during BSM checkpoint 5] and when I came back he'd built a big 3-D construction.

Researcher: Any pattern in it?

Mrs Craig: Yes, there was. (*Shows her record of it*). He could do it.

Researcher: It looks quite complex.

Mrs Craig: Yes it was. He could spot those missing pieces very quickly. It was very hard to keep him focussed. He likes to build all the time, but then he can be difficult, refuse, and reluctant to answer and you think he knows the answer but he just doesn't want to do it. I'm sure he knows lots more than he displays. (Mid Year 3)

### **Mitchell's response**

Mitchell was unable to explain why he went to different teacher at mathematics time, although he knew the names of the other two children who went with him.

Researcher: How many people go to Mrs Craig for maths? There's you, and ...

Mitchell: Rosa and Salili.

Researcher: Why do you go to Mrs Craig's room for maths? (*Mitchell looks blank*) So why don't you stay in Room 7 for maths?

Mitchell: (*Looks down at the floor*) I'm really ...(*inaudible*) (Mid Year 3)

Mitchell obviously had trouble making sense of everyday classroom routines. By the end of Year 3 his awareness of mathematics as a distinct subject was beginning to develop, but specialised vocabulary used in an abstract way such as 'more' and 'less' was meaningless to him in the context of classroom mathematics. The introduction of written mathematics using mathematical symbols for equations and expressions, appeared difficult for him:

Researcher: What's maths all about?

Mitchell: Like one plus one or something.  
 Researcher: Why do you have to learn that do you think? (*Mitchell shrugs*)  
 (*Later*)  
 Researcher: Is there anything you don't like about maths?  
 Mitchell: One plus one.  
 Researcher: Do you think you're very good at one plus one? (*Mitchell shrugs*) What do you do when you don't understand something in maths? (*Mitchell shrugs*) You don't know what to do? (*Mitchell shakes his head*). Do you do maths every day? (*Mitchell nods*) Do you do maths for homework?  
 Mitchell: Hm.  
 Researcher: What maths do you do for homework?  
 Mitchell: One plus one and stuff. (Late Year 3)

**Coping strategies**

Pollard and Filer (1999) use the concept of *coping strategies* to analyse and explain the behaviour adopted by individual children within the school and classroom. They observe that both teachers and children interact within webs of social constraint, and that in a classroom situation, they develop strategies to cope with the tensions that such constraints may produce. They use a matrix to describe what they see as the four kinds of coping strategies adopted by children, as in Figure 5:

	Conformity	Anti-conformity	Non-conformity	Redefining
<b>Career Orientations</b>	Reification	Rejection	Indifference	Identification
<b>Strategic Responses</b>	Low risk strategies (conformity to others' agenda)	High risk strategies (trouble through oppositional agenda)	Little concern or awareness of risk (because of own agenda)	High Risk strategies (influencing the shared agenda)
<b>Adaption</b>	Adaption	Deviance	Independence	Negotiation/Challenge

**Figure 5: Characteristic patterns of career orientation, strategies and adaptations** (Reproduced from Pollard and Filer, 1999, p. 28)



Mitchell's teachers had identified him as '*different*' primarily because he did not interact with them and with the other children in the ways that they expected, that is, he did not conform to school norms. While they sometimes viewed his lack of conformity as deliberate, as instanced by their use of words such as '*uncooperative*', '*difficult*' and '*reluctant*' and therefore *deviant*, Mitchell's indifference to the expectations of the group and to the mathematical tasks, demonstrated a relatively low awareness of the risks of his failure to conform. This would suggest that his coping strategies were mostly *non-conforming* in nature. He was behaving *independently* of the others, (sucking thumb instead of chanting, going to the mirror and looking at his reflection instead of gluing his pictures onto the group chart, 'in a world of his own', 'on a different plane') but not in the ways the teachers found acceptable.

Because Mitchell experienced difficulty in making sense of the daily life of school and the classroom, his learning of mathematics suffered especially when the teachers could not provide him with the 'very close monitoring' that he needed. In spite of the indications that Mitchell's understanding of mathematical ideas, and his capability to learn, were not nearly as underdeveloped as they appeared, his behaviour caused his teachers to search for explanations and strategies in order to cope with him. Having identified him as '*special*' and '*behind the eight ball*', they decided to place him with younger children for mathematics lessons. This appeared to add to Mitchell's bewilderment, and he continued to adopt non-conforming strategies, thus 'failing' to follow instructions or maintain focus on the mathematics tasks. Because the special class contained only fourteen children, the stress of working with this different child was reduced, but this approach failed to address Mitchell's mathematical learning needs.

The teachers recognised Mitchell's responsiveness to concrete materials, as Mrs Craig noted: 'he's a doing child, he needs to constantly move'; 'he loves the developmental time<sup>3</sup>' (Late Year 3). This provided a vital clue as to the methods by which Mitchell was most able to make sense of school mathematics. Whenever the appropriate materials were available, Mitchell demonstrated interest and understanding.

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<sup>3</sup> 'Developmental time' is an unstructured, free-choice session designed to develop children's skills and understanding through a process of exploration and discovery using materials such as water, modeling equipment, musical instruments and toys.

Unfortunately for Mitchell, the use of whole class chanting on the mat, BSM activities such as the one observed, and a growing emphasis on methods of written recording, were not the best ways for him to learn. During Year 3, rather than adapting her teaching approaches to cater for Mitchell, Mrs Craig continued to attempt to 'socialise' him into the listening and recording culture of her classroom mathematics programme.

When his family moved house, Mitchell was sent to another school for Years 4 and 5. For those two years, he remained with the rest of his class at mathematics time. His Year 4 and 5 teachers described Mitchell:

Miss Palliser: His capabilities I found very poor. You know, just with basic addition and subtraction, he could do them, but he was particularly slow at doing them and would take a long time to do just one. I also found his method of counting very interesting. He was using his fingers. When he counts he puts his finger to his mouth. That is what he was doing at the start but he's actually started to use counters now so he's kind of moving away from his mouth onto objects...I had him in a group of three to give him some extra help with the magic squares and two of them were saying the answers out loud and Mitchell said, 'Good, I can copy them.' So, you know, he's got this idea of just being able to copy. 'Someone will do the work and I'll just write down the answer.' ...I think he's an OK learner, but he does struggle with some of the basic skills. (Early Year 4)

Miss Palliser: Mitchell gets quite excited by numbers now. He needs instructions repeated a couple of times, and he needs lots of time on one idea. He is now not the most needy in the class and he is expected to work more independently... (Mid Year 4)

Miss Palliser: He has really settled down but still has his loopy times...He is definitely able. There was a barrier there but it's disappearing. (Late Year 4)

Ms Roche: He'll never be a scholar...underachieves in everything... has an attitude problem...uses avoidance strategies ...learned to be helpless...jeopardising others' work...pretends he can't do things this year... If I blackmail him, 'If that's not done you won't be going swimming!' he gets it done... I admit I snap at him but he has to learn... He's getting better. He gets the back of his book ruled up and the date – that's great progress for a kid like him. (Early Year 5)

Ms Roche: He's doing really well, but he forgets things – his maths, routines... By the end of last term he could rename and carry, but then forgot. It comes to the point where I don't care, just do the mechanics. A student teacher used action learning strategies with him, especially in maths, with some success. He doesn't hide under the desk any

more – he just resorts to those behaviours when he's threatened... I've modified Mitchell's programme entirely. But he's really wanting to learn. He is initiating contact with me for feedback, so there are signs that he does want to learn. He's becoming more independent. (Mid Year 5)

Ms Roche: He's got really independent, he's willing to take risks and he's willing to ask for help. He doesn't do any of that time wasting that he used to do. (Late Year 5)

For the whole of Year 5, Ms Roche had separated Mitchell from the rest of the class by placing his desk next to her worktable. All the other children were seated in desk groups. As the following observation shows, much of what Mitchell did at mathematics time in Ms Roche's class was keeping him occupied rather than teaching him mathematical ideas. For Mitchell there was little meaning in these tasks and few of the specific learning experiences he really needed.

*The class is using the National Curriculum Mathematics textbook Level 2, Book 1, (Tipler & Catley, 2001, p. 203).*

Ms Roche: Right Mitchell. *(Mitchell is sitting at his desk sucking his thumb)* Have you headed up your book? Sweetie, it's page two hundred and three. *(To the rest of the class, clapping to gain their attention)* We're going to mark page two hundred and three together at ten to ten, so you've got about five minutes. *(To Mitchell)* Write the numbers down here like this. *(Demonstrates, the first few and Mitchell finishes)* How many days in a year, Mitchell?

Mitchell: Seven?

Ms Roche: OK. Write seven down there *(points to the space beside number four)*. Right now you have to go and ask someone how many days in a year.

Mitchell: *(Mitchell takes his book over to a desk group. To a boy)* How do you do this? *(pointing to question number four)*.

Boy: Three hundred and fifty-six. *(Mitchell goes back to his desk)*

Ms Roche: What did he say?

Mitchell: Three hundred and fifty.

Ms Roche: *(Writing the answer for him)* Right, now go and ask someone else how many months in a year.

Mitchell: *(When he returns)* Twelve. *(Heads off to ask someone else how many hours there are in a day then returns)* Twenty four.

Ms Roche: *(Writing his answer for him)* Right, minutes in an hour?

Mitchell: *(On returning from asking another child)* Sixteen.

Ms Roche: No, sixty. *(Mitchell writes this)* How many days in a leap year?

Mitchell: *(Returning from asking another boy)* Three hundred and sixty six. I can't do that one.

Ms Roche: Three, six, six.

The activity revealed Mitchell's need to develop two and three digit numeral recognition, to distinguish between the 'teen' and 'ty' numbers, and to learn more about units of time with the support of real life experiences, stories, calendars, clocks and watches. He was working on the same task as the others and although he was now able to achieve a degree of behavioural conformity, very limited mathematical learning was taking place. The gap between Mitchell and the others was widening.

At times, Mitchell's 'difference' appeared to attract negative attention from other class members, as the following observation showed:

*The teacher asks a question about combinations of notes and coins to make \$10.20. Children are selected to write their ideas for answers on the board then the rest of the children vote for the answer with which they most agree. Mitchell, who has his hand up, is chosen to provide an answer. His answer (10c, 10c, 20c) indicates that either he is not yet sure of money combinations or he did not understand the question. When the time comes to consider Mitchell's answer for voting, no children vote for it and some children snigger. One boy sitting close to Mitchell says in a sneering voice, 'Mitchell never does his work.' (Mid Year 4)*

Through everyday classroom interactions such as the one above, Mitchell was beginning to develop a mathematical identity:

Researcher: Are there some people in the class who are better at maths than you do you think?

Mitchell: Yep.

Researcher: How do you know?

Mitchell: Because they're doing it right and I got some of them wrong.

Researcher: How do you know you've got them wrong?

Mitchell: There's 'exes' [Xs] by them. (Late Year 4)

Researcher: Are there any things that you don't really like?

Mitchell: Maths.

Researcher: Why is that Mitchell?

Mitchell: I'm supposed to do my times tables and I don't know them.

Researcher: How does that make you feel?

Mitchell: Sad. (Early Year 5)

Researcher: Do you get different work from the other kids?

Mitchell: Yep.

Researcher: Why do you get different work do you think?

Mitchell:           Because I'm not very good at it. (Late Year 5)

Persistence on the part of his Year 4 and 5 teachers resulted in Mitchell's becoming more involved in mathematical tasks alongside the rest of the class. By the middle of Year 5, Mitchell had been encouraged to develop less disruptive coping strategies, including using concrete materials when needed. The formalised textbook work of the Year 5 class was inappropriate for Mitchell, serving to manage rather than teach him.

Mitchell's case illustrates the problems that arise in schooling systems where all children are expected to be at a certain stage at a certain age (McDonald, 1993) and to learn mathematics in the same ways. Teachers' options are limited when confronted with children who do not readily fit into the existing structures and practices of the classroom. They are torn between two choices: the perceived social benefits of inclusion with the peer cohort, and the perceived cognitive benefits of individual remediation or keeping the child back.

Mitchell was observed in a number of situations to make sense of mathematical tasks in 'different' yet insightful ways including devising for himself an effective finger-based system for subtraction across decades that enabled him to perform the mechanics of finding the answers to simple two-digit subtraction 'sums', without relational understanding of the task (Skemp, 1978). Mitchell's skills in other areas such as his ability to memorise the spelling of quite difficult words, his love of dancing, his enjoyment of construction and working with shapes, and in Years 4 and 5, his accomplished drawing of Pokemon and other television characters, indicated a strong sense of patterning and spatial awareness. Significantly, using the hundreds board in Miss Palliser's class had helped him develop an understanding of the patterns found when counting from one to one hundred. It would seem that rather than learning mathematics through the predominant pedagogical practices of whole class chanting, individual writing and basic facts speed activities, the use of appropriate materials, supplemented with discussion from a range of supportive peers and adults, may have been of enormous benefit to Mitchell. Heavy reliance on ready-made activities and texts were also a problem, since they presumed a specialised linguistic competence and shared life experiences.

During the three years of observation, Mitchell's difficulties were never satisfactorily 'diagnosed'. The separation of his parents, which, his teachers noted had had a marked effect on his behaviour at school, problems with his hearing as a younger child, and strained socioeconomic circumstances, must all be considered as contributing factors. Mitchell's mother was aware of his learning difficulties. By comparing her son with a niece who was a year younger than Mitchell, she had concluded that he was about a year behind children of his own age. His troubles with school learning clearly caused her considerable anxiety.

Mitchell provides for us a powerful example of a child for whom the social and cognitive demands of the primary school mathematics classroom made little sense. In turn, the other members of the classroom community, including his teachers, had trouble making sense of him. The inability of others to recognise, understand, appreciate and accommodate his ways of perceiving, led to his being marginalised by the group, and created in Mitchell a profound sense of failure.

Equity for all students, including those who, like Mitchell, experience difficulties in learning mathematics, has become a strong component of recent curriculum statements on mathematics. *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) says:

'It is a principle of the *New Zealand Curriculum Framework* that all students should be enabled to achieve personal standards of excellence and that all students have a right to achieve to the maximum of their potential. It is axiomatic in this curriculum statement that mathematics is for all students, regardless of ability, background, gender or ethnicity.' (p. 12)

*The National Numeracy Strategy* (Department for Education and Employment, 1999) dedicates a section of its introduction to catering for pupils with special educational needs which notes:

All teachers will have in their class some children whose progress warrants special consideration. Their difficulties may have physical, sensory, behavioural, emotional or neurological causes...but as a general guide, you should aim to include all these pupils fully in your daily mathematics lesson'... (p.23).

The National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000) states:

Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students. ...The vision of equity in mathematics education challenges a pervasive societal belief in North America that only some students are capable of learning mathematics. ... Low expectations are especially problematic because students who live in poverty, students who are not native speakers of English, students with disabilities, females and non-white students have traditionally been far more likely than their counterparts in other demographic groups to be victims of low expectations. Expectations must be raised – mathematics can and must be learned by *all* students.' (pp. 12- 13)

If equity is to be achieved for children like Mitchell, mere inclusion in the daily programme alongside the other children is insufficient. Schooling systems that base teaching and learning on levels and stages tend to compare and rank children's achievement as assessed through a narrow range of methods. They use deficit models of learning to explain children's 'underachievement' and emphasise what children *can't* do. With this view of learning, children like Mitchell are seen as 'behind the eight ball'. Cross and Hynes (1994) suggest that instead, teachers use assessment methods that enable children to show what they *can* do, and *how* they do it. Goldman and Hasselbring (1997) propose an alternative teaching model for learners of mathematics who are identified as 'disabled', in which important skill learning is embedded in meaningful contexts based on authentic tasks. Such an approach, they suggest, helps *all* children to make sense of mathematical activity in personally meaningful and fulfilling ways. Seen in this way, it is not Mitchell who has failed to learn mathematics, rather it is schooling practices of mathematics that have failed to recognise and embrace Mitchell's personal mathematizing.

### **Jessica: 'a middle kid'**

Jessica began to experience ability-grouping in Year 3, when Mr Loch, who usually grouped by year level, but additionally by the results of pre-testing:

Mr Loch: I do a little bit of readiness beforehand usually just a simple sort of test which I give to them so I can group the kids, you know, I try to have two or three groups operating at a time and she, for some reason, fractions was something she obviously felt



confident in and she ended up being on the top group, whereas the rest of the time she has always been in that second group. (Late Year 3)

When explaining her self-rating on the scale that was used for the children to indicate how good they thought they were at mathematics (Appendix 6), Jessica provided some interesting insights into the ways in which she made sense of the classroom practices of grouping and ranking in mathematics. In Year 3, this had already begun to contribute to her developing mathematical identity.

Researcher: So why do you think you're here (*pointing to 4 on the self-rating scale for mathematical ability*) and not here (*pointing to 8, 9 and 10*) ?

Jessica: 'Cause I'm kind of in between I think.

Researcher: OK. How do you know who's good at maths in this class and who isn't so good?

Jessica: 'Cause sometimes Mr Loch, he gets us to show our work to the class. So we know. (Early Year 3)

Researcher: Where do you think you'd fit on there? (*on the self-rating scale*)

Jessica: Number one.

Researcher: Why do you think that?

Jessica: Well, because I'm not really in his band, [development band group] so I don't know any ...(*pauses*)

Researcher: How do you know you're not very good at it?

Jessica: Well sometimes I get everything wrong and stuff like that.

(Later)

Researcher: Are you in a maths group?

Jessica: Yes. Group 1.

Researcher: Is that the best group?

Jessica: Group 2's the best group.

Researcher: How many groups are there?

Jessica: Two.

Researcher: How do you get into Group 2?

Jessica: Well, you kind of like get everything right.

(Later)

Jessica: I told them [her parents] I'm hopeless at it [maths]. (Late Year 3)

By the end of Year 3, she rated herself at 1. At the beginning of Year 4 Jessica was transferred to another school. She found herself placed in the middle group of the class. She reported feeling 'hopeless' at mathematics.

- Researcher: Do you know some people who are better than that? (*Jessica nods. She has just rated herself 3 on the self-rating scale for mathematical ability*) How do you know they're better?
- Jessica: 'Cause, although they're a lot smaller [younger] than me, they're a lot better at maths. They're smarter than me.
- Researcher: How do you know they're better?
- Jessica: 'Cause they've always got their brains on. When the teacher tells us to get something out, they're the first to get them out.
- (Later)
- Researcher: So you're in Group 2? How do you think she [teacher] came to put you in Group 2?
- Jessica: She just puts you in a level. It think it's 'cause we had these mathematics tests and that's how she found out.
- Researcher: OK. Is Group 2 the top group, the bottom group or the middle group?
- Jessica: The middle group, and there's an A and a B because the group's so big.
- Researcher: Right. Are you in the A or the B?
- Jessica: The A.
- Researcher: So which is the top group then out of 1, 2 and 3?
- Jessica: Group 3.
- Researcher: Does Group 3 get harder work?
- Jessica: Sometimes Group 3s and Group 1s get different sheets from us. (Early Year 4)

In Year 5, all the children in the syndicate were cross-grouped<sup>4</sup> by ability for each mathematics topic throughout the year. At the beginning of each mathematics session, Jessica would take her book and pencil to another classroom for mathematics.

- Researcher: What makes a person good at maths do you think?
- Jessica: If you practise quite a lot. And the little girl Marnie with the curly hair and glasses, she's really good at maths, she got a hundred out of a hundred this time [basic facts speed test] 'cause she's really smart, but I don't think it's, like she practises or anything, well she probably does, but I don't think it's really that reason, I think it's because she was just born like that. And some people are born differently than others. (Early Year 5)
- Researcher: What makes you think you're a 5? [on the self-rating scale for mathematical ability]
- Jessica: 'Cause I don't think I'm perfect or anything like that, but I don't think I'm bad either. So I'm kind of about there.

<sup>4</sup> 'Cross-grouping' is the term used in New Zealand primary schools for grouping by ability within a syndicate of several classrooms. It is the equivalent of the terms 'setting', 'tracking' or 'banding' used in other countries.

- Researcher: Would there be some people who would be 1s and 2s?
- Jessica: Hm.
- Researcher: So how can you tell that?
- Jessica: 'Cause when we do this numeracy skills mastery programme, some people are doing a different sheet, because they're not as up to the others.
- (Later)
- Researcher: How do you think they've grouped you?
- Jessica: Because we do tests and the different marks – they just put you into groups.
- Researcher: So you were in Ms Washbourne's last time weren't you?
- Jessica: Right, yeah, I've been in Miss Moana's and Ms Washbourne's.
- Researcher: So it changes each time you do a test?
- Jessica: Not every time but just sometimes:
- Researcher: The ones that went to Ms Mere's class, were they the ones that did best or OK or worst at the test?
- Jessica: The worst.
- Researcher: Did they say that?
- Jessica: No, but ...
- Researcher: How did you figure that out?
- Jessica: Well, because of all the other groups. My class teacher, she's got the highest group...and I don't know about Miss Moana and Ms Washbourne, but...
- Researcher: You know that Ms Mere's got the worst group?
- Jessica: Hm.
- Researcher: How did it make you feel when you were in Ms Mere's group?
- Jessica: I felt pretty annoyed with myself, how I did, but when I got in there I thought it wasn't so bad, it's just work to me, no big deal, and we're just a little bit slower than everyone else. (Mid Year 5)
- Jessica: At the moment I'm not really happy with myself. I was in the top group [second to top, according to the teachers] for this topic, I think it's called geometry, with all the shapes and everything and like, you know, those equilateral kind of things...I was with Ms Mere, I'm pretty sure Ms Maine's the lowest, Ms Mere's the highest and there's Mrs Washbourne and Miss Moana, and I've no idea which is the highest of those.
- Researcher: So you're not feeling happy with yourself?
- Jessica: No, because I went down one or two groups.
- Researcher: How come you went down? What made that happen?
- Jessica: Well, you know how I said you have to be good at something to enjoy it, well I was really enjoying it in Ms Mere's group because I was good at that topic [geometry] But now ...for measurement, I'm not that good at it.

Researcher: So where would you put yourself on the scale now, then? [Self-rating scale for enjoyment of maths].

Jessica: Probably between four and five.

Researcher: How did you feel when you were in Ms Mere's doing geometry?

Jessica: Well I didn't feel that great about it because, well most of the other people had been in Ms Mere's, well there's a girl who has been in Ms Mere's and no other group ever. Well Miss Maine used to be the highest and they swapped, and there's this boy he's been in the highest ever since we started rotating for maths. And there's one other girl from my class and we think she's also been in the highest.

(Later)

Researcher: Why do you think you're here (*between 6 and 7 on the self-rating scale for mathematical ability*) and other people might be here? (*pointing to 8,9,10*)

Jessica: They just happen to be better than me at maths. I might happen to be better at, I don't know ... it's a gift thing. (Late Year 5)

Jessica's self-efficacy rating seemed to correspond partly to her achievement in tests basic facts tests, and partly to the changes in her group placement - 0 when placed in a 'lower' group, 5 or 6 when in a 'higher' group. Many children, including Jessica, reported receiving little feedback from teachers about their achievement in mathematics. When asked where the teacher would place them on the scale, most of the children indicated the same place they had positioned themselves, which suggested that their self-rating was closely linked to how they imagined the teacher might perceive their competence. Dominic was a notable exception.

### The teachers' views of Jessica

The teachers were asked about Jessica's progress in mathematics:

Mr Loch: I would place her, she's round about a middle kid. Before, [the fractions pre-test] I would have said she was probably a bit down on the average child in this class. (Late Year 3)

Ms Seager: She's at the top end of average (*Looking at the P.A.T.<sup>5</sup> mathematics results*) You see I've got 12 children operating at that level in my class. It's a sort of bell-shaped curve really isn't it? (Early Year 4)

<sup>5</sup> P.A.T.s are Progress and Achievement Tests, standardised national multiple-choice tests for the core subjects, and administered by many schools in the 6<sup>th</sup> week of the school year to determine individual and cohort percentile rankings by age and by year group. Mathematics P.A.T.s begin at Year 4.

Ms Seager: She's doing really well apart from the basic facts tests. She's still using fingers, so they're not automatic yet. She's on the upper end of average I would say (Late Year 4)

Researcher: What group is Jessica in your grouping system?

Ms Washbourne: The third group. I don't think she's as secure as the other children that are in here. You know, the top group has fifteen students, the bottom has fifteen, then Ms Moana and I have got around twenty two, so I think she wasn't as secure, I think she may have been better one group down (second to bottom group).

Researcher: So you've got four groups, and she's in a middle one?

Ms Washbourne: Yes, she's beyond the lowest group but she's not up with the top group.

Researcher: So you'd see her as...?"

Ms Washbourne: `Well, 'average' is a nasty word. She's basically working at, comfortably at the beginning of Level 3 objectives. That would be my estimate of her. I don't know whether her number pre-test showed that, but that's where I see her.

Researcher: And that's where you would expect to see her at this age?

Ms Washbourne: Well, for a Year 5 that's absolutely fine. There are a lot of children who are working in the bottom group and that's not a nice word either, but the children who need extra assistance often are working on Level 2. And even the next group up is consolidating Level 2 and then moving up. Whereas my group's pretty much solidly working in 3 without any extension into 4. (Early Year 5)

Ms Mere: My group is the 'challenged' you might say (*laughs*). Jessica is pretty good but there are a few from that class who were down low to begin with. (*Jessica has only recently been placed in this group*) I don't think maths would be her first love... She's a bit slapdash, untidy, but it might be her general way. (Mid Year 5)

Ms Washbourne: She was in the second group for geometry, now she's in the third group – for measurement. From the pre-test, yes, it goes according to numbers [results of the test]. (Late Year 5)

Jessica's mother reported on the schools' grouping system and her interpretation of it.

Jessica's mother: Jessica was confused about the maths grouping. They worked out the highest group from the one that this girl Gemma was in...Changing the groups has made a big difference to Jessica. It's school policy, you know, working on the children's strengths...But I think Jessica might be more of a left-brainer. (Later Year 5)

Jessica's rating of herself in terms of mathematical ability was almost identical to the ways the teachers viewed her. This may indicate that it was largely through her experiences of mathematics assessment and grouping at school, that Jessica derived her sense of a mathematical self. Her mathematical identity was reinforced by interactions with peers and the views of her mother. By the end of Year 5, Jessica seemed to have accepted that she did not have the gift for mathematics and had linked levels of enjoyment to her variable success. In their review of literature on ability grouping, Sukhnandan and Lee (1998) found that research indicated that pupils' self-esteem, school involvement, and friendship patterns were affected by ability grouping. It can be seen from Jessica's comments that she had experienced at least some of these effects.

Jessica: I always say you have to be good at something to enjoy it.

Researcher: When you get to High School do you think you're going to enjoy maths?

Jessica: Probably not. (Late Year 5)

### **Rochelle: A 'Super Smarty Pants'**

During every interview, Rochelle consistently reported that she enjoyed mathematics and found it fun. She was seen as a conforming child in all subject areas at school, her teachers commenting on the way she got on with her work. Her determined effort to learn the basic facts, her pleasure in knowing them, and being able to get most of her maths work right, provided Rochelle with a sense of satisfaction and accomplishment. Rochelle was placed in the middle or top groups for mathematics for Years 3, 4, and 5, based, it appears, on her compliance, her diligence, and her accuracy in basic facts recall and computational tasks. Teachers, her mother and Rochelle herself, tell the story of her grouping history during these three years at school.

Researcher: Were you satisfied with Rochelle's level of prior knowledge on arrival in your class?

Mrs Joiner: I had her last year, so...basic facts good. She needs to consolidate tens and ones. She doesn't get it instantly, always. She could go up to the top group but the children in that group get it quickly and they are a big group already. (Early Year 3)

Rochelle's mother: Well, Rochelle will come home and tell me what happened in the day, and they've just had a test and like yesterday she brought home a certificate to

say she'd done excellent with her basic facts, so I mean that was really, really good. (Early Year 3)

Rochelle's mother: I was speaking to the teacher the other day, and she tells me Rochelle's gone up a maths group. She's very accurate in Level 1. Every time the home book comes, she gets more and more of her basic facts right, yeah, she's bettering herself each time, but she doesn't do the whole hundred yet. But the teacher said she deserves to go up because she's working really hard. (Late Year 3)

Mrs Joiner: I've put Rochelle into the Squares group now, on her last assessment, she got 19 and a half out of thirty. (*Looking at the class results*) She's a way behind the top five – look, they're on twenty-nine, twenty-eight, but ahead of these others, so I thought she should go up. (Late Year 3)

Researcher: Why do you think you're pretty good at maths, Rochelle?

Rochelle: 'Cause in the *Daily Twenty* I can get nearly all the questions right.

(Later)

Researcher: What maths group are you in, Rochelle?

Rochelle: I'm in the middle group.

Researcher: How come you're in the middle group do you think? (*No reply*) Who put you in the middle group?

Rochelle: Mrs Ponting.

Researcher: Is that where you would put yourself?

Rochelle: Yes. (Mid Year 4)

Rochelle: I'm in the middle group – that's Circles.

Researcher: Yes? How do you think Mrs Ponting put you in Circles?

Rochelle: Um, 'cause we done a test and she just saw where we were at. (Late Year 4)

Mrs Ponting: Rochelle appears to enjoy maths, she's coming out of herself but she's a quiet little mouse... She works better sitting away from her little friends... Rochelle is in the middle of my three groups. She's got it in her but lets others do the thinking for her... She's very conscientious at schoolwork. (Mid Year 4)

Researcher: What tells you that you've got better?

Rochelle: I've got more ticks in my maths book than last year.

(Later)

Researcher: (*After Rochelle places herself at 8 on the self-rating scale for mathematics ability*)  
What makes you think you're an 8, Rochelle?



- Rochelle: Because I have ticks and crosses.
- Researcher: Are there some people who would be 9 or 10?
- Rochelle: Some people would be 9s.
- Researcher: How can you tell?
- Rochelle: There's a Standard 2 that's a 9. Because when we done this sheet, he got a hundred out of a hundred.

(Later)

- Rochelle: Ms L. has a working group, and that's the people who aren't so good at maths, but I'm not in that, and they work with blocks and all that. (Early Year 5)

- Ms Linkwater: Her recall is really good [basic facts]...And some like Rochelle's group, and I've got a couple of Year 4s in that because they're really, really good at it, I call them the Super Smarty Pants now and again...SSPs I put on the board and they could work it out (*laughs*). But it's a really nice way of saying those kids who really work hard with a good attitude, good setting out, yeah it's just that positive attitude and cope with the work. The reason I call them that is they have got the concepts, they work independently, and to work independently you have to be Super Smarties. (Early Year 5)

At the end of the first term of Year 5, the syndicate was reorganised into a cross-grouping system for mathematics. Rochelle was put into the group that went to Mrs Ponting (Rochelle's teacher in Year 4) for mathematics.

- Researcher: How come you got to go to Mrs Ponting do you think?
- Rochelle: People who done really well in the test got to go with her.
- Researcher: Are you in a maths group?
- Rochelle: Hm.
- Researcher: What's your group called?
- Rochelle: The Pentagons, which is the highest group.
- Researcher: Why are you in the Pentagons group?
- Rochelle: Don't know.
- Researcher: You're not sure how she put you into groups?
- Rochelle: I don't know about the Triangles and Circles, but Rectangles, they're in Room 3, and Pentagons, they're much better than Room 3.
- Researcher: Are you the only one from Room 3 in the Pentagons group?
- Rochelle: I think there's two others.
- (Later)
- Rochelle: (*After placing herself at 6 on the self-rating scale for mathematics ability and being asked why*) You take your work to Mrs Ponting.
- Researcher: Does she say anything?

- Rochelle: No. She marks your work.
- Researcher: So you've decided you're a six. What things do you think you could do better?
- Rochelle: Learn to do the three steps [multiplication algorithm] (Mid Year 5)
- Mrs Ponting: Rochelle is great at basic facts and computation. She gets them all right.
- Researcher: Where would she sit compared with the other children?
- Mrs Ponting: I'd put her right up there. She's a very hard worker, never talks or complains, and just gets on and does it. (Late Year 5)

During a number of classroom observations, it was seen that while Rochelle was able to follow rules and memorise facts in mathematics, she ran into difficulties on the rare occasions when tasks were open-ended or called for problem solving and communication skills such as lateral thinking, logic, spotting patterns or explaining and justifying methods. Since little of mathematics time appeared to be spent on these kinds of learning experiences during Years 3, 4 and 5, Rochelle did not appear to be daunted by this. Her teacher's opinion of her mathematical ability, and therefore her group placement in the middle or top group, continued to be based almost exclusively on her performance in routine basic facts and computation tasks and her compliant hard work. There were indications that Rochelle was aware of maths becoming harder and was expressing apprehensions about her future achievement in mathematics. Peers might also have contributed to Rochelle's feelings about mathematics.

- Researcher: How do the other kids in the class feel about maths?
- Rochelle: They don't like it.
- Researcher: Would that be most of the others?
- Rochelle: Yes.
- Researcher: Why is that do you think?
- Rochelle: I don't know. They say, 'Can we please not have maths today?'
- Researcher: Can you see yourself enjoying maths when you get to high school?
- Rochelle: No.
- Researcher: Why is that?
- Rochelle: It'll get harder.
- Researcher: OK, what about intermediate?
- Rochelle: No. My sister has really hard work.
- Researcher: Do you think you're going to be good at maths at high school?
- Rochelle: Um ...I don't know.

Boaler (1997b, 1997c) has noted the pressure and discomfort that can be experienced by children such as Rochelle, who are placed in higher groups.

### Children's experiences of being sorted

Mitchell, Jessica and Rochelle provide us with examples of how the children were identified, labelled, separated and treated differently according to their teachers' perceptions of their mathematical 'ability'. The grouping procedures they experienced were by no means unusual, as Table 3 shows. Arrows indicate promotion or demotion during the year.

**Table 3: Groupings experienced by the children during Years 3, 4 and 5**

Grouping system	Year 3	Year 4	Year 5
<b>Whole class teaching - occasional groups</b>	Fleur	Rochelle Jared Peter	Fleur Toby
<b>Within-class ability groups:</b>			
<b>Top</b>	Rochelle (later)	Liam ↓↑	Liam ↓
<b>Middle</b>	Liam, Rochelle ↑	Toby, Jessica	Liam (later)
<b>Bottom</b>	Georgina, Jessica	Georgina,	Georgina
<b>Across-class ability groups:</b>			
<b>Top</b>			Rochelle
<b>Middle</b>		Dominic	Dominic, Jessica ↑↓
<b>Bottom</b>	Dominic	Fleur for one unit	Jared
<b>Within-class mixed ability groups</b>	Toby		
<b>'Extension'</b>	Peter		Peter (later)
<b>'Special needs'</b>	Jared Term 4 placed in special class Mitchell to younger class	Mitchell with class , some individual work	Mitchell with class, individual programme

Classroom environments often reinforced mathematics ability grouping in visible ways. These sometimes took the form of a chart or list on the wall with names of the children who belonged in each group, such as the one in Georgina's Year 3 room. Basic facts achievement charts (Fleur's Year 4 room) or graphs (Jessica and Rochelle's Year 4 rooms) were another means by which children compared achievement. On the board, or a chart, teachers frequently noted the day's timetable including activities for each of the ability groups. (Dominic's Year 5 Room, Rochelle's Year 4 and 5 Rooms, Georgina's Year 5 Room). In Year 5, Jessica's

syndicate teachers typed a list of ability groups for each topic. In Mid Year 5, the children were observed crowding around the new list which the teacher had just posted on the classroom wall, to see where they had been placed. Stickers, stamps and ticks were other signs used by teachers to reinforce achievement in mathematics. The children used these signs to assist them in a process of sociomathematical 'ability mapping' – identifying who was better/worse/ the same as themselves at mathematics, and where they fitted in relation to the others.

### Teachers' explanations of grouping systems

As Table 3 (p. 164) shows, in most classrooms, the children were grouped by ability in some form for mathematics. In almost every case, the grouping decisions were based on the results of time-constrained written tests.

Syndicate cross-grouping for mathematics was an everyday occurrence in four of the ten schools. Teachers explained how and why the children were grouped in this way.

Researcher: So in this school you change around for maths. How was that decision made?

Ms Torrance: Well actually I instigated it in this subject and the thing is that we were already doing that [in the senior part of the school]. I just thought it was so wonderful not to have a whole thread, [a variety of different groups to teach] and I actually think the standard is quite high at this school, so I thought it worked really well, and it certainly works well from a teaching point of view...I've just been to a course on educating the gifted and talented and we were having this big debate about whether you should enrich them or withdraw them or just...they [course tutors] don't think that streaming is a great plan. (Late year 3)

Miss Awatere: (*Explaining syndicate cross-grouping and why Dominic was put in the middle group*) I think it's just the logistics of accommodating children in a comfort zone to maintain their confidence in a school environment. (Late Year 3)

Mrs Ponting: We changed to the cross-grouping – it was on staff recommendations. We wanted to change groups because that way they are a more manageable size, and, you know, it's less workload for us. (Mid Year 5)

Researcher: Do you have a group system?

Mrs Wai: I don't usually have groups, no. Because my special needs child goes out of the classroom and if I find I have anyone to cope with, we have within the school, we

have facilities for them to go to. So if I had, say, three children who were away ...not mixing in with the others, I would send them to another teacher and they would be able to cope with them. (Early Year 4)

Researcher: You've told me how you regroup for maths [across the syndicate]. Do you do that for any other subjects?

Mr Waters: No, that's the only one we do, but we'd have the arts, they get into syndicate groups, and for syndicate sport but that's the only [other] time we do it, it's got nothing to do with ability. Math's the only one.

Researcher: So how many classes are involved with the swap?

Mr Waters: Three. Next door is Alan, he's got [Years] 5 and 6, and Sue's 7 and 8, so I've got people going [from his Year 5 and 6 class] into Alan's and my Year 6, my more capable ones going right up to Sue's. Oh, in actual fact there's four, 'cause Mark takes the extension classes.

Researcher: Right, that's interesting, so did you use the P.A.T.s to decide who went where, or just looking at them [the children]?

Mr Waters: Basically looking at ability, the P.A.T.s were done afterwards...it is just a consolidation basically, to keep them with their peers, basically, and then get them going, just to consolidate, understand where they're going, so they won't be completely lost. (Early Year 5)

Researcher: So how did you decide who went where?

Mr Ford: Just on their last year's achievement, across the board, in testing, in observational stuff, in their book work, P.A.T.s, you know, just looking at the whole thing and saying really where they fall in terms of where they're at in maths. And so we can give them the idea that we can give them the kind of support as a whole group, that they're all at. (Early Year 5)

Teachers would group children within their classrooms, most often into three ability groups. They explained why and how the grouping was done.

Ms Torrance: I've got them in three groups and I'll just teach one group at a time, but I think particularly with number they need more intensive one-on-one thing.

Researcher: Are they ability groups or random or ...?

Ms Torrance: They are ability groups, but I mean I don't regroup them for every strand so it's pretty stupid really. They are probably number groups, ability and number groups. (Early Year 3)

Researcher: How did you form the groups?

- Mr Solomon: That's a pre-test for each unit.
- Researcher: So the groups would vary?
- Mr Solomon: Yes, they do.
- Researcher: Much?
- Mr Solomon: Mostly the same. Almost the same all the time in each group but yeah, they do vary slightly...That top group are mostly year 4s...and there are some of the brighter Year 3s that are working with them there, then there's a middle group which are closer to that lower group, so there's a huge gap between that top group and the middle group that I've got.
- Researcher: And the middle group would be a mixture of Year 3s and 4s?
- Mr Solomon: Yes, and then there's that smaller group [Georgina's] which have never really seen the sort of strands before, or don't relate to them in terms of the more structured maths that we're doing, so that's the group that I'm using lots of resources, you know, hands on resources with. (Early Year 3)
- Mrs Cayo: I would quickly go over one objective and do it together so they understand, then send that group, those who understand properly, set them work while I work with the children who are not ...who didn't understand well, who are below average and then make them understand and then send the second lot away. I've got my maths groups but I haven't used them as much. Eventually they will blend into their ability groups and work at their own level. (Early Year 4)

While ability grouping *across* classes was rarely found in any other subject area, it appeared to be a relatively new innovation for mathematics teaching. Four of the fourteen schools were using this system consistently and had recently converted to this form of organisation. One more was using it occasionally. A trend towards increased ability grouping across classes in the primary school has also been noted by Boaler et al (2000) in the UK and by Harlen and Malcolm (1997). Boaler et al state that 'concerns with educational equity have been eclipsed by discourses of 'academic success', particularly for the most able, which has meant that large numbers of schools have returned to the practices of ability-grouping' (p. 631).

Ability grouping *within* classes was common practice for mathematics teaching and also appeared to be widespread in English instruction, particularly for reading. For other subjects, however, ability grouping was far less common. This might have indicated that teachers viewed instruction in mathematics and English as more important than in other subjects and that they believed children's learning would be

enhanced by grouping according to perceived 'need'. Another possible explanation is that teachers' felt more comfortable working with children in groups that were perceived to be homogeneous. Mrs Ponting points out the reduced workload and greater manageability of the cross-grouping system. The few teachers who, like Mrs Cayo, said that they taught the whole class together, had built in flexible systems for catering for diverse and changing learning needs.

Extension groups for mathematics were found in several of the schools. While teachers may have been hesitant to use 'bottom group' to name the children who were deemed to be at a less-developed stage of learning compared with the others, they used the terms 'top' or 'extension' group without hesitation. Peter's experience of the extension group was viewed positively by his teacher, but not so positively by Peter himself.

Researcher: How do you think Peter feels about being in the extension group?

Ms Summers: I think he's really enjoyed it. He hasn't communicated to anyone – just picking up. His smile. But it's amazing that you put a kid in that situation who perhaps wouldn't feel very confident about maths and suddenly that pupil rises. (Early Year 3)

Researcher: Do you like the extension group better [than maths with own class] or not so much?

Peter: Not so much.

Researcher: Why don't you like it so much?

Peter: Don't know. (Late Year 3)

An observation of the extension group confirmed Peter's discomfort. He did not interact with the others, he laboured over tasks that the others seemed to complete with ease, and he did not appear to be enjoying himself.

In spite of a growing body of evidence suggesting that ability grouping in mathematics has a negligible positive effect on students in the higher ability groups, and appears to inhibit the learning of students in the lower groups, (Boaler, 2000; Hallam & Toutounji, 1996; Hoffer, 1992; Ireson & Hallam, 1999; Linchevski & Kutscher, 1998; Slavin, 1990; Sukhnandan & Lee) most of the teachers in this study appeared to take it for granted that ability grouping was not only beneficial, but that it was the *only* way that they could successfully cater for what they viewed as the



widely differing learning *needs* of the children. Zevenbergen (2002) suggests that it may be the dominant epistemological view of mathematics as a sequentially-arranged body of truths that lead to what she sees as a pervasive and entrenched belief that ability grouping in mathematics benefits learning. In explaining how this may be justified she postulates 'if it is believed there is a hierarchy in the complexity and demands of the discipline, then it would be logical that students be mapped against this hierarchy' (p. 514).

### **Teacher's views of the children**

Through their descriptions of the children's progress in mathematics, the teachers revealed much about the ways they viewed children's mathematical ability, their beliefs about the reasons for success or lack of it, and their labels for children of differing achievement.

#### *Fleur*

- Mrs Field: She's not up there, you know, way up the top. She's down a fair bit but she's quite enthusiastic about what she's doing. (Early Year 3)
- Ms Fell: She was in the lower group. (Mid Year 4)
- Mrs Meadows: She's middle of the range, you know. She'll never be top notch. I don't know what the parents' expectations are – is she expected to be higher at home, I wonder?

#### *Georgina*

- Mr Solomon: I would have expected her to know more. There were four or five of them that that I thought might have been of a higher level. Simple number problems she can do but again it's the very slow adding on the fingers and going right back from the start to add all the fingers we've held up... She actually enjoyed and did really well on geometry, shapes and things like that so maybe she's a, um, spatial type person. (Early year 3)
- Her negative attitude is not just to maths, it's right across the board. She's definitely capable but there's a blockage. (Late year 3)
- Mrs Cayo: Yeah, she's not too bad, I think she can fit into ...I could even make a Year 4 group and put her into the lower group. Some kids are very, very smart, got high ability and perhaps she can come to the next group. (Early Year 4)
- Mrs Isles: She's really not competent even with the 2 times table and by this stage she should be able to do the twos, fives, tens, so, no, um, and generally with maths tests and things, I mean we have pre-tests and mastery tests and there you can see she's struggling. (Early Year 5)

*Dominic*

Ms Torrance: He's in the middle to up the top of my class. He's coming out with very good assessments – he's got double ticks in just about everything. (Mid Year 3)

Miss Awatere: He's top of the pile in terms of ability, I mean not *the* top, but in the top, you know, twenty percent. He would be in the top eight children and he probably knows that because he knows the answers pretty much. (Late Year 3)

Mr Swift: (*Describing Dominic's group in the syndicate cross-grouping system*) Towards the bottom end. The two middle classes do a lot of similar stuff so we're towards the bottom end. He's a boy that's been exposed to level 2 so he can, for some of the subject areas, some of the strands, go up to level 3.

Mr Ford: He's in the middle bunch. He's average for his age.

Researcher: What was his P.A.T. score for maths?

Mr Ford: About 65% I think. (*A later check shows that Dominic scored in the 86<sup>th</sup> percentile for his age. He is one of the youngest children in his class*)

*Jared*

Ms Flower: I would say he would be a bit below average, but not...not being sort of fast anyway. (Mid Year 3)

Mrs Wai: I was quite surprised when his mother approached me at the [parent teacher evening] barbecue and she was worried about him academically. She said he was struggling. I see him sort of in the middle, within the normal band. (Early Year 4)

Mr Waters: He's right at the top end of the scale, basically, in this class ['lowest' of the syndicate cross-grouped mathematics classes]. (Early Year 5)

*Liam*

Miss Peake: He's in the Triangles group. Cycle 9 [the 'middle' of her three groups, based on the BSM system]. (Mid Year 3)

Ms Sierra: He's in the top maths group again – he went to the middle but now he's back. He's topped the *Quick Twenty* for 2 weeks now, so he's the class Maths Champ. (Late Year 4)

Mrs Matagi: I don't think he's, ah, brilliant at everything in the sense that he doesn't always get it first pop every time. I think he's quite comfortable on Level 3. (Mid year 5)

*Peter*

Ms Summers: (*Talking of Peter's work with the extension group*) He's quiet in the group, and perhaps a few steps behind the other children. I always think he needs to be buddied up with someone who's an energetic thinker. (Late Year 3)

Mrs Waverley: I've no idea of his mathematical ability. If I'd known you were coming I would have got everything out and had a good look. [She had forgotten about our appointment]...he would be one of those quite affable children whose hand would

never go up and would never volunteer and would sit very quietly, hopefully happily, and wait to escape. (Early Year 4)

Mrs Waverley: He's well behind other class members in basic facts – very variable results. (Mid year 4)

Miss Sanderson: He's got a good grasp of where he should be now. Making steady progress.

Researcher: Will he be in the extension group?

Miss Sanderson: Yes. Oh. (*Checking his P.A.T. results in which he scored in the 71<sup>st</sup> percentile*) The benchmark is actually 75%, so no. (Early Year 5)

### *Toby*

Ms Firth: I would say he's between middle and top, I would say between that sort of range. In general I've got quite a capable class when it comes to maths, so even with the majority of the Year 2 children, we have been working at early Level Two. Whereas I creamed some of those more capable Year 3 children and we've met them in the middle of Level 2 in the curriculum...yes, so I actually think Toby is quite capable because he's very quick to grasp concepts and that's because he has got such a good base knowledge. (Early Year 3)

Mrs Kyle: He's in the middle group, he's obviously not struggling. (Late Year 3)

Mr Cove: I just think he's an able student. (Mid Year 5)

### **Teacher's beliefs about ability**

As these examples show, the two most common metaphors used by the teachers in relation to ability and achievement in mathematics related to speed ('slower and faster') and altitude ('top, middle, and bottom groups', 'high and low' achievers). Given the emphasis on speed in classroom mathematics, this is perhaps not surprising. However, altitude metaphors for achievement were by far the most frequently used: 'high and low', 'top, middle and bottom', 'up and down', 'above or below' average. Again, this is not surprising, since scores and test results were also described as 'high' and 'low' by teachers and children.

Evidence for hierarchical views of mathematics learning is found in the diagrams used in government documents in which mathematical learning stages are arranged vertically. *Mathematics in the New Zealand Curriculum* (1992, p.17) shows levels of achievement rising in an ascending staircase. The *Numeracy Project* (2001) depicts its strategy stages in a mountain diagram (Te Maunga Tau – The Number Mountain),

with the highest stage at the narrow peak. The widespread assignation of altitude metaphors to mathematical grouping and to the children when describing their mathematical achievement would suggest that in the minds of the teachers, the stratification of the groups creates a ladder-like mathematical hierarchy in their classrooms.

Other common metaphors were used such as 'smarter' and 'brighter'. Equally labelling were their terms such as 'able', 'capable', 'competent', 'needy', and 'struggling'. A child can only be described as 'able', 'competent' 'needy' or 'struggling' within systems, structures and ways of thinking where children are routinely compared and ranked in relation to one another, where their 'capability' is measured in terms of their success at performing tasks in the ways that their teachers *expect* and *accept*, and where ability is regarded as an immutable part of the child's personal makeup. Labelling theory, developed by a number of sociologists in the 1950s and 60s (e.g. Lemert, 1951; Becker, 1963), has been used to explain the observed phenomenon 'that pupils tend to perform as well, or as badly, as their teachers expect. The teacher's prediction of a pupil's or group of pupils' behaviour is held to be communicated to them, frequently in unintended ways, thus influencing the actual behaviour that follows' (Meighan and Saraj-Blatchford, 1998, p.309). This phenomenon has been demonstrated in grouping for mathematical learning where students of a similar initial 'ability' are placed in different ability groups, and whose later differing achievement has been attributed to the grouping effect. However, it has also been suggested that it is the differing curricula delivered to students in their respective ability groups that produce the widening of gaps in such systems (Ruthven, 1987; Boaler, 1997a).

Lim and Ernest's (2000) study of public images of mathematics showed that the vast majority of a sample of adults in the UK (94%) believed that certain types of people are better at mathematics than others. Fifty percent of them regarded this mathematical ability as genetically derived, the quality of teaching was the next most quoted explanation, followed by effort and perseverance. The two most strongly-held beliefs about mathematics that the sample revealed was that mathematics is difficult and that mathematics is only for the 'clever ones'. This is consistent with the research of Burton (1988), Cesar (1995), and Vanyan et al (1997).

The teachers' systems for categorising, grouping and labelling appeared to be well-entrenched. Where the children did not consistently and neatly fit, as in the case of Georgina who 'got them all right' in geometry, but 'struggled' with number, they presented difficulties for the teachers. Georgina was recognised as having geometric skills - 'a spatial type person' - but this seemed somehow insufficient to the teacher as though this alone was not enough to be 'mathematically able'. Even where there was apparent flexibility in grouping, for example regrouping for each topic, the groups appeared to remain fairly static. Once categorised early in the school year, children were seldom re-labelled. The most movement occurred with the 'middle' children. Teachers' views of the children remained remarkably consistent across the three years, indicating that 'ability' in mathematics as defined by the school system, was fixed from a fairly young age, contrary to the literature which suggests that children learn mathematics at vastly varying rates and in many different ways. This would in turn suggest that 'mathematical ability' is difficult to define and therefore even more difficult to 'detect' or identify in a child.

Some teachers, Ms Washbourne for instance, expressed discomfort about using labels with 'nasty' connotations such as 'average' and 'bottom', referring to the hierarchical levelling system of the curriculum statement instead, but most teachers used labelling terms freely and in a manner that indicated that such classifications were a commonly understood, accepted and unchallenged language.

### **Parents' views of their children**

Parents were invariably interested in their children's progress in mathematics and expressed their beliefs about their children's achievement in a variety of ways.

Fleur's mother: I think she's somewhere at the top of the second group for maths. She tries her best but just doesn't have the confidence. We had to help her a lot with the times tables. She would sit there saying, 'I'll never be able to do this'. Yeah, she's more confident in other subjects. (Mid Year 4)

Georgina's father: I'm frustrated because Georgina doesn't seem to be getting it (basic facts). I could do it easily - I've got a scientific mind. (Mid Year 5)

- Jessica's mother: They've either got it or they haven't, haven't they? Genetics must play some part. (Early Year 3)
- Maths is just not her favourite subject. She loves writing and always has done. She's in the bottom group for maths this year, apparently, but some of the children in that group are the brighter ones, so...hm, I don't know. She often finds the maths they are doing easy but she's scared to say – she'll plead ignorance in case she gets harder work. (Mid Year 5)
- Rochelle's mother: I feel she's not pushed, that's my main concern. I feel she should be pushed a bit more because she's got the ability. She comes home and says maths is too easy, and she's getting bored. (Mid Year 5)
- Dominic's mother: His teachers for the last couple of years have worked out that he seems to have a bit of a gift for maths as opposed to reading and writing.  
(Early Year 3)
- Dominic's father: He is aware now of where he sits in comparison to others, which he was blissfully unaware of before, which was really neat, so he is now ...being in the last maths group. He's aware now that the kids who go off to Ms Torrance for maths are the ones who are the lowest, whatever that means to him...When he talks about maths he sounds quite confident that he can do it. His only reservation is that the others think he's thick or slow because he has to go to Room 5 to do it. (Late Year 3)
- Dominic's mother: With reading they just do a running record and work out how comfortable they are with a similar grouping, but with maths I don't know how they do it, whether they go by age or how they do it. (Late Year 3)
- Jared's mother: I thought he was taking a long time to get it, but it's starting to fall into place for him... I sometimes sort of get him to add, and he just guesses a number in the vicinity rather than actually knowing how to get to it, whereas Aaron [Jared's brother, younger by two years] will go forward and grab it and come up with the right answer. (Mid Year 3)
- Liam's mother: He's a little smarty. We'll say to Chantelle [Liam's sister, older by two years] 'What's such and such?' and he'll go like, (*clicks her fingers*) not a problem really. (Early Year 3)
- Liam's father: He'd probably be quicker, if anything he might be quicker than Chantelle. Yeah, but they're different people. He's not going to be a reader or story teller like Chantelle is. (Early Year 3)

- Mitchell's mother: It must be genetic, mustn't it? I'm not that great and nor is his Dad. (Late Year 5)
- Peter's mother: He's never been a kid who's really good at something, until he's mastered it. He said, 'I don't like maths,' and that's a sure sign that he feels just a little bit out of his depth...I was talking to his teacher and she was surprised when I told her that - as far as she's concerned, Peter's really good at maths. I was actually astonished that he was chosen for the maths extension group. (Early Year 3)
- Toby's mother: I think he's doing quite well – for his age. ... You do get some information but you don't like to ask how they compare with other kids. But yeah, he'd be around the top group. (Late Year 3)

### Children talk about their ability

In the cases of Mitchell, Jessica and Rochelle, being assessed, labelled and grouped by the teacher, or being judged and ranked by peers or parents, had marked effects on the children, particularly in the development of their mathematical identities. The following conversations reveal something of how the other children interpreted the ways in which others regarded their performance in mathematics.

- Fleur: I'm a bit of a slow learner. They're quick [other children]. I'm quite a bit slower. Because I struggle. They know a bit more and what they're doing. They get the point of it all. (Mid Year 4)
- Fleur: (*Having rated herself at 5.5 on the scale*) Well, I find it hard, so that kinda puts me back, so if I find it hard that means that I'm no good at it.
- Researcher: Are there some kids who find it easier do you think?
- Fleur: Definitely.
- Researcher: How do you know that?
- Fleur: They like maths, they enjoy it. (Mid Year 5)

By early Year 4, Fleur was developing a sense of who was and wasn't 'good' at maths, and this conversation shows some of her reasoning, including ideas from the television programmes she had been watching:



- Researcher: Is there someone you know about who is really good at maths like your family or friends?
- Fleur: Carly.
- Researcher: How did you find that out?
- Fleur: Because I sit next to her and she's usually, when I look at her work, it's like, so good.
- (Later)
- Researcher: What about in T.V. programmes or anything like that, that you think is good at maths? *(Pause)* What about Bart Simpson? Is he good at maths? *(Fleur has said earlier in the interview that she likes The Simpsons)*
- Fleur: *(Laughs)* Lisa is.
- Researcher: Is there anyone you know who's no good at maths?
- Fleur: Hm. Leanne.
- Researcher: How do you know that?
- Fleur: Because she's always in a group where she's trying to find out more things about the maths.
- Researcher: Anyone else you can think of who isn't very good at maths?
- Fleur: Hm. Mary-Kate and Ashley Olsen.
- Researcher: Are they in the same group as Leanne?
- Fleur: *(Laughing)* No, no, no! They're off, um, *It Takes Two*. Have you seen that programme? [Current television comedy for children]
- Researcher: No, I haven't. How do you know they're no good at maths?
- Fleur: Because, um, in the first one [episode] she said, 'You know in real life you can use a calculator?' They never do their homework. (Early Year 4)

Georgina did not feel good about being apparently less able than most of her peers. The class grouping system reinforced her sense of failure through exclusion. Ruthven (1987) argues that ability grouping leads to differential treatment of children by teachers, and this may have been a factor in Georgina's unhappiness with her group placement.

- Researcher: How do you know Justin's really good at maths?
- Georgina: Because he's in the highest group and he's always getting everything right.
- Researcher: How do you know he's getting everything right?
- Georgina: Because he always, like, puts up his hand every time, 'cause he's really fast and goes 'Shp! Shp! Shp!' *(Georgina acts out the way Justin puts up his hand quickly time after time)*
- Researcher: So he's in the highest group. What group are you in?
- Georgina: Squares.
- Researcher: How did you get into that group?

Georgina: It's the second to last group I think. I want Mrs Cayo to put me up because I'm actually meant to be higher 'cause I get everything right, 'cause I'm in a Year 3's group.

Researcher: And you're a Year 4.

Georgina: Yeah. 'Cause there's only two Year 4s [in that group] and that's Erena and me. (Mid Year 4)

Georgina: I don't like it when I have to stay on the mat [for extra work with the teacher] and the other kids can go off.

Researcher: Does that happen very often?

Georgina: Yeah. (Early Year 5)

Researcher: Why do you think you're around 5 at maths [on self-rating scale]?

Georgina: Well, when we were doing fractions it was really hard for me and I didn't get it and they [other children] were just going, 'Oh yeah, I know that one!' and they got it straight away and they were correct. (Mid Year 5)

Researcher: What makes you think you're a 5 [on self-rating scale].

Georgina: Some people would be about 8 or 9.

Researcher: How can you tell?

Georgina: I dunno. 'Cause most tests they get them all right and in, like, two minutes, they get them all right. (Late Year 5)

Dominic said that he felt that the teacher would rate him less favourably than he would rate himself. As described earlier, Dominic's Year 5 teacher rated him as average when according to his P.A.T. score he showed considerably more mathematical skills than the 'average' child of his age.

Dominic: Mr Swift would probably put me at about 9 [compared with his self-rating of 10]. Because he usually doesn't see things, but I always get it right, but the first time he saw it [all correct] he said, 'Gee Martin,' and he's [Martin] about 10. Now he doesn't even check me...I'm one of the best in the class.

Researcher: How do you know that?

Dominic: 'Cause every basic facts test, I get eighty out of eighty.

Researcher: Would there be quite a lot of people that would be 10s in your class?

Dominic: No, only four, they are me, Eden, Whitney and Martin...We practice a lot and the other people don't practise much...I'm smarter than them... (Late Year 4)

Jared's mathematical identity changed markedly after he became part of the syndicate cross-grouping system. When comparing himself with others in the 'bottom' group, he considered himself to be one of the best.

Jared:                    (*Rating himself 10 on ability scale*) There's one person in the class who's the same as me. We both know all the answers. (Mid Year 5)

This contrasted with his lowest rating of 6 during the previous two years. This phenomenon has been noted by Pollard et al (2000) who found that for the lowest achieving children in their study, 50% said mathematics was their most liked subject. They suggest that these children 'saw it as a refuge from more open-ended tasks like writing. They enjoyed the security its routines could provide and with differentiation they enjoyed a degree of success'(p. 101).

Liam had been in the top mathematics group for around a year and a half. He had come to regard himself as good at mathematics. However, an event occurred which shook that belief. Based on the results of one of his topic pre-tests, he was demoted from the highest group, with a concomitant adjustment in his self-rating from 8 to 6.

Liam:                    I'm not the best. They get higher scores than me. (Mid Year 5)

Peter and Toby also rated and ranked themselves according to their *marks* and *scores* as compared with peers.

Peter:                    (*Rating himself 9 on the scale*) Because I can get most of it.

Researcher:            Would anyone be a 10 do you think?

Peter:                    Yeah, because some people in the class always get top marks and stuff. (Mid Year 5)

Toby:                    (*Explaining why he rated himself 8*) Because there are people in my class who would be a 10 because they're quite a lot better than me - Jasper Thompson who knows all of his times tables...(*Explaining his group*) Um, well, I think that, um, the Squares are the highest, the Circles might be the second highest and the Triangles might be the third highest but I don't know if Mrs Kyle meant to put me in the Triangles because there are some people in the um, the Circles that I'm about as good as. (Late Year 4)

Social mechanisms of peer ranking for mathematics operated amongst the children as revealed in the following extracts:

Researcher: *(To a group of boys in Rochelle's class who have asked about the purpose of my visit)* How do you feel about maths?

Eli: Good. I'm the best in the class.

Brad: No you're not.

Eli: Who is then?

Brad: Maria.

Tim: No, she doesn't know four times nine.

Eli: It's, um, thirty-six!

Josh: *(Sneeringly)* You think you're better than Corey at maths! *(The debate continues in this manner for a little longer, with Eli becoming visibly upset.)*

Eli: *(Finally in a distressed voice)* Shut up! *(The teacher intervenes by asking the boys to get ready for maths.)* (Late Year 5)

*(Georgina sits with a group of girls who are discussing those who are good at maths in the class.)*

Charlotte: There's this boy in our class. He's really brainy, he knows his times tables and division and all that.

Hayley: What about Tara? She's good too. She's too brainy to be a Year 3. (Early Year 3)

Meighan and Siraj-Blatchford (1998) suggest that 'pupils assessing pupils' is the most prevalent form of assessment in school settings, yet educators pay it the least attention. The children's statements show that self-rating as a comparison with their peers, was of deep significance in their everyday lives. The primary methods by which children gauged their own and others' mathematics ability were:

- comparing themselves in mathematics speed activities, including games;
- comparing mathematics test results;
- comparing the difficulty of work they were given by the teacher;
- comparing speed of task completion;
- comparing the feedback they were given by the teacher including stamps, stickers, the proportions of ticks and crosses on the work in their mathematics exercise books, and verbal feedback;
- comparing the groups by determining, if not apparent, which were the top, middle and bottom.

- Researcher: Do you ever talk about maths with your friends?  
 Dominic: Just probably after our basic facts tests, when we talk about our scores and stuff.  
 (Late Year 4)

It was through this system of peer assessment, that Dominic overrode the teacher's opinion of him and drew his own conclusion that he was one of the four best in the class.

Where the structures of mathematics teaching encouraged peer assessment and ranking, such as speed tests, games of *Around the World*, and grouping based on pre-test results, there were always 'winners' and 'losers'. Fleur, Georgina, Jessica, Jared, Mitchell and Peter had all experienced being losers. They had each developed unique coping strategies. Even the more confident children such as Rochelle, Dominic, Liam, and Toby who generally regarded themselves as 'winners', though positive about their mathematics achievement, had doubts about its lasting nature, as the following extracts show:

- Researcher: How do you feel when you do the tests?  
 Rochelle: Sometimes a little bit worried. Because I think they're going to be hard. (Mid Year 5)
- Dominic: I feel quite nervous, well, 'cause you don't know which group you will be in.  
 Researcher: You could change groups?  
 Dominic: Yeah...He changes the groups all the time but I've always been in the highest group.  
 (Late Year 4)
- Liam: Sometimes I get really nervous, 'cause I might get a real bad score. I feel like my legs would shake... Kids might say, 'That sucks, you should've got higher than that.'  
 (Mid Year 5)

### **Defining and identifying mathematical 'ability'**

Mathematics in the New Zealand Curriculum (1992) is unique among New Zealand's current curriculum statements in providing a 'development band' for children it perceives as 'more able'. It says that:

Some students develop *faster* in all aspects of mathematics than most of their peer group...

The intention of the development band is to encourage teachers to offer broader, richer and more challenging mathematical experiences to *faster* students. Work from the development band should allow *better* students to investigate whole new topics which would not otherwise be studied and to work *at a higher conceptual level*. *Talented* students should have their interest in mathematical ideas stimulated. (p. 19) [Italics added]

This differs from the curriculum statements of other essential learning areas such as *English in the New Zealand Curriculum* (Ministry of Education 1994b) which simply says 'the aims and objectives in this curriculum statement provide goals and challenges for all, including *gifted and talented students*. Teachers should adapt learning contexts to stimulate and extend these students' (p. 15). [Italics added]

*Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) also says that 'students of *lower ability* need to have the opportunity to experience a range of mathematics which is appropriate to their age level, interests, and capabilities' (p. 12) but makes no special provision for these students. *English in the New Zealand Curriculum* (Ministry of Education, 1994b) on the other hand says 'there are a significant number of learners for whom the acquisition of skills in formal English is difficult. The English language programme must offer *students with communication difficulties and disabilities* every opportunity to develop their communication skills' (p. 15). [Italics added]

*Development Band Mathematics* (Ministry of Education, 1996) was published as a guide to assist teachers in catering for the '*faster*' and '*better*' children. Throughout the handbook these children are variously named, '*with special mathematical abilities*' (p. 5) '*talented*', '*an exceptional few*' (p. 8) '*very able*' (p. 9), '*really able*' (p. 16) '*extremely able*' (p. 17) and '*very bright*' (p. 20). The authors are at pains to explain that they are 'avoiding where possible' the terms 'gifted' and 'talented' in the handbook, 'because of the implications and expectations the words carry for those who are given these labels' (p. 17). However, these labels *are* used (pp. 8, 18), presumably because avoidance was impossible. The handbook urges teachers to 'identify' these children, and provides a range of diagnostic 'tools' for this purpose.

The fact that the national mathematics curriculum included a special band for the 'talented' students and provided a guidebook for teachers to cater for such students

when curricula for other subject areas did not, could be interpreted as an indication of a widely-held belief that mathematical 'talent' requires a kind of special treatment not required for 'talent' in other domains. Although a handbook was intended for the mathematically needy, this has yet to be written. If all social acts including curriculum development are grounded in shared meanings, then the unique construction of *Mathematics in the New Zealand Curriculum* and its handbook, *Development Band Mathematics*, reflect dominant beliefs about the nature of mathematics and of mathematical 'ability'.

The writers of *Development Band Mathematics* have struggled to reconcile two seemingly incompatible beliefs:

- education should be inclusive and that it should provide equal opportunities for all students;
- education should cater in special and exclusive ways for the individual needs of students, particularly the 'very able.'

This contradiction is illustrated when the authors attempt to promote the labelling and segregating processes of identifying development band students, while simultaneously espousing the principles of equity:

The purpose of identifying students capable of development band work is to meet their individual needs. Since students' interests and apparent abilities change and develop, identification should be an ongoing process. ...It is especially important that equity is guaranteed in the identification process. Development band students will be both male and female and from all cultures ... The identification of the majority of development band students is straightforward. They are the ones who do their work quickly and achieve good results in tests and assignments. (p. 16)

This contradicts the statement found in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992):

Traditional time-constrained pencil and paper tests have proved unreliable indicators of Maori achievement in the past. ( p. 13)

It is highly unlikely that children from all cultures will be fairly represented in development band identification. A conundrum exists here, and the dilemmas are linked to the pervasive and overriding social compulsion to identify, rank and label.



The teachers in the study have also shown how these same dilemmas drove them to seek ways to accommodate their paradoxical views of the nature and purpose of mathematical educating.

In *Development Band Mathematics* (Ministry of Education, 1996) the idea that children's 'apparent ability' is a fluid and mutable thing, is proposed. While the impression is given that all students could and should at some time be included in development band activities, the handbook states: 'At some time during their school years, about twenty-five percent will take part in a development band activity of some kind.' (p. 8). In spite of the equity rhetoric, it appears that the authors expect, as some kind of universal axiom, that only 25% of our children will ever make it to those 'higher cognitive levels', no matter how well we teach, and even then, not all of the time. Catering for 'Development Band' students would appear to rest, therefore, on taken-for-granted beliefs about a 'natural distribution' of mathematical ability. The consequential denial of access to 'higher' level mathematics for 75% of the student population is conversely also taken for granted.

Only one of the study schools appeared to make use of either the material for development band students in *Mathematics in the New Zealand Curriculum* or *Development Band Mathematics*. Apart from Lake School where Jessica said 'I'm not in his band thing' (Late Year 4), the term *Development Band* was never heard. The study schools preferred the terms *extension* or *enrichment*.

Boaler and Wiliam (2001) remark that 'in the UK there is a long tradition of grouping students by ability, particularly in mathematics. This practice is founded on the widespread belief that ability grouping raises attainment' (p. 77). They provide evidence from extensive classroom observations and interviews with 11 year old pupils that ability grouping was a negative experience not only for those in the lower sets, but for those in the higher ones, where children complained of excessive expectations and pressure to succeed (as Rochelle, Dominic and Liam also commented) and of more formal and faster paced lessons with less time for exploration and consolidation of new learning, and separation from friends from whom they felt comfortable in asking for help. The study concluded that teachers teach children in the 'higher ability' groups in quite a different way to children in the

'middle' or 'lower' groups. Classroom environments of the 'top' groups were characterised by faster pace, more procedural pedagogy and competition between students. The other key issues arising from their study of ability grouping were the lower expectations and limited learning opportunities of the less able children, the apparent homogeneity of children in 'ability' groups leading to inflexible placing of students (once children were placed, movement between groups was rare) inflexible pacing (all children in the group had to work at the same speed) and restricted pedagogy. Teachers in schools where children were grouped by ability were far more likely to rely heavily upon a textbook approach to teaching of mathematics. Although the children of the present research were of a younger age than the students discussed by Boaler and Wiliam, their findings resonate with those of this study.

### **Mathematical ability as a social construct**

Postman (1996) argues that ability or 'smartness' is not something people 'have' but something they 'do' in a particular place at a particular time:

In schools, for instance, we find that tests are given to determine how smart someone *is* or, more precisely, how much smartness someone *has*...The people I know sometimes *do* smart things and sometimes *do* dumb things depending on what circumstances they are in, how much they know about the situation and how interested they are. Smartness, so it seems to me, is a specific performance, done in a particular set of circumstances. It is not something you *are* or *have* in measurable quantities. In fact, the assumption that smartness is something you *have* has led to such nonsensical terms as *over-* and *underachievers*. (pp. 176 – 177)

Mr Loch was surprised when Jessica *did* a smart thing – she scored well on the fractions pre-test. He had previously believed she *was* a 'middle kid', even a bit below average. He tried to find explanations for this apparent aberration, while his view of her as a middle kid remained. Many of the other teachers talked of the children as though their ability were fixed – something they *were* or *had*.

Parents too sometimes held views that suggested they believed ability was something their children *had* for example Jessica's mother thought Jessica *was* 'a left-brainer', implying therefore that she was less likely to be able to *do* certain things.

Oakes et al (1997) regard 'ability' as a social construction built on an equally constructed view of 'knowledge'. They use the work of Berger and Luckman (1966) to explain the social construction of ability as a 'reality'. They see human knowledge of everyday life as socially constructed rather than scientific fact, that conceptions of intelligence are socially constructed rather than scientifically discovered, and that schools' responses to differences in intelligence, such as ability grouping, are themselves social constructions, rather than self-evident implications from established scientific knowledge. Dowling (1998) also challenges the notion of 'ability' as fixed, and argues that schools contrive to categorise and separate children, especially through the teaching of mathematics:

Schooling comprises cultural institutions, practices and beliefs which are constituted by and are constitutive of the relations which characterize the social. Specifically, schooling in general, and school mathematics in particular, is organized on the basis of the distribution of pedagogic content and action in terms of student attributes. In the early stages of mass schooling the principles of this distribution were commonly explicitly stated in terms of social class and gender. More recently, the rhetoric has tended to background these social considerations in favour of categories such as ability, achievement and needs. Nevertheless, it seems that the differentiation of the curriculum remains more or less closely associated with the social stratification of the student population. In stark terms, my position is that there is no such thing as 'ability' or 'achievement' or 'needs' insofar as these are interpreted as substantive predicates of individual students. Rather, these are variables which are constituted in and by the practices of schooling. (pp. 68 – 69)

Berger and Keynes (1995) argue similarly:

Separation creates an underclass which receives inferior treatment...Each year of schooling is a filtering process for students of mathematics. Only the top group is assumed to have passed through the filter. (pp. 90 - 91)

Zevenbergen (2001) notes the close connections between social class and supposed mathematical 'ability', and the results of differentiation by ability as translated into differentiated curricula for each of the identified groups.

In speaking of teachers of mathematics, Drew (1996) says 'our approach to teaching depends upon whether we assume that (1) virtually everyone can master the material and the challenge is to present it in a manner that allows them to do so, or (2) the

material is tough and only a few of the best and the brightest will be able to learn it' (p. 9). He contends that tests that supposedly identify those with special mathematical aptitudes 'can be extremely destructive if they send a message – an incorrect message – to those who are not selected that they are incapable of learning the material' (p. 10).

For children like Georgina who was never placed in a development band or extension group in spite of her apparent competence in geometry, sorting and labelling in mathematics can be a disheartening experience. However, as Fleur showed below, when she discovered that there was something in mathematics that she could do better than most of her peers, being labelled as 'slow' does not necessarily have lasting or irreversible effects and that children can shake off such labels given even minimal encouragement or opportunities for 'success'. However, success in the children's eyes was invariably equated with being better than others.

Fleur: *(After learning to perform addition and subtraction calculations in written working form with renaming)* I love maths now. I'm one of the best in the class. (Late Year 4)

Fleur: Seventy-nine was my highest score. One more point and I would've got into the top group, the ones that are at the top of the class. I bet a whole lot of people that I normally would never beat in that test. I just tried really hard. Tried my hardest. (Late Year 5)

### School Policy

In the schools' information pamphlets for parents, mention was frequently made of the importance of mathematics as a subject, and of the special provision made for top and bottom learners in this subject. Here are two examples:

'As part of the planning and assessment process all teachers regularly provide enrichment activities, particularly in the areas of language/reading and maths.' (River School)

This school offers: 'A comprehensive primary education with an emphasis on literacy and numeracy, with special help in reading, language and mathematics for children with difficulties.' (Spring School)

## Conclusion

Much of what occurred in the everyday sociomathematical worlds of the children appeared to be driven by sorting and labelling beliefs. In every classroom, some form of ability grouping was found. Extension groups were provided in a number of instances for the children who were identified as talented, but little support for those who were perceived as lagging behind. There appeared to be a widespread belief that effective teaching could not take place without some kind of differentiation by ability. Ability was usually determined by written tests, contrary to the clear directive of *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) that a variety of assessment techniques be used. As discussed in Chapter 3, regular speed activities were also used by the teachers and children as a major vehicle for sorting tops, middles and bottoms. Number skills were the prime determinant of mathematical ability, privileged over all other kinds of mathematical facility.

The children developed awareness of their own mathematical 'abilities' through the explicit and implicit everyday grouping practices of the classroom. For some, this produced feelings of anxiety, and exclusion, for others, pride and satisfaction. Furthermore, grouping practices denied a significant proportion of the children access to the broadened mathematical curriculum provided for others, resulting in alienation, marginalisation, and impoverished learning. The children's perceptions of mathematical ability were shaped and reinforced by the events and practices of everyday life such as speed games, tests, teachers' marking of their work and grouping. Teachers provided minimal verbal feedback to the children about their mathematical strengths and needs. Such feedback might have given them a more realistic idea of their mathematical learning development, than the pictures they were forming based on everyday events such as the easily comparable results of daily basic facts speed tests.

The examples of Georgina, Dominic, and Mitchell, whose mathematical understandings were not fully recognised by their teachers, illustrate how pervasive reliance on traditional assessment methods such as timed written tests, resulted in the sorting, labelling and differential treatment that was entirely inappropriate for these

children. In the cases of Mitchell and Georgina, this resulted in impoverished learning, marginalisation, and severe disaffection with school mathematics.

*Ability* implies an innate quality, while *attainment* refers to demonstrable and measurable achievement. These concepts became blurred in the discourse surrounding mathematical learning. *Ability* and *achievement* were found to be taken-for-granted social constructs, and highly resistant to alternative perspectives, such as found in the introduction to *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992), where the pedagogical and epistemological bases for such judgements about children's learning of mathematics are questioned.

The following chapter shows how the children, constructed as winners or losers through competitive speed activities, and differentiated into tops, middles, and bottoms by means of written tests where mathematical ability was equated chiefly with number skills, were inducted into the world of classroom mathematical 'work'.

### **'DOING MATHS': ESTABLISHING A MATHEMATICAL WORK ETHIC**

Researcher: So what things do you usually do in maths time?

Georgina: Get out our maths books and do our maths. (Early Year 4)

This chapter examines the children's views of *doing maths* based on their everyday experiences of the 'working' phase of mathematics lessons. Through classroom routines, teachers presented *doing maths* as a specialised kind of *work* involving the production of written answers to questions from the board, worksheets, or textbooks. Teachers' interactions with the children established and reinforced entrenched protocols of setting out, neatness, and the 'rules' and expectations of 'doing maths'. The chapter demonstrates the establishment of an academic mathematical work ethic through the predominant classroom norms of individual written recording, the dwindling use of concrete materials, and the suppression of discussion. It is shown how this view of *doing maths* was endorsed by the wider social structures of school management, family, textbook authors and curriculum developers. The chapter discusses the impacts on the children of the compulsory classroom operational modes through which the mathematical activities of school were constructed as work.

#### **Children's views of 'doing mathematics'**

During their first interview (Early Year 3), the children were provided with a blank page headed 'This is a picture of me during maths time' (Appendix 10). They were encouraged to draw themselves in any way that best showed what they usually did during this part of the school day. The drawings of these seven-year-olds revealed much about what they perceived as 'doing maths' (Figures 6 – 15).

Eight of the children drew themselves seated at a desk or table, pencil in hand and their maths book or worksheet in front of them. Liam was the only child to draw himself actively engaged with others. He depicted himself with his friends, naming each one as he drew, constructing a tower of wooden blocks (Figure 15). Dominic



drew himself at a table with other children, all working individually in their maths books (Figure 11). Toby drew other (childless) desks with worksheets to indicate classmates, but showed himself to be working alone (Figure 8). Mitchell was the only child who was not able to distinguish 'maths' from the other activities he was expected to do at school. He drew himself skipping, the activity in which he had been engaged a short time before the interview, and drawing, the activity he said he most liked (Figure 14). Jared's drawing is notable for its action and movement (Figure 6).

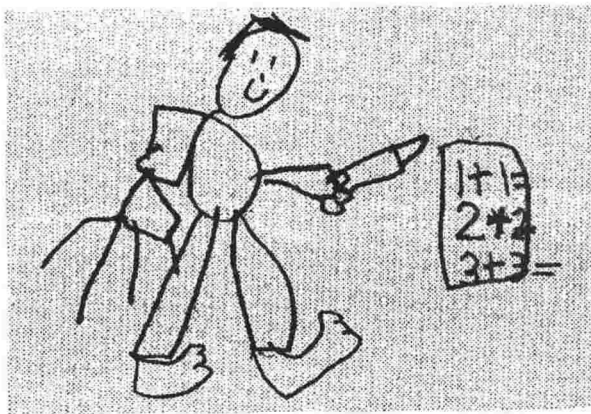


Figure 6: Jared (Early Year 3)

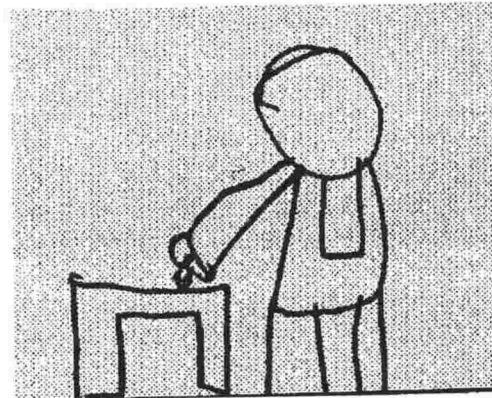


Figure 7: Peter (Early Year 3)

The children were asked to explain their drawings:

Toby: This is the table and that on there is the worksheet. (Early Year 3) [Figure 8].

Researcher: And what's that you have just drawn? [Figure 9].

Rochelle: It's my desk.

Researcher: So what's this here?

Rochelle: Book.

Researcher: Is that your maths book? (*Rochelle nods*) (Early Year 3)

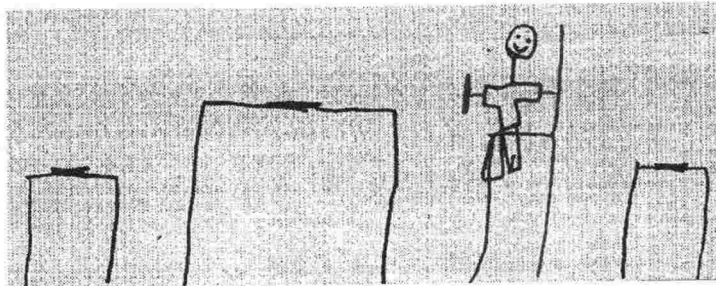


Figure 8: Toby (Early Year 3)

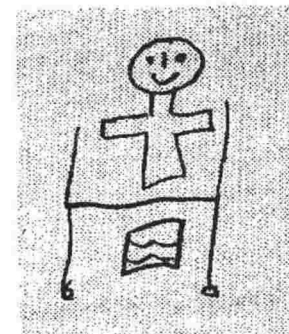


Figure 9: Rochelle (Early Year 3)



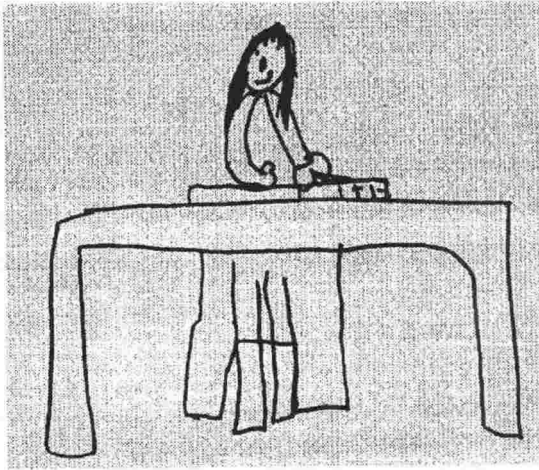


Figure 10: Georgina (Early Year 3)

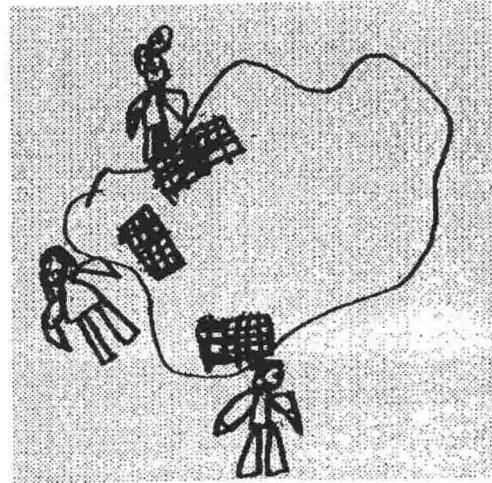


Figure 11: Dominic (Early Year 3)

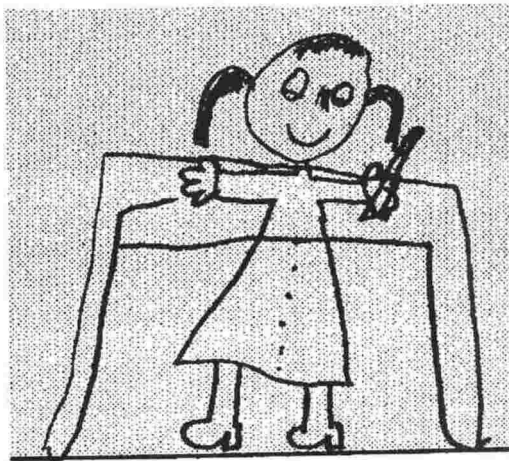


Figure 12: Fleur (Early Year 3)

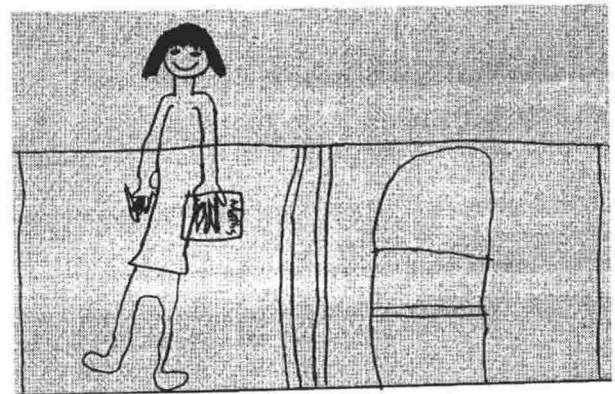


Figure 13: Jessica (Early Year 3)

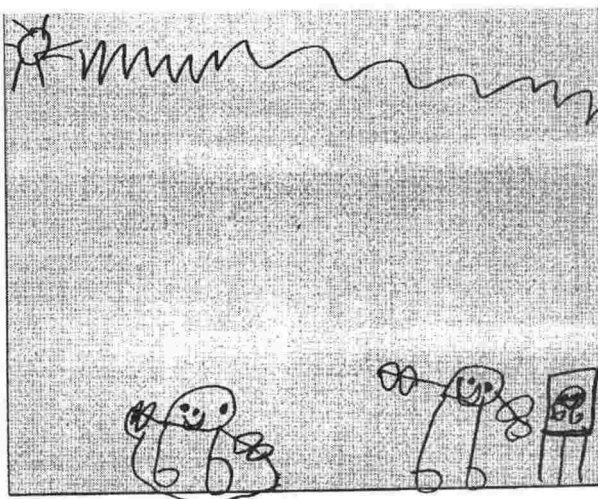


Figure 14: Mitchell (Early Year 3)

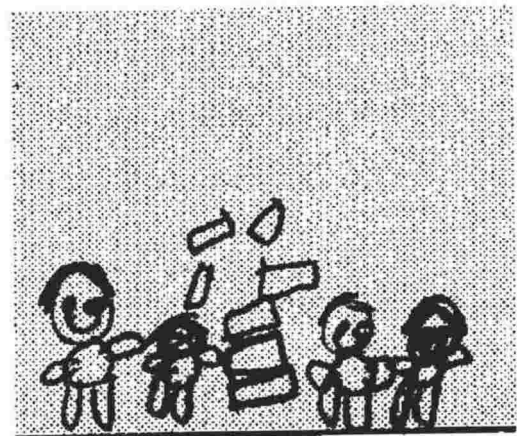


Figure 15: Liam (Early Year 3)

At the beginning of Year 4, the children were again asked to draw themselves during maths time (Figures 16 – 25). By this time, Mitchell was able to talk about what happened at maths time and how to identify maths as a distinct subject.

- Researcher: How could you show me that you're doing maths on your picture?  
 Mitchell: I've got a desk.  
 Researcher: And what's that?  
 Mitchell: My maths book.  
 Researcher: And it's got a tick on it, has it?  
 Mitchell: No, it's a 'seven' [See Figure 16] (Early Year 4)



Figure 16: Mitchell (Early Year 4)

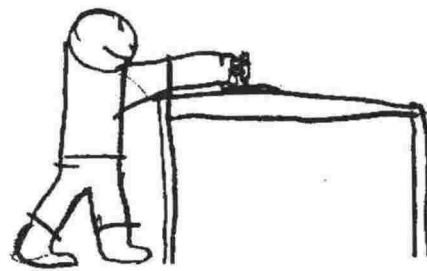


Figure 17: Peter (Early Year 4)



Figure 18: Dominic (Early Year 4)

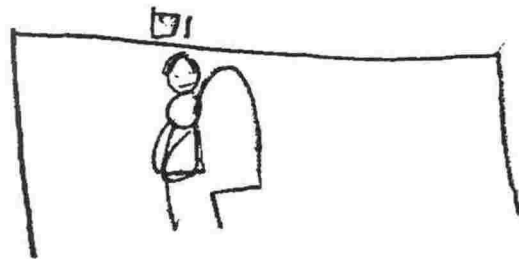


Figure 19: Liam (Early Year 4)

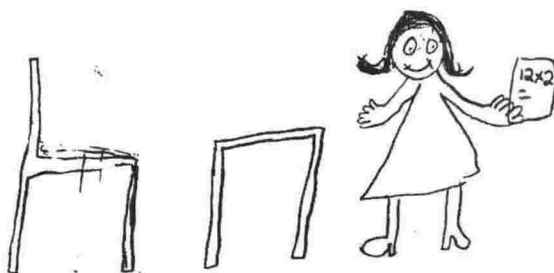


Figure 20: Fleur (Early Year 4)



Figure 21: Rochelle (Early Year 4)

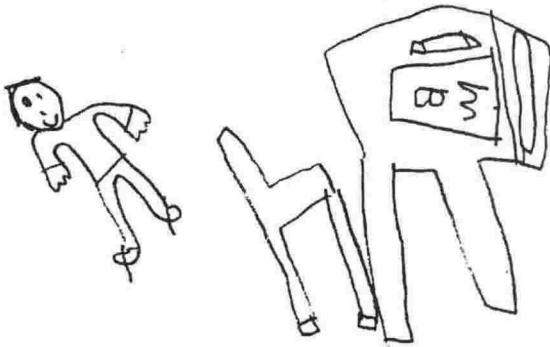


Figure 22: Jared (Early Year 4)



Figure 23: Toby (Early Year 4)

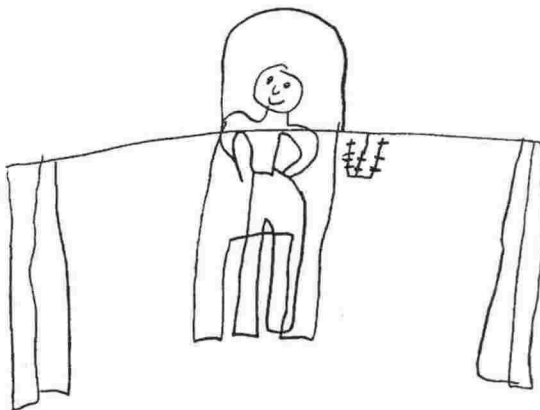


Figure 24: Georgina (Early Year 4)

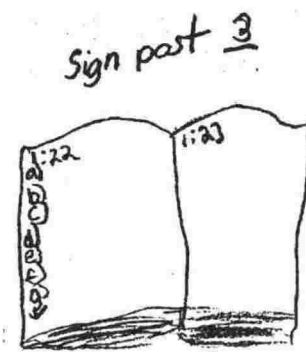


Figure 25: Jessica (Early Year 4)

While nine of the ten children drew themselves engaged in a writing task, Georgina drew herself with a three-bar abacus, (Figure 24). Earlier in the interview she explained that using the abacus was one of the few mathematics activities she had really enjoyed. The fact that she drew this instead of what usually happened at maths time was the result of comments made during the drawing process:

- Researcher: Here's a place for drawing a picture of yourself during maths time. So what would you usually do?
- Georgina: Shall I draw a table?
- Researcher: Yes. *(After Georgina has drawn herself with a big smile)* You're looking pretty happy. *(She has earlier rated herself at only 1.5 out of 10 on the self-rating scale for how happy she feels at maths time)*
- Georgina: I'll put the abacus.
- Researcher: So what things do you usually do in maths time?
- Georgina: Get out our maths books and do our maths. (Early Year 4)

Jessica was not keen to draw herself. Instead she drew her maths exercise book (Figure 25).

- Jessica: Do I have to do it of me? Can I just do it of my maths book?
- Researcher: It's hard drawing you is it? (*Jessica nods*) How would you want to draw yourself if you could? How would you imagine yourself, what would you be doing with the maths book?
- Jessica: Um, well, what I could do is I could do us standing looking at the maths book and then you could see a little bit of the writing.
- Researcher: Sounds great. Away you go.
- Jessica: Then it would be the one we work out of. (*Draws the her maths exercise book opened at a page of exercises*)
- Researcher: What's the book called?
- Jessica: We usually put the label, Signpost 1, Signpost 2.<sup>1</sup>
- Resreacher: Which one would you usually use?
- Jessica: Signpost 3. (*Writes this label above her exercise book.*) (Early Year 4)

Liam's Year 3 and Year 4 pictures differ markedly. Classroom observations revealed why. In Year 3, his teacher conducted an activity-based programme using *Beginning School Mathematics*. Discussion and direct experience with concrete materials were the norm in this classroom, with children recording as necessary on worksheets or paper, while the teacher recorded on a small blackboard. When Liam moved on to Years 4 and 5, mathematics exercise books were introduced and used almost daily, while peer collaboration and the use of equipment became less and less frequent.

### What the children saw as 'doing maths'

There was an overwhelming prevalence in the children's representations, of 'doing maths' as lone seatwork, with an emphasis on number tasks, such as completing equations. This distinctive common feature of their drawings indicated that individual written work was repeatedly experienced by the children at maths time, and what they most identified as 'doing maths'. Observations of mathematics sessions, teachers' and children's descriptions of a typical lesson, and examination of children mathematics exercise books, supported these suppositions. Written work as depicted in their drawings, was the most common activity experienced by the children at mathematics time. Because of this, the children attached the most significance to it, so that less frequent kinds of mathematics activities such as using equipment for

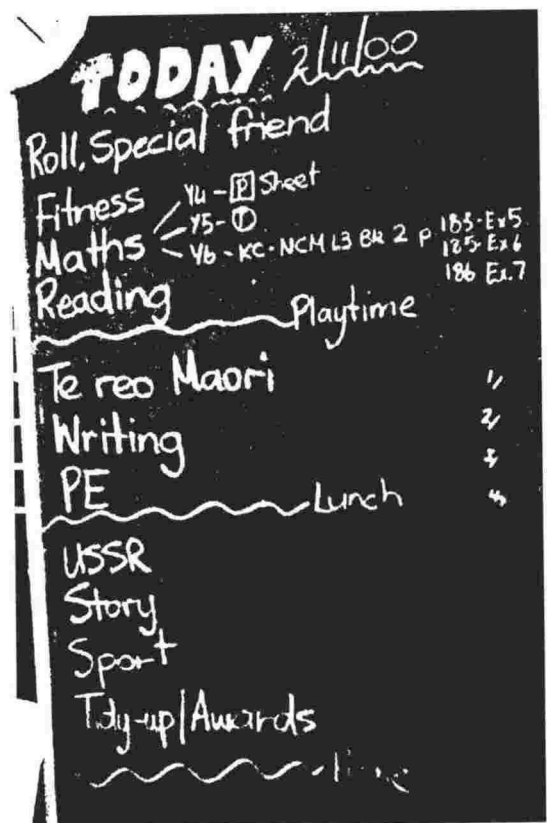
<sup>1</sup> *Signpost*: Textbook series, Parker and McSeveny (1994).

measuring, or gathering statistical data, were considered by the children as less typically 'maths'. Although the children were regularly seated on the mat at mathematics time either as a whole class listening to the teacher, or in a teacher-guided group learning situation, this did not feature in their drawings, and seldom in their verbal descriptions of doing maths. The teacher is notably absent from all the children's drawings.

### Maths time

'Doing maths' was a daily occurrence in the majority of the study classrooms. In only one of the thirty-two classrooms visited was mathematics taught on four days of the week rather than five. 'Maths time' usually occupied between 40 to 60 minutes of the daily programme and was without exception, a morning activity. In the majority of the classrooms, mathematics was taught before morning interval, usually between 9:30 and 10:30 am. In four classrooms, mathematics was taught just after morning interval, and in two it was scheduled just before lunchtime.

The regularity of mathematics teaching, the percentage of teaching time it received, and its morning placement, believed by teachers to be the time of day when most learning is likely to occur while the children's minds are 'fresh', showed that mathematics was a highly valued school subject. 'Maths' usually appeared on the daily programme on the board, as shown in Figure 26. This particular timetable specified each year group task, P meaning 'practice', T meaning a session with the teacher, and KC meaning 'keeping clever'. Apart from Y5 who were with the teacher for part of the time, all the other children in the class were engaged for most of this mathematics lesson in individual writing tasks using worksheets or the textbook.



### Doing maths – the typical lesson

A typical lesson consisted of a starter activity involving the whole class (see Chapter 4). This was often followed by some form of learning associated with the current mathematics topic, or reinforcement of recently learned concepts. Children were usually placed in ability groups for this part of the lesson, either within or across classes (Chapter 5). Where children were grouped within classes, teachers would sometimes withdraw one group to teach while the others worked independently on written tasks based on assigned pages of a textbook, a worksheet, or questions on the board. The following extract in which Ms Fell describes a typical lesson in her classroom, illustrates the place of independent written work in her daily programme:

Ms Fell: I'll bring everyone down on the mat and we'll talk about what we're doing that day. If it's something new, quite often we won't be doing anything in our books, we'll be talking about a lot of things, get in a circle, and you know, talk, and then send people off for ten or fifteen minutes to do some work in their books so I can get around and work with people individually... We've just purchased halfway through last year, that AWS<sup>2</sup> series of books where there's one for every strand and they've been excellent... we've been able to photocopy off class sets. (Pukeiti School, Mid Year 4)

As described in Chapter 4, the children were asked what usually happened at maths time. Their accounts of the start of the session were given, with some kind of speed activity most commonly cited. When asked what usually happened next, typical responses from the children were:

Fleur: We go into our book. Our green or red books. [NCM<sup>3</sup> textbooks] (Mid Year 5)

Researcher: Does she explain it first or do you just go and do it?

Fleur: She explains it. (Mid Year 5)

Georgina: We get into our groups and do the worksheet. (Mid Year 4)

Jessica: It would usually be out of a textbook and once we've finished that we would do a sheet. (Late Year 5)

<sup>2</sup> AWS : mathematics worksheets and teacher guides (Stark, A.W., 1997 –2000)

<sup>3</sup> NCM: *National Curriculum Mathematics*: Textbook series, Tipler & Catley (1998 onwards).

- Rochelle: A group goes on the mat. Then the group that was on the mat does the group sheet.  
(Late year 3)
- Rochelle: We do these. (*Shows exercises in her maths book*) (Mid Year 5)
- Dominic: Then we do NCM. Do you know what that is?
- Researcher: Yes, one of those textbooks.
- Dominic: Yeah, or *Figure it Outs*. (Late Year 5)
- Jared: Write stuff. (Mid Year 3)
- Jared: Work. Yep, working in our maths books. (Early Year 4)
- Jared: The teacher says, 'Go and get your maths books out.' And she writes stuff on the board for maths. (Mid Year 4)
- Researcher: What is maths, Jared?
- Jared: Maths is heaps of work. (Mid Year 4)
- Liam: We do sheets and we work with Miss Peake. (Early Year 3)
- Mitchell: We go back to our desks.
- Researcher: Do you do work in your books or does she give you a sheet or...?
- Mitchell: In the maths book.
- Researcher: Does she write stuff up on the board?
- Mitchell: Hm.
- Researcher: So at maths time it's usually writing? (*C nods*) (Late Year 4)
- Mitchell: You have to sit down and do some times tables or pluses or take away. (Late Year 5)
- Peter: Just do worksheets ...finishing all the worksheets and sticking it into your book.  
(Late year 4)
- Toby: Then we mostly turn to the front of our book and do proper maths.
- Researcher: Does she put the work up on the board or give you a worksheet or do you have a book with the questions?
- Toby: No. Mrs Kyle gets the questions out of a book, and we have to get the answers.
- Researcher: How does maths time finish?
- Toby: It just finishes after we've done our proper maths, like you put your maths books away and sit on the mat. (Mid Year 4)

A cumulative picture of the everyday experience of mathematics was established through child and teacher descriptions of typical lessons, and ninety-five classroom observations. It was revealed that a high proportion of mathematics time was spent on written tasks in the form of worksheets, textbook pages, or work from the board.







As evidenced by their drawings, the establishment in the children's minds of 'doing maths' as written recording of answers to questions, had become an entrenched norm of the everyday life of the classroom from as early as Year 3. This part of mathematics lessons was referred to as 'doing their work', or 'doing maths', an expression not used when referring to other kinds of mathematical learning, such as using concrete materials in new learning situations with the teacher.















































Smith and Glynn (1990) noted from their observations of thirty-three mathematics lessons in Year 7 classrooms, that around a third of each mathematics lesson was spent in individual written work and a half was spent in listening to the teacher, with negligible time spent on group activities or discussion. This contrasts with the findings of Higgins (1998) and Thomas (1994) whose observations of Year 1 and 2 classrooms showed the children working independently of the teacher on *Beginning School Mathematics* programmes were engaged in some kind of group activity involving active experience with materials.

Young-Loveridge (1991) found a significant group of low-achieving girls in her longitudinal study of the mathematical learning progress of children aged from five to nine years. In seeking explanations she suggested that 'the maths taught during the third year of primary school changes in important ways from that taught in the first two years...there is certainly greater emphasis on symbolic representation as children learn how to carry out operations and have to use written symbols to record their actions. Problems become more abstract and are often taken out of the everyday contexts that they were in previously' (p.127). From her interviews with the nine-year-olds, she notes 'the impression given from the children's responses is that much of the work in maths at this level is on operations and that it involves a lot of worksheets or working from the textbook' (p. 72).

While each classroom in this study differed slightly, from Year 3 onwards, 'doing maths' became increasingly established as the practice of individual written *work*. Table 4 summarises the 'maths work' experienced by each of the children.

Table 4: Type and frequency of mathematics 'work' experienced by the children

Key: **D** = daily, **F** = Frequently, **S** = sometimes
 = Textbook,  = worksheet,  = exercises from the board,  = using concrete materials

Child	Year 3	Year 4	Year 5
Fleur	F  S ●	D  F  AWS	D  NCM
Georgina	F   S ●	F  S ●	F  NCM 
Jessica	F  SM2 S 	F   <i>Signpost</i> S ●	D  NCM or  AWS
Rochelle	F 	D computation cards   SM3	D computation cards F  FIO
Dominic	F 	F   S ●	F  FIO, NCM
Jared	D  S ●	D  <i>Wellsford</i> <sup>4</sup> S  SM2	D  <i>Wellsford</i> S ●   SM3
Liam	D - Group Box <sup>5</sup> ● from 'maths shelves'	F  	F  S  FIO
Mitchell	D BSM Group Box ● S 	F  S ● teacher-designed tasks	D  F  NCM S ● Mitchell allowed to use counters
Peter	F teacher- designed written tasks ●	F  S  SM3	F  
Toby	F  ●	F  	D   <i>Longman</i> <sup>6</sup>

Textbook key:

AWS = *AWS Mathematics Resources*, (Stark, 1997 onwards).FIO = *Figure it Out* series (Ministry of Education 1999a, 2000a).NCM = *National Curriculum Mathematics* (Tipler & Catley, 1998 onwards).SM = *School Mathematics* series (Department of Education, 1983)<sup>4</sup> 'Wellsford' refers to *Topic-Based Maths, Levels 2 and 3* (Adams, 1996).<sup>5</sup> The Group Box is a collection of activities, stored in a box labeled with a group name, and designed for the children belonging to that group to use independently of the teacher.<sup>6</sup> *Longman Primary Maths* teaching programme, (Patilla et al, 1996)

## Maths books

In the majority of classroom settings the most important accessory in mathematics books were used frequently for core subjects including mathematics. The first thing that most of the study children did when the teacher announced that maths time was about to begin, was to take out their 'maths books' and turn to the place where the quick questions were to be answered. Mathematics exercise books differed from those of every other subject where exercise books were required. Dominic's Year 3 picture (Figure 11) showed his view of these books. He went to some trouble to depict the pages of graph paper. Universally in Year 3, 1H5 exercise books with 9mm squares were used. In Years 4 and 5, these were replaced by 1E5 books with smaller 7mm squares, requiring finer writing. This phenomenon was paralleled by the reduced gap between the lines in their writing books, as the children became older.

'Exercise' as associated with the children's mathematics workbooks and the examples they were expected to complete, is a word with particular connotations. It may hint at the historical view of teaching and learning in general, and of mathematics in particular, as a process where children are shown specific procedures, which they then practise by working through a series of similar examples, known as 'exercises', until the skill is thoroughly 'mastered', much as an athlete exercises to increase fitness and perfect techniques, or a musician plays 'exercises' to perfect a skill.

### 'Setting out' and 'neatness'

Mr Loch: (*Speaking of Jessica*) She's one of the children who seems to have been able to go, to make that transition from juniors to Standard 1.<sup>7</sup> It's a big step, and they have to start writing in books and setting out to a certain, [standard] you know, date and page number, number with answers, those sorts of things and she seems to be able to do that. But she can be a little untidy at times. (Early Year 3)

Ms Sierra: (*When asked how the children find the transition from year 3 to year 4*) Quite hard at the beginning but slowly getting there. Sitting with a book and a pen, using a ruler, something new to them, sitting down and ruling a book. (Early Year 4)

<sup>7</sup> The term *Standard 1* was formerly used for Year 3 of primary school

Ms Fell: *(Commenting on the children's current skills)* With BSM, they haven't done a lot of number. They don't know how to set things out, they have not added vertically.  
(Late Year 4)

Mr Loch, Ms Sierra and Ms Fell expressed a belief that was found commonly among the teachers in the study - that setting out in mathematics exercise books was an important skill for the children to acquire as they moved into the middle primary school. No justification was ever given for this belief. It was taken as self-evidently true. In most of the classrooms, the teachers put considerable effort into training the children to set out their mathematics books in a precise and standardised way. 'Setting out' usually consisted of ruling margins on the left and in the centre of the pages. These were used to enter question numbers. The 'short date' was written at the top of a new piece of work and underlined with a ruler. The children were told to enter only one digit per square on their page, to write neatly, and to space out their answers. Work was to be 'ruled off' at the end of each session. This made learning of early mathematics particularly difficult for left-handed pupils and others with handwriting difficulties. None of their other exercise books were 'set out' in this way, so this particular practice was peculiar to mathematics.

Georgina explains how she had to learn a new way of setting out her mathematics book when she changed schools.

Researcher: Do they do maths differently here?  
 Georgina: Yeah, sorta. 'Cause like they set up their books differently.  
 Researcher: All right. Can you show me? *(Georgina takes out her maths book)*  
 Georgina: They set it up like this, *(points to ruled margin down the left side and down the centre of the page)* like that, and at my old school...  
 Researcher: You didn't set it up like this?  
 Georgina: We just, like, put a rule down there *(points to margin on left side only)* (Early Year 5)

It was observed that the teachers frequently reinforced setting out and neatness during mathematics sessions, as shown by the following examples:

Mr Loch: *(Handing out the children's exercise books).* I took your books in yesterday. Some of you are doing a really good job of setting out. You got a stamp if your work was

good [ Jessica's book has 'Neat work' written in it and she has been given a stamp]... I want the numbers to be clear, and you have to put up the number of the [text] book... *(To a boy as he looks at his work)* The only problem is setting out neatly... *(To another child)* This is too crowded when I mark it... *(To another child who has brought his work to be marked)* ...I don't want to look at this book, it just makes me feel ill! You won't get a stamp... *(To another child)* That's better. I like that setting out...*(Erasing a child's work)* Start again! (Early Year 3)

Mr Solomon: Rule off after the last work. Put the date and underline it. *(To a child)* Can you get rid of that little mark there, please? (Early Year 3)

Ms Summers: Good girl, Theresa, you're really improving with your neatness. (Late Year 3)

Mrs Kyle: Space answers, Toby. It's a good idea to space answers, isn't it? Then we can read what you've written. (Mid year 4)

Mr Ford: Remember when you do your numbers, it's one numeral per square. I'm looking around and a number of you have forgotten that. (Early Year 4)

Mrs Ponting: Remember how to do it boys? Nice and neat! ...You can borrow my ruler, 'cause I like rulers in books...Beautiful work girls – must be all those vegetables you've been eating. (Mid Year 4)

Ms Fell: *(Roving and checking on children's work. To one child)* Ooh, I can see a book that's not ruled up.*(To another)* That's not how we set out our books, is it? Rub it out. (Mid Year 4).

Ms Washbourne: Quickly, quickly, quickly. *(To children who have been told to take out their maths books. Begins demonstrating setting out on the board)* Now, we have our side margin, and our middle margin, then we underline our day's work. We put the short date, underline it and put the name and number of the book you're working from...I'm coming around to check that everyone has nicely ruled up, neat books...We have some very snappy-looking maths books here. Well done. I've told you a million times, mathematicians are neat and orderly. (Late Year 5)

For children who were able to work neatly, ruling margins, writing dates, underlining and ruling off were satisfying, as their proud displaying of their books showed, and earned praise. For others, achieving the expected standard of neatness was difficult or even stressful, as shown in these examples:

*Toby was leafing through his maths book explaining the maths work he had done. Two of his classmates were looking on. Toby came to a page where Ms Firth had written beside an exercise, 'Very neat'. Toby and his friends discussed this activity.*

- Toby: This was really hard [an exercise in copying numbers].
- Marshall: Yeah, she told us she would rip our page out if it wasn't neat enough.
- Researcher: Did anyone get their pages ripped out?
- Pita: No, not that time but three people did in spelling, and this boy, he had his page ripped out two times! (Mid Year 3)
- Georgina: When I first got my book, I did it really neatly and Mr Solomon wrote, 'A really good effort but, one - get it marked, two - stick your sheet in, and three - ', what was the other thing? ..oh yes, 'Needs to be underlined'. And Mum said he's a dickhead because I did it really neatly. (Late Year 3)

The pressure and anxiety experienced by certain children through this kind of insistence on setting out and neatness, and a strong sense of injustice when genuine attempts to achieve it were perceived as not having been recognised by the teacher, were evident in these accounts. So strong was Georgina's apparent feeling of hurt, that she had scribbled over a later teacher comment in her mathematics book which read 'only one number in each square!'

Teachers' feedback to children through their mathematics exercise books was of three main types:

- 'marking' of 'correct' answers with ticks, often in red pen, and 'wrong' answers with crosses, although a number of teachers preferred to place a dot or the correct answer beside the 'wrong' or simply left wrong answers unmarked;
- written praise comments such as: 'Good work.' 'Excellent', 'Well done'. Stickers and stamps were also used for praise – smiley faces, stickers saying, 'Perfect', 'Special', 'Hooray!' 'Ka Pai';<sup>8</sup>
- written presentation comments such as: 'Neat work'; 'Good work, neat'; 'What happened to your margin?'; 'Set out carefully, please'; 'Go back and fill in the spaces over the page.'

<sup>8</sup> Ka Pai is a Maori expression for 'That's Good'.

Presentation comments were extremely common, and constituted almost the entire portion of teachers' written feedback to the children.

Mrs Waverley: (*In Peter's maths book*) 'Getting hard to read, Peter';  
(Later) 'Some lovely work, Peter, thank you – Mrs Waverley. (Year 4)

Mrs Meadows: (*In Fleur's maths book*) 'Try to keep your work neater' (Early Year 5)

Mrs Isles: (*In Georgina's book*) 'Please rule your book up correctly – page number!'

Ms Torrance: (*During a mathematics lesson*) Good ruling, Dominic! (Early Year 3)

Comments referring to the mathematical understanding of the tasks were virtually non-existent. 'Good thinking!' was seen once written by the teacher in a child's book, and was heard twice in teachers' interactions with children in the three years of observation. In none of these instances did the teacher elaborate by explaining why the thinking was 'good'. This is consistent with the findings of Smith and Glynn (1990) who observed in the classrooms they visited that 'there was very little written or spoken feedback from the teacher except regards to work presentation' (p. 133). This imbalance might very well convince a young person that mathematics work was more about form than substance. Clark (1999) and Hungwe and Nyikahadzoi (2002) examine the quality of teachers' written feedback on pupils' mathematics work and note the prevalence of what Hungwe and Nyikahadzoi term *superficial endorsement*, that is, marking of right and wrong answers without detailed, instructional, content-related comment to guide their students' learning.

The emphasis on neatness was not confined to mathematics, but applied in other subject areas where bookwork was required, such as handwriting. Teachers regarded neatness and setting out as part of the general focus of teaching in the early years of primary school, and considered this essential in the development of good work habits. Particular modes of setting out in mathematics have been promoted for decades e.g. a New Zealand Department of Education publication (Duncan, 1959) stresses that 'children should be required to set out their work neatly' (p. 12). However, setting out and working neatly, it could be argued, have little to do with a child's conceptual facility at mathematical tasks. In the classrooms observed, written mathematics tasks



usually required children to produce answers but not solutions. Neatness, therefore, was to be achieved simultaneously with mathematical thinking and working out. Being able to work neatly is not necessarily the same as being able to think mathematically. 'Untidy' workers attracted teacher criticism both publicly through verbal comments and privately, via written feedback in their books. Hungwe and Nyikahadzoi (2002) found that such negative comments were off-putting for students.

Children naturally develop at greatly varying rates in terms of the manual dexterity and fine motor control required for writing tasks. As we can see from the children's drawings, some children showed far greater control than others. The same can be seen in their handwriting. The expectation that all children should be able to set out and write neatly in mathematics exercise books, reinforced through frequent comment on the tidiness of children's mathematics 'work', would appear to be setting unreasonable standards for a proportion of children.

### **'Finishing'**

Teachers often cited the ability to work neatly in maths books, work quickly, and complete set written mathematics tasks within the given times, as indicators of the children's overall progress in mathematics. They seemed to value the completion of mathematics tasks above mathematical understanding.

Mr Loch: At the moment I'm finding it's taking time for some kids to settle down, settle into a routine. I've inherited some problems I think from other years. Standards haven't been set and kids just don't complete work and they're not used to getting, not used to actually getting through something. Finishing it off. That's something I'm very tough on. I like things to be completed. (Early Year 3)

Completion featured highly when they discussed children's progress in mathematics.

Ms Summers: (*Talking of Peter*) He's quite meticulous about the work he produces, he's quite methodical and I think he settles to a task quite quickly, he doesn't have to be encouraged, and he works consistently to complete a task. (Early Year 3)

Ms Summers: He's methodical, and he's always perhaps a little slow to complete. (Late Year 3)

Mr Solomon: Georgina, I had to separate out from the others, for about four or five weeks I think it was. I gave her a desk over there by herself. (*Points to corner of classroom*) She was just far too distracted and didn't finish or get on with her work. (Mid Year 3)

Mrs Wai: (*To the children who are slow to complete the examples given*) Who's up to number three? There are lots of people who are up to number three. (Mid Year 4)

Completion also featured in teachers' reporting to parents.

Mrs Joiner: (*Writing about Rochelle*) She is showing enjoyment with her maths and reading work and she is making excellent progress in these two areas and needs only a few reminders to complete set tasks. (Progress report for parents - Early Year 3)

Completion was reinforced during the teachers' interactions with the children during mathematics time:

Ms Summers: (*To Peter*) You've finished! Doesn't it feel good when you've done it? (Late year 3)

Mrs Kyle: How many finished? (*Looking around at the show of hands*) Most of you didn't finish. You must learn to put 'DNF' - did not finish, at the bottom.' (Early Year 4)

Ms Torrance: We have some amazing speedsters who have got on their rollerblades and got their two sheets done already. Will I be cross if you don't finish these? (*Indicating some extra worksheets she has given to the 'speedsters'*)

Children: (*together*) No.

Ms Torrance: No, these are just for speedsters (Class observation, early Year 3)

Children were frequently heard to compare their work rate with that of others when engaged in their written mathematics tasks, typified by this example:

Jessica: (*To Angela, sitting beside her*) I'm up to 10. What are you up to? (Early Year 3)

Teachers' expectations children's work output increased year by year.

Jessica: (*Explaining the difference in Year 4 mathematics*) We get harder work. We've got to get it done. (Early Year 4)

## Working solo

'Independence' was often mentioned by the teachers as something they valued highly in children's behaviour generally, and especially at maths time. Independence during mathematics sessions referred to the ability to sit alone, concentrating on the given task, asking few questions, and completing set work within the time allotted.

Mrs Joiner: Rochelle is developing good independent work habits. (Progress report for parents Early Year 3)

Ms Torrance: (*Talking of Dominic*) He's getting work habits that are really ...the standard's really high. (Late Year 3)

Ms Summers: I think Peter's very focused, a very focused learner and he's able to work independently. (Early Year 3)

Mrs Ponting: I'm using this new resource – *Figure it Out*. It's great because it gets them doing work for themselves. (Late Year 4)

Children were expected to work alone and in silence at mathematics time. 'Chatting' with friends was often viewed as being off task, and teachers expressed the belief that talk during mathematics time would 'distract' others or prevent them from thinking. When reporting on the children's progress, their comments illustrate this.

Ms Torrance: He's [Dominic] quite happy to talk about it [mathematics] and he enjoys the hands-on, but the recording thing much less so. I think he would prefer working in a group, but he can work well on his own. I would prefer him to work on his own. Independent tasks, he's not the best; he's very chatty. (Mid Year 3)

Mrs Ponting: She [Rochelle] works better sitting away from her little friends. She's got it in her but lets others do the thinking for her. (Late Year 4)

Mr Cove: He [Toby] sits in that corner over there with three other boys. Let's say I have to have my eyes on that corner because they're very sociable. (Mid Year 5)

Toby's mother: The only problem I hear [from the teacher] is sometimes he chats a bit with his mates. (Late Year 3)

Talking while 'doing maths' was often actively discouraged by the teachers.

Mrs Ponting: Excuse me!(*To two boys who are sharing one textbook because there are not enough to go around, and whose 'talk' has been observed to be entirely involved in the mathematics of the task, with extremely effective exchange of ideas to clarify and reach solutions to the questions*) What are you two boys doing together? I want to know what you can do on your own! (*To another child*) Here's another boy who didn't do his own thinking and copied off his neighbour. (Mid Year 4)

Ms Sierra: You're supposed to do your own work, OK?...I don't want you talking, I want you to concentrate.  
(Later) Charlotte cannot concentrate because you people are too noisy. (Early Year 4)  
(*To the group who have been sent to work independently*) You are getting too loud. Well done to you people who are sitting at your desks doing your work. (Mid Year 4)

Ms Fell: If you're busy chatting, you can't be working. (Mid Year 4)

Mrs Waverley: Do your own, please. It's your brain. (Late Year 4)

The sign on the Keeping Skilful Box in Rochelle's classroom (Figure 27) provided a strong message about how the teacher expected the children to work at independent mathematics tasks:

<p><b>Easy! Easy! Easy!</b></p> <p><b>Keep Silent!</b></p> <p><b>Be neat!</b></p>
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**Figure 27: The sign on the Keeping Skilful Box, Bridge School, Mid Year 5**

Children themselves were sometimes observed to object to sharing with others, interpreting this as 'cheating'.

Joel: (*Angrily, to Liam whom he suspects of looking at his worksheet*) Don't copy off me, you cheater!

This unwillingness to share (or be exposed) sometimes extended to children's shielding of their written work with their free arm. Dominic for example was observed doing this often in Year 3.

However, not all the children said they wanted to work with others.

Mitchell: By myself. Because it's fun by myself. (Late year 4)

Peter: By myself. Because they're more noisy. (Early Year 5)

When the children were asked whether they preferred to work alone or with others, the majority said that in certain circumstances they preferred to work with others and explained why.

Researcher: Do you like doing maths by yourself or having someone to talk to?

Fleur: Well I don't really cooperate that good. When it's something hard, I like doing it with someone who knows how to do it. If someone's, like, really poor at it, and I know I'm really poor at it, well I don't want to do that, I want someone who knows what they're doing, or if they've just got a tiny bit of an idea. (Mid Year 5)

Georgina: With other people. 'Cause when I work by myself I get really bored. 'Cause I don't really get it, like what ...I ask them the question. (Mid year 5)

Jessica: I like doing it with, um, someone else. Just one person.

Researcher: What happens if it's more than one?

Jessica: Well, it would work but I don't really like working in groups.

Researcher: Are you usually allowed to work with someone else?

Jessica: No. We mostly have to do it by ourselves. (Late Year 5)

Rochelle: With other people. Every time I don't know the answer, I just ask them. (Early Year 5)

Dominic: Probably with someone else, so they can help me. But that's only sometimes that I need to ask someone for help. (Late Year 5)

Jared: With someone else, 'cause if I get one wrong, they might know it. (Early Year 5)

Liam: By myself, unless it's hard then someone can help me. (Early Year 5)

Peter: Talking about it.

Researcher: Why do you like talking about it best Peter?

Peter: Because it's easier than writing it down. (Late Year 4)

Researcher: Do you like maths best when you're talking about it or writing about it?

Toby: Talking about it.

Researcher: Are you allowed to talk about it when you're working?

Toby: I'm not sure.

- Researcher: Does Mrs Kyle say, 'I'd like you to talk about the maths while you're doing it?'
- Toby: No, she doesn't say that. Sometimes she says, 'People can't get on with their work so ten minutes' silence.'
- Researcher: I see.
- Toby: That's when you *never* get to talk. (Mid Year 4)

These comments showed that the children valued the support that could be gained from working with others, particularly when they were unsure of the answer or method. In many classrooms, this support was routinely denied. In explaining the development of mathematics anxiety in three case studies, Seaman (1999) reflects that 'math has often been treated as a solitary subject' in which one is 'relegated to working in relative personal isolation' (p. 2). Lemke (1990) also notes the ways in which teachers 'ignore students' needs to communicate with one another' (p. 78), and adds that 'viewing learning as an essentially individual process, and ignoring social dimensions, helps rationalize holding individuals solely accountable for their own right and wrong answers, their own success or failure at learning' (p. 79). The insistence on the part of the teachers that talk would inhibit rather than enhance the children's learning is not supported by the views of learning theorists such as Vygotsky (1978) whose research led to his conclusion that language was an integral part of a child's approach to solving problems. He noted that 'children solve practical tasks with the help of their speech, as well as their eyes and their hands' (p. 26). He believed that for children there is a 'fundamental and inseparable tie between speech and action in the child's activity' (p. 30).

Although children were usually grouped for mathematics, the purpose of the groups was not to enable children to work collaboratively, rather it was to allow for the teaching of homogeneous ability groups who were set to work on common group tasks but as 'independent' individuals. It seemed that children's working together at maths time was not an everyday feature of most of the classrooms. In five of the thirty-five classrooms visited, cooperative learning of mathematics was observed, although this was far from usual. Mrs Linkwater explained that while she often used cooperative learning in other subject areas, this was the first time she had tried it in mathematics. Later interviews with Rochelle revealed that this approach was used only once that year. In another classroom where a cooperative group statistics activity

was observed, the children said that they had never done maths like that before, so it seemed likely that this teaching approach had been chosen because the lesson was to be observed. On one visit, Liam's group was sent out to measure the perimeter of the school grounds. In the final interview, Liam said that this was the only occasion in the year that he had ever worked that way. Another classroom used a group problem-solving approach only during the annual Maths Week. In Dominic's classroom, activities from the cooperative learning mathematics resource *Get it Together*<sup>9</sup> were being used for several weeks early in the year as part of a three-group rotation system. Dominic said he enjoyed this kind of collaborative problem solving. When this activity was observed, it was found that there was no apparent establishment of practices to enable the children to share their thinking and justify their ideas, and very little teacher interaction during the group problem solving process.

Verbalising mathematical ideas is strongly advocated by *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) in the belief that it is an essential process in the learning of mathematics. It states that children must be provided with opportunities to 'become effective participants problem-solving teams, learning to express ideas, and to listen and respond to the ideas of others' (p. 23). Many studies have examined the place and effects of discussion and collaboration in mathematics learning (e.g. Gooding & Stacey, 1993; Wood & Yackel, 1990). Thomas (1994) examined the types of discussion that occurred when children in junior classrooms were working on independent group tasks and found that much of their talk was related not to the mathematics of the tasks, but to the requirements of social interaction. Such authors believe that effective discussion depends upon the quality and requirements of the tasks and the establishment of norms that promote the kinds of talk that enhance mathematical learning.

Research suggests that cooperative group work has significant positive effects on children's learning in mathematics. (Webb, 1982; Davidson, 1985; Slavin, 1988; Jacobs et al, 1996; Leikin and Zaslavsky, 1997). The infrequency of cooperative group work for mathematics in the study classrooms suggests that although there were occasions when children were permitted or encouraged to work together, these were 'special' times rather than the norm, and something the children regarded more as

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<sup>9</sup> (Erickson, 1989).



having fun than doing real maths. Higgins (1998) has documented the arguments about independent group work in children's first two years at school, concluding that teachers' non-interventionist approach to independent group work can result in masking of mathematics learning for the children, through the non-mathematical demands of the tasks and the unwillingness of the teachers to 'intervene'. However, *Mathematics In the New Zealand Curriculum* (Ministry of Education, 1992) notes that 'techniques that help to involve girls actively in the subject include ...assigning co-operative learning tasks'(p. 12).

Mendick (2002) found that secondary students she interviewed believed discussion was unnecessary in learning mathematics. As one student stated 'well, you do the discussion but if you know the answer, you know the answer and then there's nothing to discuss' (p. 383). She points to epistemological reasons for this view. The dominant approach to mathematics, she argues, holds that mathematics is an external body of knowledge and answers are therefore more important than processes. She suggests that 'it is only by moving to an understanding of maths as a social practice that discussion becomes an integral part of doing maths. Oral contributions are not judged whether they are right or wrong, but in terms of their value in furthering the collaborative social activity of doing maths.'(p. 383). The dichotomous right/wrong view of mathematics was also a feature of the classrooms of this study, and will be more fully discussed in Chapter 7. Heibert et al (1997) are strongly critical of the 'work alone' tradition and see that assessment practices may predicate mathematical activity as solo.

Traditional forms of instruction often encourage, and even require, students to work alone. Working together or using the suggestions of a peer have been discouraged. Students are supposed to do their own work and not rely on others. This concern may result, in part, from the importance that has been placed on individual performance. We believe this concern, which has sometimes become an obsession, has had a destructive effect on the climate and culture of mathematics classrooms...doing mathematics is a collaborative activity. It depends on communication and social interaction. (p. 44).

### **Working from textbooks**

Early in Year 3, textbooks were becoming a regular part of mathematics sessions in some classrooms, and by Year 5, were widely used. They were often, although not

always, used by the teacher as follow-up to some kind of group teaching. In many instances, teachers would write the work to be done on the board, noting the book and the numbers of the pages the children were to complete, as in Figure 28.

- |   |
|---|
| <ul style="list-style-type: none"> <li>* <i>Number Nibbles</i></li> <li>* <i>Flower Power</i></li> <li>* <i>NCM Page 259 - Exercise One plus practical</i></li> <li>* <i>NCM Page 262 - Exercise Two</i></li> </ul> |
|---|

**Figure 28: Textbook exercises set by the teacher (from the board, River School, Late Year 5)**

At other times, usually after a new process had been demonstrated, the teacher would verbally direct the whole class to work from a page in the book, as the following example shows:

Ms Washbourne: *(After she has demonstrated a place value concept on the board)* Turn to page one hundred and seventy-one, and you can do these in your books this time. Have a look at the numbers in the book. (Late Year 5)

Teachers explained their use of textbooks in the following ways:

Mr Cove: *(Explaining what he would do after he had taught the children a new concept)* Then maybe they would have understood it and I would be able to say, 'If you get that book out, on page 21 there's the next part of what we did yesterday', and they'd be able to do it. Then I'd try to find another book or some more sheets for children who needed some extra pushing, or maybe some children who were having trouble. (Mid Year 5)

Researcher: What are the resources that you'd use most?

Mrs Ponting: Quite a few ones that I've made myself like the computation cards, a little bit of the School Mathematics booklets, but not a lot. It's very airy fairy. I quite like the old M.S.M.s<sup>10</sup> for pages for practice, hm, and quite a few worksheets.

Mr Waters: They love using the books [textbooks], you know, those old books I would have used at school. It does sort of trick them into learning maths and then you give them a book and they think now they're actually doing some maths. (Early Year 5)

<sup>10</sup> *Modern School Mathematics*: Department of Education (1983).

Mr Waters reflected the children's belief that such work was 'actually doing maths'.

The following examples from Jessica's Year 3 class, Rochelle's Year 4 class and Fleur's Year 5 class, illustrate the different ways in which textbooks were used, and the questionable effectiveness of textbook work in children's learning of mathematics.

### **Jessica and *School Mathematics 2* – a 'group' activity**

From videotape, Lake School, Mid Year 3

*Jessica's group is working on the concept of multiplication on the mat with the teacher. Jessica is working with a partner placing plastic counters into circles drawn on small chalkboards. Mr Loch asks the children to write the 'adding' sentence to match the arrangement of counters in the circles, then the 'times' sentence. At first he records these on the board then asks the children to write on pieces of paper. Jessica's partner does the writing. Mr Loch hands out copies of the *School Mathematics 2* textbook and asks children to turn to page 74. He then asks them to turn to the next page in their maths books. He checks that they have ruled off after their last work, and gets them to write SM2 and the date at the top of the next section of work. The children then return to their tables. They have to sit in the same place each day. It would seem that moving around is not encouraged as their places at the table are marked with stick-on name tags. The first examples from page 74 of *School Mathematics 2*, match the activity that the children were doing on the mat. Jessica quickly completes the addition, then the multiplication sentences that match the pictures of bottle tops in circles in the book. (Questions 1 – 10) From question 11 onwards, the pictures change. Groups of coloured blocks are shown on a number track. Jessica is clearly confused and unable to continue. She looks at the work of Angela, the girl opposite, who is writing quickly, now up to number 15. Jessica then looks at the book of Charles beside her, who has also reached number 11. Harry on the other side has worked out how to continue and is working busily. Jessica, still unsure of how to proceed, marks time by writing the question numbers 11 to 17 down the page. She looks around once more, then places circles around all the question numbers. Still uncertain, she records the questions themselves beside each number, but leaves out the answers. She looks once more at the others' work.*

Jessica: (To girl opposite) Angela, can you do that? (Indicates the picture of the blocks)

Angela: Yeah, it's easy. I'm up to there. (Points to question 18 in the book and goes back to her work).

*Jessica reads the instructions in the book, frowns, mutters to herself, looks around, plays with her rubber, drops the rubber on the floor and picks it up, talks briefly to Charles next to her, looks at her book once more. Jessica has been stuck now for 15 minutes. Arlo who has been working alone at another desk comes over to tell Harry that he has finished. By this time, Harry has almost finished as*

well, although Charles is also stuck at the same place as Jessica and losing focus. At no stage does Jessica directly ask for help from other group members or the teacher. Helping one another does not appear to be either encouraged by the teacher or practised by the children. Mr Loch, who has been working with Group 2 on the mat during all of this time, does not ask Group 1 how they are getting on, or come to check their work. Jessica has become restless and appears bored.

Mr Loch: OK, I want Group 1 to pack up now please. (*Jessica looks relieved and quickly closes her books.*)

In this instance, the teacher was using the textbook work primarily as a management device, enabling him to concentrate on teaching another group. Not all the children understood the task or could follow the written instructions, and because collaboration was discouraged, children such as Jessica were left to flounder. It may have been more productive for the children to continue their exploration of multiplication using the chalkboards and counters, recording their addition and multiplication sentences on paper for later discussion or display. This record of their learning could have been used for future reference. Instead, their work remained concealed inside their books.

### **Rochelle and *Figure it Out* - a 'whole class' activity**

From fieldnotes, Late Year 4

*10.00 am. The children have finished the daily starter activities of Quick 20 and individual computation cards. Mrs Ponting now asks the whole class to take out their Figure it Out Level 2-3 Algebra textbooks, and get on with page 6. This page is entitled Fair and Square. There are 5 different sequential coloured tile patterns drawn in boxes on the page, with questions based on the relationships between the coloured squares. There are five questions on the page. Mrs Ponting does not introduce or discuss the ideas.*

Mrs Ponting: Those clever ones, when you've finished page six you can go on to page seven.

*Rochelle is up to number 3 on page 6. She has solved numbers one and two by drawing pictures. She now stares at the book, frowning. Number 3 involves a pattern of blue and pink square tiles orientated with their corners pointing to the top and bottom of the page. Three elements of the pattern have been drawn, and underneath it says: 1 blue square, 2 pink squares; 2 blue squares, 3 pink squares; 3 blue squares, 4 pink squares. If there are 21 pink squares, how many blue squares?*

*10.10 am. Rochelle is still puzzling over number 3. She is stuck. Emily beside her has now almost finished, and shields her book from Rochelle with her arm. The teacher has been helping a group on the other side of the room, with a commentary that can be heard around the class, for example:*

Mrs Ponting: *(Loudly, to a boy)* Excuse me! You're not even thinking to make a stunning error like that.

10.20 am.

Rochelle: *(Looks at Emily)* Emily, would you show me how to do it? *(Emily briefly uncovers her book so that Rochelle can see it, but says nothing. Rochelle now draws some squares like Emily's. She then rubs them out)*

10.25 am:

Mrs Ponting: *(Roving around to see how children are progressing)* Right Rochelle, how are you going?

Rochelle: *(In a whisper)* I can't draw diamonds.

Mrs Ponting: You haven't got enough room? Never mind. Have a go at that one *(Points to number 4. Loudly to the whole class)* Well, some people have finished that page. Rory, wake up!

10.26 am. *(Researcher asks Rochelle if the class has any coloured squares blocks, pointing to the top of the page where it says: 'You need: - Square tiles'. She says they don't. At researcher's suggestion, Rochelle turns her maths book so the squares on the page now look like 'diamonds.' She can only draw 13 pinks/12 blues across the page. Researcher asks Rochelle how many blues she thinks there would be if she could keep drawing until there were 21 pinks.*

Rochelle: Twenty!

10.30 am:

Mrs Ponting: Right everyone, put your books away now would you? It's playtime.

Without the researcher's interaction, Rochelle would have achieved little during the 30 minutes' independent work time. By maintaining a 'work' pose - head down, not talking, looking at the book - she had convinced the teacher that she was on task. Conversations with Rochelle show why the textbook session may have been an ineffective learning experience for her:

Researcher: Which of these three ways helps you learn best do you think? Writing it, talking it through or using real things?

Rochelle: Talking with other people about it.

Researcher: Are you allowed to talk to other people about your maths?

Rochelle: Sometimes.

Researcher: Would you rather do maths with other people?

Rochelle: Yes.

Researcher: What real things have you used in maths this year? Have you used any real things?

Rochelle: Um ... *(Thinks)* No. *(Late Year 4)*

Researcher: What do you do when you don't understand something in maths?

- Rochelle: I don't ask the teacher. I take it home and ask Mum.  
Researcher: Why do you do that?  
Rochelle: Because it's embarrassing asking the teacher. (Early Year 5)

Given the opportunity to model the tile pattern with coloured squares as the instructions suggested, and to talk it through with a partner, would certainly have enhanced Rochelle's understanding of the mathematical ideas involved in the problem, including her view of the squares as 'diamonds'. Throughout the three years of the study, Rochelle routinely adopted unobtrusive conforming behaviour at maths time and received teacher praise for her work habits. Mrs Ponting described her as 'a quiet little mouse' and noted that she was reluctant to engage verbally at maths time, but was generally very satisfied with her progress. Rochelle was successful at learning and applying routine mathematical operations, and felt safe doing this. However, when presented with unfamiliar mathematical tasks in written form, such as the one above, she was hampered by her limited experiences of doing maths.

This situation is very similar to that of Jessica. Both children came to a standstill when faced with textbook questions that had not been introduced by the teacher and which they could not interpret alone. Neither received help. In both cases, because working collaboratively and using real materials were discouraged, the children were left without strategies to enable them to continue.

These events were by no means isolated. Throughout this study, many similar situations were observed where children who had been instructed to work 'independently' at written tasks, experienced difficulties in comprehending those tasks because there were inadequate structures built into the classroom procedures and management for briefing, checking, guiding or providing support. In many instances, the teachers were quite oblivious to the children's difficulties.

As Table 4 (p. 199) shows, textbook use was encountered by most of the children in the study, and had become a regular feature of eight of the ten classrooms by Year 5. The purpose and basic format of mathematics textbooks have changed little in more than a century. They are usually designed to reinforce knowledge about mathematical concepts and procedures that are considered essential for the student, and to provide exercises or examples for student practice and application of these skills.

Mathematics textbooks support and perpetuate a number of assumptions about 'doing maths.' They not only reflect but also reinforce traditional and time-honoured protocols of mathematical work. One of these assumptions is that the pupils will work on their own, and that they will record answers in their mathematics exercise books. The popular and recently published *National Curriculum Mathematics* (NCM) textbooks for example, interspersed with pictures and the occasional 'investigation', replicate the format of textbooks from the early twentieth century. Numbered lists of 'exercises', in the form of questions are provided for the children to answer and constitute the bulk of 'work' children are expected to do. It is not expected that the children will copy the questions. This creates the need for the book and page number to be recorded so that teachers or children can mark these exercises from the answers that are provided as part of the textbook resource. Children must be trained, as Jessica and Rochelle demonstrated, to record the question number in the margins of their exercise books, with the answers alongside. Because questions and answers become separated in the process of working from a textbook, it is difficult for children to crosscheck questions with answers in the marking process, let alone recreate the thinking that produced the answers in order to analyse errors. In addition, if answers only are recorded, the exercise book has little usefulness as a record of learning, either for child, teacher or parents. This method of teaching and learning mathematics from at least Year 3 of primary school has remained largely unchanged for decades.

Mathematics textbooks can be regarded as significant agents within the social worlds of schooling. Until recently, little attention seems to have been directed towards the role of texts in mathematics teaching, apart from studies that have analysed the representations of gender in mathematical texts. (e.g. Department of Education, 1980; Northam, 1982). Bierhoff (1996) compared the textbooks from Germany, Switzerland and England, concluding that the German and Swiss texts differed from the English in their use of problems designed to promote children's learning of mathematical concepts. Dowling's (1998) sociological analysis of mathematical texts focussed on 'the ways in which school mathematics is established as a set of practices ...and the divisions and distributions within mathematics and between mathematics and other practices'(p.1). From an international survey of the images presented in mathematical textbooks Harries and Sutherland (cited in Harries and Spooner, 2000), concluded that textbooks reflect predominant views about the nature of mathematics,



the parts of mathematics that are important for children to know, how mathematics is learned, and how it should be taught. Harries and Spooner (2000) comment that 'textbooks play an important role in influencing the ways in which teachers think about teaching and learning mathematics...what appears in a mathematics textbook does not appear by chance. It is influenced by the multifaceted aspects of an educational culture' (p. 46). As cultural artefacts, textbooks reinforce dominant approaches that construct 'doing mathematics' as academic work.

The recently-published *Figure it Out* series of curriculum support texts, developed through the New Zealand Ministry of Education and intended for use from Years 3 to 6 (1999, 2000) continues the tradition of the mathematics textbook. While this resource does include a number of activities that require a partner or use of concrete materials, it also contains many lists of examples that imply formal recording of answers, and in many instances, explicit instructions are given to the children to do so in their 'books', assuming that every child from at least Year 3 onwards, will have such a book. Writing in 'your book' in response to externally imposed tasks can clearly be seen in these instructions, as one of the taken-for-granted of the mathematics classroom. Here are just a few examples:

Copy the tables into your book and fill in the blanks. (Level 2-3, *Algebra* p. 7)

Using the addition array above, copy the following equations into your book. (Level 2-3, *Basic Facts*, p. 7)

Write down the facts you do not know in your books. (Level 2-3, *Basic Facts*, p. 12)

Besides encouraging the continuation of the individual writing tradition of mathematics 'work', textbooks assume that children will read and interpret instructions and diagrams as the author intended. Campbell (1981) and Santos-Bernard (1997) cited in Harries and Spooner (2000) conclude that children interpret textbook illustrations differently from adults, and that this can lead to confusion where children may take illustrations literally, rather than as the representations intended. This confusion was well-illustrated by the experiences of Jessica and Rochelle (pp. 213-216), who were bewildered both by the language and the diagrammatic representations in their textbooks. Rochelle was baffled because she viewed the squares in the textbook question as diamonds. Jessica could not make



sense of the picture of rods in a number track, having never seen them used to model the concept of multiplication.

Textbooks presume a shared understanding of the purposes and procedures of 'doing maths'. They assume a special role in the classroom, 'speaking' to the children in the authoritative 'voice' of the sage and the imperative tone of the taskmaster. Because textbooks are written by adults who inevitably possess entirely different views of the world to those of the children using the texts, confusions are inevitable. Bishop (1991) asks the question:

'whose are these books? Who writes them and for whom, and why? Does the author know the learners who will use them or the teachers who will teach from them? ... That the textbook controls is well-known ... The control by the textbook therefore effectively prevents the teachers from knowing their learners and thereby prevents them from helping their learners effectively.' (pp. 10-11)

The children described their views of textbook work:

Georgina: It's OK [working from the textbook]. And we can go to the back of the book and we say, 'there's the answer.' (Mid Year 5)

Researcher: What is it that you like about working out of the book [NCM]?

Jessica: Well, it's like the book's already there and all you have to do is write the answer. (Mid Year 5)

Researcher: And you work out of those red [SM] books. How do you feel when you do that?

Rochelle: Sometimes it's really hard because she gives us different pages every day. (Early Year 5)

Researcher: What's the most important part of maths time do you think?

Dominic: I would say the most important part is *Figure It Out* or NCM.

Researcher: Why is that, Dominic?

Dominic: Because that isn't like games or easy stuff, it's getting right into it.

(Later)

Researcher: What don't you like about maths?

Dominic: Sometimes NCM because it's just really boring.

Researcher: Which bits are boring?

Dominic: Usually the stuff where we have to make sentences and copy stuff. (Late Year 5)

## Worksheets

Worksheets from a variety of commercial sources were frequently used in the classrooms. Teachers also designed their own worksheets. Most worksheets seen in the classrooms took the form of lists of closed questions with gaps to be filled with answers. In some of the classrooms, worksheets were used almost every day. The following comments typified the children's views of these worksheets.

Fleur: I like those... Usually they're fun. (Late Year 3)

Georgina: Worksheets? Good, 'cause some are easy. (Late Year 3)

Researcher: Which is the most important bit [of maths]?

Jared: Worksheets.

Researcher: Why do you think that's the most important bit?

Jared: Because it gives you more to learn.

Researcher: OK. So how do you learn off those worksheets, Jared?

Jared: They don't say the answers and they test you.

Researcher: Yes? And what if you don't know?

Jared: We just put in an answer. (Early Year 5)

Liam: *(When asked how he feels about worksheets)* Good, because I'm allowed to ask for help [from the teacher]. (Early Year 5)

Toby: We're doing worksheets for graphs now and I like that. (Mid Year 4)

Like textbooks, commercially produced worksheets were not tailored to take account of the interests, experiences or even the physical characteristics of the children in the classrooms. As a result, problems arose, as the following observation illustrates.

From fieldnotes, Island School, Mid Year 4

*Mrs Cayo has given the children in Georgina's group a worksheet instructing them to draw the 'mirror image' of a snowman. The snowman is pictured on the left-hand side of the page with a dotted vertical line separating it from a space on the right-hand side where the mirror image is to be drawn. The teacher has provided the children with small rectangular mirrors. Holding the mirror on the line Georgina tries to draw the reflected image behind the mirror but abandons this method and draws freehand. Mrs Cayo approaches.*

Mrs Cayo: *(Sharply, to Alan who is sitting beside Georgina)* Put it there. *(Places the mirror on the dotted line)* Keep it there. *(To Georgina)* That's not the way to do it. Rub that one

off please. (*To the whole group, demonstrating*). Hold it with one hand and draw with the other one. You *must* hold the mirror there all the time.


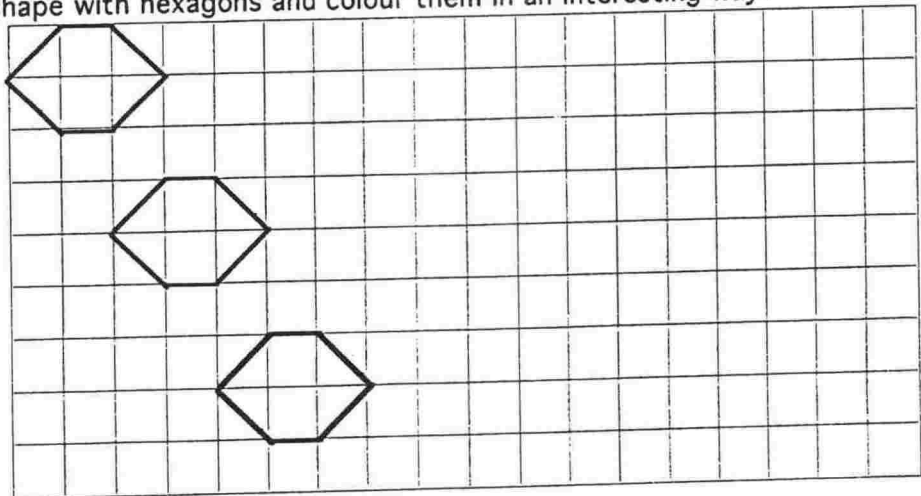
*Mrs Cayo leaves and Alan is now crying. It quickly becomes obvious why this exercise is so difficult for him. He is left-handed as is another boy in the group, who is also finding the task virtually impossible. The worksheet has not been designed with lefthanders in mind, and the teacher has not picked up on why these children are unable to complete the task as instructed.*

It was observed that worksheets were often given to groups of children to complete alone while the teacher was engaged with another group. Little guidance could therefore be given to the children, and, as the example above shows, even when teachers did check, the underlying mathematical ideas were rarely discussed.

Teachers had not always come to grips with the mathematics of commercially produced worksheets before assigning them to the children, it was noted. They were sometimes observed to be struggling to understand the tasks, resulting in misunderstandings or limited learning for the children.

From fieldnotes, Spring School, (Late Year 5).

*Each of the children in Jared's class have been given a worksheet (Figure 29).*

Pinada Blackline Masters Level 3 Book 2 Geometry I page 3...	Name:- _____
	
<b>Tessellations</b>	
Hexagons are fun to make tessellating patterns with. Can you fill in the shape with hexagons and colour them in an interesting way?	
	

**Figure 29: Tessellation worksheet, Spring School, Late Year 5**

*Jared ignores the instructions and looks at the work of the boy sitting next to him who has started to draw a hexagon to the right of the top one, with only corners touching and sharing no sides. Jared copies this. He repeats this process all the way across the grid and then does the same with the other two hexagons. The three rows of hexagons he has drawn are touching only at the corners and the spaces between the hexagons are not of a regular shape. Because the design he has produced does not follow the instruction 'fill in the shape with hexagons' it is not the tessellating hexagon design intended but a repeating pattern created by horizontal linear translation of the hexagon*

Researcher: (To Jared) Is that the only way to do it do you think?

Jared: Yes.

*Most of the others have done the same, including the teacher, who is helping one of the children. One child, Danielle has begun to draw hexagons dovetailed together, creating the beginnings of the tessellating pattern. Noticing that the work of the three boys at her group is different, she begins to erase her design, no doubt believing that her method is wrong. The researcher asks her a question.*

Researcher: (To Danielle) Which is better, your pattern or theirs do you think?

Danielle: (After some thought, pointing to her own) This, because there's no gaps.

Researcher: What might your pattern look like if you keep going?

*Danielle regains faith in her method and starts once more to dovetail the hexagons. Soon she has finished the design of tessellating hexagons and is visibly excited at the honeycomb effect. Other children come to look at it.*

Sarah: Oh cool!

*Jared goes over to look at Danielle's design. When asked how his compares with hers, he doesn't respond. By this time he has 'finished' the worksheet and seems unwilling to reflect on what he has done. Jared takes his work to the teacher who directs him to colour it in.*

There were problems for both children and teacher with this worksheet. Neither the resource nor the teacher suggested that the children initially experiment with tiles or blocks to develop an understanding of 'tessellating'. Without the opportunity to trial different hexagon configurations, the children were confined to the difficult process of drawing and erasing their attempts.

By Year 3, worksheets were such familiar fare for Jared that they had become synonymous with doing mathematics as his drawing showed (Figure 6). His view of the purpose of the worksheets is shown in the following responses.

Researcher: What was it you were doing this morning? [addition worksheet]

Jared: Filling in gaps.

(Later)

Researcher: What do you do when you don't understand something in maths?

Jared: I fill in the gaps. (Early Year 3)

Because worksheets did not encourage or allow for children to record their thinking, Jared had interpreted mathematical activity merely as a gap-filling exercise, a view he continued to hold over the following three years.

Commercially produced worksheets of this type were designed to by-pass the use of concrete materials. As this example shows, illustrations or diagrams of concrete materials are not able to provide children with the kinds of practical experience that would foster mathematical understanding. Dominic, Fleur and Toby were all observed at different times, to be engaged in worksheet activities picturing rings on the three-bar abacus. When asked whether they had actually used the abacus beforehand, they said they had never seen one.

Researcher: (*Looking at Dominic's maths book.*) OK, you've drawn some abacuses. Did Mr Ford get out some abacuses for you?

Dominic: No, we just drew them, then he just drew them up on the board. (Mid Year 5)

In all cases, these children found the illustrations of this unfamiliar apparatus confusing. Georgina on the other hand, had experienced the use of a three-bar abacus as her self-portrait shows (Figure 24). Her enjoyment of the activity produced an enduring positive memory as well as retention of the mathematical understanding she had developed in the process.

### Working from the board

From videotape, Spring School, Late Year 5

Mr Waters: First of all this morning we're going to put up the title (*Writes 'Problem Solving' on the board*) Underline it and miss a line. See if you've got your brains into gear. (*Writes the first pattern on the board: (1) 2,4,6,8, , ,*) A nice easy one to start off with. What you're going to do is complete the number pattern. (*Writes: (2) 3,6,9, , ,*), Fill in the numbers and continue it on. Maths is patterning, that's all it is. Complete the whole number pattern. (*Writes: (3) 5, 25, 45, 65, , ,*) They're going to get harder and harder. (*Looking at a child's work*) There's no need to write the boxes, the boxes on the board represent the ones that are in your book.

Make sure you have the most important piece and that is the comma between, if you don't, your numbers will represent something else. You must set them out properly.

*(The lesson continues in this way, the teacher explaining and writing examples on the board, the children writing in their books, individually, in silence. When they have finished, Mr Waters calls for answers and the children mark their own work.)*

Besides emphasising the protocols of setting out, Mr Waters was reinforcing the rules, routines and nature of mathematics learning as 'work.' Through his use of the terms 'you're going to', 'you must', 'you don't', 'make sure', he emphasised that all the children were expected to follow the same very particular rules.

This lesson was typical of the whole-class mathematics teaching observed. Teachers were able to maintain tight control during these sessions, and delivered powerful messages about what was meant by 'doing maths'. This management of classroom work is consistent with the observations of Doyle (1988) who described work in mathematics classes as a process in which, 'teachers affect tasks, and thus students' learning, by defining and structuring the work that students do, that is, by setting specifications for products and explaining processes that can be used to accomplish work' (p. 169). He argues that much classroom mathematics work is of the structured and familiar variety, and that 'such work creates only minimal demands for students to interpret situations or make decisions within the content domain' (p. 173). Doyle expresses concern about the meaning of the work students do in mathematics classrooms, by arguing that teachers often emphasise production at the expense of understanding, claiming that 'meaning itself is seldom at the heart of the work they [students] accomplish' (p. 177). In an earlier study Doyle (1983) explained 'doing mathematics' as an induction into the world of academic work. He estimated that 'in general, 60 to 70 percent of class time is spent in seatwork in which students complete assignments, check homework, or take tests' (p. 179).

### **Concrete materials**

At the beginning of Year 4, the children were asked to rate, on a 0 – 10 scale, how frequently they experienced different work modes at maths time, 0 indicating 'none', and 10, 'lots'. Over the three years of the study, they were also asked to rate

mathematics activities on a 0 -10 enjoyment scale, 0 being 'really hate' and 10, 'really like'. Their responses are presented in Table 5.

**Table 5: Children's perceptions of work modes: classroom frequency and personal preference**

**Key:** ✍ = individual written work, 👂 = teacher-led session, 🙋 = putting hand up to answer a question, 😊 = group work & discussion, ● = using concrete materials

Child	High frequency activities (Early Year 4)	✓ Most preferred ✗ Strongly disliked		
		Year 3	Year 4	Year 5
Fleur	✍ 👂 🙋	✓ ✍	✓ 😊 ●	✓ 😊 ●
Georgina	✍ 🙋	✓ ●	✓ 😊 ●	✓ ✍
		✗ ✍	✗ 👂	✗ 👂
Jessica	✍ ●	✓ ✍	✓ ● 👂	✓ 😊
		✗ 🙋	✗ 🙋 😊	✗ 🙋
Rochelle	✍	✓ ✍	✓ ✍	✓ 😊 ●
		✗ 🙋		
Dominic	👂 ✍ 🙋	✓ ●	✓ 😊 🙋	✓ ✍
		✗ ✍	✗ ●	✗ 😊
Jared	✍	✓ ● 😊	✓ ✍	✓ ● 😊
		✗ 🙋	✗ 😊	
Liam	✍ 🙋	✓ ● 😊	✓ ✍	✓ ✍
		✗ 👂	✗ ● 😊	✗ 😊
Mitchell	✍ 🙋	✓ ●	●	✓ ● 😊
		✗ ✍	✗ 👂 🙋	✗ 👂 🙋
Peter	👂 ✍	✓ ✍ 😊	✓ 👂 ●	✓ ●
		✗ 🙋	✗ 🙋	✗ 🙋
Toby	✍ 👂 🙋	✓ ✍	✓ ✍ 😊	✓ ✍ 😊
				✗ ●



Children's reports of frequency of work modes were relatively consistent across the ten classrooms. The use of concrete materials was rated 0, 1 or 2 in eight of the classrooms. The activities reported as most frequent were individual written work, raising hands to answer questions, and teacher-led sessions. Restricted use of concrete materials and discussion while the children were 'doing maths' was found in the majority of the classrooms, yet these were the work modes most commonly preferred by the children.

While two of the Year 3 classes were found to be extremely well-equipped with mathematical apparatus which was well-organised and readily accessible, in the other classrooms mathematics equipment was scarce, worn, unattractive, or poorly organised. Where concrete materials were used, it was mainly during the teacher-directed group learning sessions. Table 4 (p.199) shows the gradual disappearance of equipment from mathematics work times as the children got older, as Dominic noted:

Dominic: We don't use them [concrete materials] much any more (Late Year 5)

Table 5 shows that while a number of the children preferred to work in this mode, they often associated use of equipment with the 'bottom' group.

Rochelle: Ms Linkwater has a working group, and that's the people who aren't so good at maths, but I'm not in that, and they work with blocks and all that. (Early Year 5)

Jessica: When we were in Ms Maine's [lowest group] we did, [use concrete materials] but if you're in Ms Mere's, [middle group] I doubt we will. (Mid Year 5)

On the occasions where children were seen to be working with equipment, there was almost always an accompanying written component to such lessons. As their reflections show, teachers believed that the written form of mathematics was the most important part, or even that this *was* the mathematics, and the materials merely a motivational tool or vehicle by which children would arrive at the symbolic form.

Mr Solomon: Yes, and then there's that smaller group [Georgina's] which have never really seen the sort of strands before, or don't relate to them in terms of the more structured maths that we're doing, so that's the group that I'm using lots of resources, you know, hands-on resources with. (Early Year 3)

- Mr Solomon: She [Georgina] enjoys hands-on stuff, like blocks, but she doesn't always use it in the way intended. She is still counting on her fingers – I'm trying to get them to put the biggest number in their head – she's still at the concrete stage, abstract is not part of her repertoire. (Late Year 3)
- Mrs Linkwater: I have concrete materials for maths. Now Rochelle doesn't need this but it's always available if children actually need it to prove, and just to work through what they're actually doing. (Early Year 5)
- Mrs Ponting: (*Complaining that so many children in her class need their fingers to work out basic facts which they should know*) I suggest the ruler so they can see the relationship rather than fingers because what's going to happen when you run out of fingers? Use my toes?
- Ms Summers: In their junior years it's almost a developmental session most of the time, a lot of tactile, kinaesthetic learning that goes on and I guess at Year 3, suddenly the children are starting to use those experiences, to give them that meaning, it's got to be in a really supportive way but also quite a structured way, so they actually have that time and are able to focus. I've seen situations where they are busy in little groups all over the place all the time. I actually feel with maths they do need some quiet time to actually process. They are learning to record, I think that it's important to be able to record your ideas. There are a lot of skills to be taught at this level. (Early Year 3)
- Ms Flower: They [Year 3s] are starting to use their books more. Year 1 and 2 seems pretty much hands-on. (Early Year 3)
- Mrs Matagi: Some children love to be making things or just love geometry. Liam, he does enjoy the success that he gets out of, you know, writing a page of work and finding it's correct. (Early Year 5)

Written mathematics in formalised and prescribed form was seen by many of these teachers as 'structured', an apparent advance on using concrete materials which were necessary only for the children who were slow to catch on. To them, a child's 'needing' concrete materials was a sign of an earlier stage of mathematical proficiency. This indicates that teachers' viewed children's mathematical learning as a linear sequence of developmental stages following a Piagetian model. Teachers frequently based their understandings of children's mathematical learning on their observations of behaviour during individual work time, and on their assessment of the

child's written product. Throughout the lessons, children's thought processes seemed to receive the least attention from the teachers.

Although teachers did not say, nor were they asked, why the decline in provision of concrete materials during mathematics lessons, there are a number of possible explanations. Teachers may have found it easier to photocopy worksheets, or turn to the page of the textbook, than locating, distributing and managing the appropriate concrete materials. These resources may not have been available in the schools, or the teachers used to incorporating them into their lessons. It appeared that teachers did not regard concrete materials as necessary, believing that children should be able to 'do maths' without them.

The children's view of concrete materials differed from that of the teachers.

Researcher: What suits you best – writing your maths, using equipment or talking it through?

Georgina: Using equipment.

Researcher: Why do you like using equipment best, Georgina?

Georgina: 'Cause it makes it easier. (Mid Year 5)

Fleur: Equipment makes it easier so I like that.

Researcher: Do you use it very much?

Fleur: Well, no, not much. (Late Year 5)

Researcher: Would you rather do writing in your book, or be using some equipment like blocks or rulers or string – those sorts of things?

Rochelle: Using the blocks or the string. (Mid Year 5)

Researcher: Do you use equipment much in maths?

Peter: No. (Early Year 5)

Researcher: Do you like it best when you write your maths or when you do things like you did this morning using rulers and string and things? [Perimeter measurement activity with student teacher]

Peter: Using rulers and string and things. (Mid Year 5)

Bruner (1964) identified three ways in which ideas can be presented to learners:

- the *enactive* mode where learners physically experience the concept;
- the *iconic* mode where the concept is represented by models or images;
- the *symbolic* mode, where specialised signs are used to signify the concept.

Apparatus is essential for learners working in the enactive mode and the iconic mode where modelling is used. Lack of apparatus confines learners to either to the iconic mode using imagery such as pictures, or the symbolic mode.

By providing a daily diet composed mostly of written work in mathematics, the teachers obliged all children to operate largely in the symbolic or iconic/imagery mode, even though this may not have been the most appropriate or effective method for certain individuals to develop sound understandings of particular concepts. As previously noted, children such as Mitchell and Georgina worked much more effectively in the enactive and iconic/modelling modes than the symbolic mode. Because they were frequently denied the support or appeal of apparatus, and were confronted instead with symbols that were both less engaging and made less sense, this diminished their enjoyment of mathematics and eroded their confidence. Dominic and Toby, however, were comfortable working in the symbolic mode, experiencing little difficulty in comprehending the mathematical recording codes and conventions they encountered in their written work.

Displays of children's mathematics work were rare in the classrooms visited. This lack of visual reinforcement of children's mathematical ideas contrasted sharply with the work they produced in other learning areas. Children's writing, art, and topic work (science, social studies and health) could be seen prominently and colourfully exhibited on classroom walls. The few displays of children's mathematics work seen in over ninety classroom visits, were statistical graphs, models of 3-D shapes, and geometrical and algebraic patterns. These were rarely labelled with the mathematical ideas involved in their production. Evidence of children's mathematical thinking such as writing about their understandings of mathematics, or recording their methods and solutions in various ways, were almost non-existent. Much of the mathematics work that the children did was not considered as something that could or should be shared.

### **School Policy**

Schools placed great emphasis on children's generic 'work' skills. The progress report form for parents of Pukeiti School typified the work attributes that were seen to be

valued in the study schools. Under a heading '*Personal and Social Growth*' eight of the thirteen categories related to 'work habits'. These categories were:

- is organised and prepared for learning;
- responds to instructions promptly;
- completes set tasks on time;
- completes homework effectively;
- works well independently;
- displays perseverance;
- takes responsibility for own learning/work;
- works well with minimal supervision.

For each child, teachers were to place C (commendable), S (satisfactory), or N (needs improvement), beside each category when reporting to parents.

In the context of the mathematics classroom, teachers' focus on children's work standards and independence took precedence over the teaching and learning of mathematics itself, and appeared to be aimed at reinforcing the schools' and communities' values regarding academic 'work'.

### **Homework**

By Year 3, most children in the study were receiving regular written homework tasks that usually included some mathematics. The mathematics homework activity most frequently cited by the children and the teachers was learning basic facts, especially times tables.

Miss Palliser: Usually they just have some basic addition and subtraction and multiplication they do each week. (Early Year 4)

Mrs Ponting: It's usually just, um, quite basic, something they know. On a sheet with other work. It's basic facts and tables and ...at the moment it's simple addition.

Researcher: Is that school policy?

Mrs Ponting: Yes, and the parents like to see their homework. (Early Year 4)

Fleur: *(Remembering some recent maths homework activities)* We've been doing magic squares, we did *Ten Quick Questions* which had things like big pluses like five hundred and three plus eight hundred and fourteen, and we had division... and there was place value and then we had writing whole numbers like sixty-three and ninety-five. A whole lot of maths questions, piles of everything and the questions 'How much would it cost for three hot dogs if they were two forty cents each. (Late Year 5)

Georgina: Times tables and stuff like that. (Early Year 4)

Dominic: It'd usually be tables. We have to practise our times tables. (Mid Year 4)

Most parents reported that homework was a regular activity in their households. They believed that homework was good for their children, and that it was important that their children develop homework routines.

Ms Flower: They [parents] always ask about times tables and that sort of stuff, and I just tell them they can do that easily by themselves at home, they don't need me to do it. (Early Year 3)

Most parents said that their children were conscientious about completing mathematics homework tasks. Failure to complete homework tasks generally resulted in some form of penalty at school. Some children found homework a positive experience, especially where parental support led to feelings of success:

Georgina: 'Cause at the start of the year, Dad showed me all of my [mathematics] homework. He showed me how to do one then I did it all by myself and I got them all right.

Researcher: How did that make you feel?

Georgina: Really good when no one was helping me. (Mid Year 5)

Not all children felt positive about mathematics homework:

Peter's mother: I leave him well alone but when he got his first times table to do, he has to sit here *(indicates kitchen table)*, and we'd hear nothing from him until he started whimpering, then we'd work out, yes, he was having problems with it. (Late Year 3)

Dominic: I hate homework.

Researcher: What is it you hate about homework, Dominic?

Dominic: Like, when I just get back from school I have to do like about four questions of homework and that really pisses me off. (Late Year 4)

Because it was mostly based on written number tasks, mathematics homework reinforced children's views of doing mathematics as a solo endeavour, consisting of producing written answers to externally imposed, written questions. Family members reported that the children mostly worked alone on mathematics homework tasks, and that parents or siblings became involved only when asked for help.

### **The curriculum view of 'doing maths'**

In the 1980s, a change took place in the teaching of written language. The focus shifted from learning language as a repetitive performance based on exercises and 'stories' neatly written into books, to a purposeful *process*, where children were encouraged to 'brainstorm' their initial ideas, create draft copies and edit their drafts. Only in the final publishing stage was presentation such as setting out or neatness considered important.

A similar change has not been observed in mathematics classrooms, in spite of the fact that process is also strongly emphasised in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992). While working strategically and systematically are stressed, at no point does the curriculum indicate to teachers that written work in mathematics exercise books, particularly neatness or setting out, are desirable or even helpful in the learning of mathematics. On the contrary, it states:

'Students learn mathematical thinking most effectively through applying concepts and skills in interesting and realistic contexts which are personally meaningful to them. Thus mathematics is best taught by helping students solve problems drawn from their own experience...The characteristics of good problem-solving techniques include both convergent and divergent approaches. These include the systematic collection of data or evidence, experimentation (trial and error followed by improvement), flexibility and creativity, and reflection – that is, thinking about the process that has been followed and evaluating it critically. (p. 11)

Teachers are provided with practical examples of how this process might happen in their classrooms in the handbooks *Implementing Mathematical Processes* (Ministry of



Education, 1995) and *Developing Mathematics Programmes* (Ministry of Education, 1997).

Baker and Baker (1990) also present a strong argument for a 'process' approach to mathematics teaching, as found in the teaching of writing. They comment that:

On the whole, the process by which they [mathematicians] arrive at results or methods of finding a proof, the rough calculations, the data generated to find examples of a theory, the diagrams drawn and discarded, are all hidden. What is presented is the finished, polished result. This near obsession with hiding the process used to pervade maths teaching. Layout and presentation, at least in our day, used to score as highly as correctness – always the prize – and rough work was rubbed out, even discouraged. (p. 26)

As shown earlier in this chapter, layout and presentation still scored highly in the classrooms of this study, indicating that this traditional classroom practice is alive and well.

*Mathematics in the New Zealand Curriculum* makes a strong statement about the value of using of concrete materials at all levels:

The importance of the use of apparatus to help students form mathematical concepts is well-established. Using apparatus provides a foundation of practical experience on which students can build abstract ideas. It encourages them to be inventive, helps to develop their confidence and encourages independence. ...Junior school teachers are used to choosing an appropriate range of apparatus to focus students' thinking ...such an approach is equally valid with older students and should be used wherever possible. ... At all levels, students should be introduced to new ideas by having their attention drawn to examples occurring in their natural environment, and then modelling them with apparatus. (p. 13)

Contrary to this directive, apparatus was observed to become increasingly scarce as the children became older, as previously discussed.

### **Children's responses to 'doing maths'**

Winter (1992, p. 90) provides an insight into children's sense-making regarding adult-imposed mathematical tasks by reporting his conversation with a five-year old who explained that the opposite of choosing is 'work... but you can choose to work'. He

argues that work is what adults choose to do for themselves, and what teachers tell children to do. He believes that the construction of school mathematics as compulsory 'work' is unfriendly to children. When asked how maths time could be made better for them, the children in this study were often quite clear about what it was about 'doing maths' that did and didn't work for them.

Some children imagined that an increase in the frequency of the types of mathematics activities they most enjoyed would improve mathematics time for them. The activities they cited were mostly social or creative in nature.

Dominic: Just playing a bit more games.

Researcher: Is that the way you learn best?

Dominic: Yes. (Late Year 3)

Researcher: What things would make maths better for you?

Jared: Easy work.

Researcher: If you were the maths teacher what sorts of things would you have at maths time?

Jared: Easy work.

Researcher: Would you have worksheets?

Jared: No.

Researcher: What would you have?

Jared: Playing games. (Late Year 3)

Liam: Playing *Around the World* and games like that. (Early Year 5)

I wouldn't really do it [maths work] I'd just play the games. (Late Year 5)

Peter: Um, probably more maths games and, um, more drawing things. (Mid Year 5)

A number felt that they needed more individual teacher assistance.

Fleur: Mrs Meadows helping me individually 'cause she doesn't really help us.  
(Late Year 5)

Jessica: If there's only one person that needs help then the teacher should help them even if it's until the end of the maths session, at least the other people have learned something new and that person is up to that stage. (Late Year 5)

Mitchell: Help.

Researcher: Get some more help?

- Mitchell: Yeah.
- Researcher: Who would you like to have helping you?
- Mitchell: The teacher. (Early Year 5)

The isolation and pressure of individual work was off-putting for some.

- Jessica: I'd like it if we did it together, like, not every single person because you don't get a turn to say something, but, like, three people and you all get a turn...that's what I would like to do and if that happened I think it'd be quicker and easier. (Late Year 4)

Time constraints were seen as a problem. Some children suggested more flexibility.

- Georgina: Have more time, like we have half an hour on maths and we don't hardly have any time to do it. (Georgina, Mid Year 5)
- Jessica: Well, long enough for me to get stuck into it and start enjoying it. And then once I've started getting a bit bored, I think 'I want to finish this.' (Mid Year 5)

Two of the children were unable to suggest any positive changes, indicating either that they were comfortable with the conventions of doing mathematics in their classrooms, or that they couldn't conceive of doing mathematics in any other way.

- Researcher: How could maths be made better for you, Rochelle? (*No reply*) Could it be made better?
- Rochelle: No. (*Shakes her head and smiles*).
- Researcher: How could maths time be made better for you, Toby? (*Wait for answer*) Could it be made better?
- Toby: No!
- Researcher: You pretty much like it like it is?
- Toby: Yep!

## Conclusion

Starting from Year 3 of the children's schooling, and increasingly through subsequent years, mathematics exercise books, worksheets, textbooks and questions on the board

became the everyday tools of trade for teachers at mathematics time. They represented to teachers and children alike, the 'doing' of mathematics. Rather than fostering processes of exploration, experimentation and creativity as suggested in *Mathematics in the New Zealand Curriculum*, these tools obstructed such an approach to the teaching and learning of mathematics.

The sociomathematical worlds of the ten study children were not places where mathematics was taught or learned as a process through which ideas and possible solutions might be brainstormed, explored, trialed, presented, evaluated and recorded in a variety of ways. Instead, they were places that fostered a belief that mathematical knowledge and competence was to be gained primarily through conscientious application to solitary written work as defined through the authoritative directives of teacher, textbook and worksheet. Teacher emphasis on desirable work habits such as setting out, neatness, completion, and working 'independently' indicated that these skills were highly valued, establishing a work ethic within classroom environments that superseded concerns about children's mathematical understanding. It was assumed that by a certain age, children would benefit from the 'structure' of this kind of work.

These taken-for-granted customary practices of teaching and learning mathematics formed a significant part of the everyday worlds of the children. For them, there was no other way of 'doing maths'. As the children became older, written work increased, while active exploration and the use of concrete materials diminished. By Year 5 use of concrete materials had become largely confined to small group instruction time with the teacher, or abolished altogether for all but the most 'needy' of learners. Symbolic and abstract modes of working were privileged over the use of real objects, working in silence over group discussion, and individual endeavour over collaboration.

For many of the children, the isolation, tedium and difficulty of written mathematics tasks were off-putting and served to increase their feelings of alienation and inadequacy. For a few, written work was reassuring and satisfying. Only rarely were any of the children observed to experience 'doing mathematics' in the spirit of

changed views of mathematical learning as a social, dynamic, active, meaningful and purposeful process.

The following chapter examines how the children developed understandings through their experiences of mathematics as presented to them within a culture of speed, ability differentiation, and individual academic work, of what was meant by 'learning' and 'knowing' mathematics, and of what mathematics *was*.

### THE RIGHT/WRONG DICHOTOMY: 'LEARNING' AND 'KNOWING' MATHEMATICS

The previous chapters have examined the use of speed and competition in the teaching of mathematics, the ways in which the children were identified and differentiated according to socially constructed conceptions of mathematical ability, and the widely accepted view of 'doing' mathematics as a specialised mode of individual work. This chapter explores the fourth major dimension of the children's sociomathematical worlds: what the children understood, through everyday interactions with teachers, peers and parents, as *learning* and *knowing* mathematics. These understandings were inextricably linked to what they believed mathematics to be, why it was taught at school, and its importance as a subject.

Far from supporting the frequent claim that mathematics is a culture-free, universal and neutral body of knowledge, e.g. 'mathematics provides a means of communication which is powerful, concise and unambiguous' (Ministry of Education, 1992, p. 7), classroom practice, it will be shown, constructed and *valorised* (Abreu et al 1997) particular ways of mathematical 'knowing' to the exclusion of others. The chapter illustrates how the children equated mathematical capability with the production of right answers as defined and validated by teachers, parents and textbooks. The children's beliefs about mathematics, developed within the social contexts of home and school where authorised versions of the 'right' answer or procedure endorsed or negated their existing mathematical intuitions and reasoning, are examined. The children's experiences are linked to the views of teachers and parents, and located within the wider context of school policy and mathematics curriculum development.

#### **The dominant approaches to 'new learning'**

As discussed in previous chapters, a typical mathematics lesson in the classrooms observed consisted of a lesson starter, followed by some form of activity related to the

current mathematics topic. The teachers introduced new mathematical concepts either to groups of children differentiated by age or ability, or to the class as a whole. In describing a typical lesson in his classroom, Mr Cove drew the distinction between a *teaching* lesson and a *practising* lesson:

Mr Cove: If it's a teaching lesson then I would take step one of something and teach them that and I might say first of all, 'We're going to learn something new today,' and then I'd probably do it on the blackboard, or with blocks or whatever...depending on what it is, and then after going through it maybe myself, then I'd say, 'Why don't you sort of see if you can do it?' and they try and repeat what I've just done. (Mid Year 5)

This approach to the introduction of new mathematical concepts followed a well-defined pattern that was highly typical of the teaching practice observed. The following example from one of Jessica's Year 5 lessons highlights the taken-for-granted features of this phase of the teaching process.

### Jessica's story: The tree diagram lesson

From fieldnotes, Roto School, Mid Year 5

*Jessica's teacher was conducting a statistics unit over several weeks. In the previous session, she had taught the children to record possible outcomes using a grid, and at the beginning of this lesson, she revisited this procedure to illustrate the possible outcomes for two throws of a coin. She then introduced the new learning.*

Ms Mere: Have you seen a tree diagram?

Alice: (*Calling out*) First we have a stem and leaf graph, now a tree diagram! I've seen one but it's hard.

Ms Mere: Well, this is what it looks like with our heads and tails. (*Draws on blackboard, as in Figure 30*)

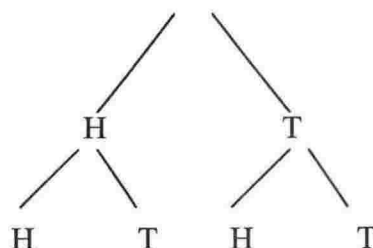
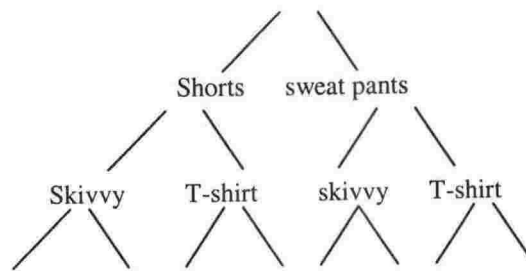


Figure 30: Tree diagram 1, on the blackboard, Roto School, Mid Year 5

Ms Mere: So you just follow the branches. (*No demonstration given*) This is just a different way of doing it. Now we're going to do the same for the clothes problem we did yesterday. (*Drawing on the blackboard as in Figure 31*)



**Figure 31: Tree diagram 2, on the blackboard, Roto School. Mid Year 5**

Ms Mere: Any questions? (*Child raises hand*) Yes, Lettie?  
 Lettie: Could you have more things?  
 Ms Mere: Of course. (*Adds sneakers and sandals to the bottom of the diagram*).  
 Alice: It looks like mountains.  
 Tess: No, houses.  
 Ms Mere: Right, now we're going to practise. (*The children take out the AWS Level 3, S5 18 worksheet they were using the day before*) Please, quiet now, have a look at the worksheet I've given you so you don't have to ask unnecessary questions. It's very important to notice that you can either do it top down or from the side like there (*Indicates example on the worksheet that runs horizontally*). I didn't do that on the board.

*(The children are now expected to work alone. Appearing unsure of what to do, Jessica glances at the book of the child on her right, then raises her hand. The teacher does not respond. She is helping others who have also raised their hands. Jessica puts her hand down. She draws the diagram sideways in her book as depicted on the worksheet. Because the page is already ruled into two narrow vertical columns, the diagram does not fit so Jessica rubs it out and reduces its size. When finished, she is unable to list all the possible outcomes because reading the 'branches' was not clearly demonstrated by the teacher. It is evident that Jessica has not understood the concept.)*

As this extract shows, not all the children understood the mathematical ideas or procedures the teacher had expected to *transfer* to them. Jessica was one of these.

The ritualised formula of this lesson was clear:

- teacher demonstrates while children watch and listen, asking questions only when invited to do so;
- alone, children apply teacher procedure using a different example;
- teacher checks, assisting individual children *where necessary*.



This pedagogical style, based on the belief that mathematical knowledge, consisting of non-negotiable facts, rules and procedures, can be transmitted from the expert teacher to the novice learner who is regarded as a passive receptacle, was typical of the teaching practices observed. As has been revealed in the previous chapter, there was also a high reliance on formal written forms of representation in this lesson to the exclusion of modelling, acting out or student discussion.

*Purpose* and *meaning* were overshadowed by *procedure* in this lesson. The reason for using tree diagrams was never established, nor was any attempt made by the teacher to link the worksheet contexts to real life experiences of relevance to this particular group of children. The spontaneous vocalisations of some of the children in response to the unfamiliar nature of the terminology and diagrammatic representations used in the lesson - 'first we have a stem and leaf graph, now a tree diagram!'; 'it looks like houses'; 'no, like mountains' - showed how children were trying to construct personal meaning from the teacher's exposition by connecting the diagrams to familiar objects.

This way of presenting new mathematical ideas to students has been described by Stigler and Hiebert (1997) as following a *cultural script* consisting of two phases: *acquisition* and *application*. They assert that:

teaching is a cultural activity. Cultural activities often have a 'routineness' about them that ensures a degree of consistency and predictability. Lessons are the daily routine of teaching and are usually organised according to a 'cultural script.'...In the acquisition phase, [in a typical grade eight classroom in the USA] the teacher demonstrates or leads a discussion on how to solve a problem. The aim is to clarify the steps in the procedure so that students will be able to execute the same procedure on their own. In the application phase, students practice using the procedure by solving similar problems similar to the sample problem. During this seatwork time, the teacher circulates around the room, helping students who are having difficulty.

(p. 18)

Brousseau et al (1986) described what they termed the *cognitive development sequence* in which 'traditional mathematics teaching ...consisted mainly of showing the students adult versions of mathematical knowledge and giving them practice using it.' They provide examples of the frequent failure of this method, as in the case of

Jessica and the tree diagram lesson, and demonstrate students' having constructed quite different ideas from those intended.

### **Dominic's classroom: closed questions/ right responses**

From fieldnotes, River School, Early Year 4

*The class are seated on the mat in front of the board. Mr Swift writes some instructions on the whiteboard as in Figure 32.*

One, ten, hundred, thousand, million.  
Put these numerals in written form.

15, 48, 480, 4800, 75,641  
Put these numbers in numeral form.

Seventy six, fourteen thousand, seven hundred and fifty-six, two hundred and five,  
seven thousand and seventy seven, nineteen thousand, one million two hundred thousand.  
Place these in order from biggest to smallest.

*Mr Swift begins a lesson on place value with a series of questions for the whole class. This question-answer session is designed to revise concepts introduced the previous week. The questions are asked in such a way as to give the impression that there is one correct answer for each and Mr Swift fires the questions in a 'brisk' voice as though he expects rapid answering. [The expected answers are shown in brackets].*

Mr Swift: What we are doing in number at the moment? [Answer: place value]  
What did we do last week? (Pause) ... beginning with 'a'?... [Answer: abacus]  
(Chooses a child to come up and draw a three-bar abacus on the whiteboard)  
Those little stick things, what are they for?  
(When the correct answer is not forthcoming) What do they do? What are they?  
What are the upright pieces on the abacus for? [Answer: for showing hundreds, tens and ones]  
What's after the thousands? [Answer: Tens of thousands]  
Which arm would you use for the number of people in a car? [Answer: the 'ones' arm]  
*The children are then directed to 'do' the 'questions' on the board as shown in Figure 34 above. This exercise also requires children to produce the correct responses.*

[Note: These children were never shown an abacus, much less given the opportunity to use one.]

Discursive practices in the mathematics classrooms observed were characterised by a distinctive kind of questioning, highlighted in the example above. Almost every one of the teachers' questions heard during the three years of observations of mathematics lessons, were *closed*, that is, they required one correct answer, such as:

Mr Swift:           How many times does five go into fifteen? (Mid Year 4)

This kind of questioning implied that there *was* only one right answer to mathematical questions, that such an answer was a non-negotiable 'fact', and that matching the correct answer to an externally posed question was indeed the point of mathematics. Mr Swift's question above ignored the children's many ways of 'knowing', 'learning' 'understanding' or justifying this 'fact'.

On a later visit, another exchange was observed between Mr Swift and his pupils. In order to revise and consolidate mathematical knowledge introduced in recent lessons, Mr Swift again posed a series of quiz-like questions, in response to which the children attempted to provide 'right' answers.

Mr Swift:           What is statistics?  
 Tina:                Gathering information? [N.B. the child's questioning tone implying, 'Am I right?']  
 Daniel:             What we did yesterday?  
 Mr Swift:          When we do a graph, what do we put across there?  
 Daniel:             You've got to put the 'x' and the 'y'.  
 Mr Swift:          What do we call those lines that we've called 'x' and 'y'?  
 Daniel:             (*Calling out*) The 'y' goes down and the 'x' goes across.  
 Dominic:           I know what the bottom one is – axes.  
 Mr Swift:          They're both called axes. Good boy.   (Late Year 4)

This pattern of teacher questioning and child answering is well-recognised (Bussi, 1998; Edwards & Mercer, 1987; Maier and Voigt, 1992). Their analysis shows that in a 'standard' mathematics classroom, the teacher *initiates*, eliciting pupil *response* and the teacher evaluates the response through *feedback*. Edwards and Mercer, citing Stubbs and Robinson (1979) refer to this as the I-R-F exchange structure. Lemke (1990) who has examined this discursive pattern in science classrooms, refers to it as the 'triadic dialogue', and elaborates on the structure of a typical sequence:

[Teacher Preparation]  
**Teacher Question**  
 [Teacher Call for bids (silent)]  
 [Student Bid to Answer (hand)]  
 [Teacher Nomination]  
**Student Answer**  
**Teacher Evaluation**  
 [Teacher Elaboration]

The bracketed steps are not always included in the exchange. (p.8)

Possible disagreement or alternative views in mathematics were tacitly negated and/or overridden in Mr Swift's interaction above by the powerful use of *we* statements. *We* statements presumed an authoritative voice speaking from beyond the classroom, and non-negotiable *group agreement* about mathematical language and procedures to be used. Pimm (1987) examines this characteristic of the traditional dialogue of mathematics classrooms. Shared understanding of specific knowledge and one correct answer or procedure, was implied by the use of *we* or *us* in many of the teachers' questions in the classrooms observed, evident in the following typical examples:

Miss Fell: [Place value] What do we call this place? (Pukeiti School, Mid Year 4)

Miss Field: [Working form addition] What's the first thing we have to do? (Hill School, Late Year 4)

Ms Seager: [Fractions] What does the top number tell us? (Roto School, Mid Year 4)

Teachers also frequently used *me* in their verbal interactions, reinforcing the I-R-F exchange structure in which they, in a controlling role, unwittingly defined the purpose of mathematical activity as pleasing the teacher, e.g:

Mr Waters: Can anyone tell me what the pattern is? (Spring School, Mid Year 5)

Ms Summers: Who can tell me about a rectangle? (Beach School, Early Year 3)

Dillon (1988) describes how questions of this sort, rather than fostering the kinds of discussion that might lead to a rich exchange of ideas, foil such communication.

Worksheet and textbook questions replicated the I-R-F exchange structure in a written form with closed questions requiring 'correct' answers. Teachers usually evaluated the children's answering in terms of accuracy alone, as in the following situation:

Mrs Meadows: *(To a child who approaches her with her maths exercise book for feedback on her work) You haven't got this one right, and you haven't got this one right and you haven't got this one right. (Pukeiti School, Mid Year 5)*

In the case of textbook questions, evaluative feedback was supplied not only by the teacher's marking, but also by the presence of the *authoritative other* in the form of 'answers' provided in the back of the book.

Within the classroom climate of closed questions, teachers were often heard to 'fish' for the one correct answer, then praise the child who could produce such a response. In many instances, teachers did not recognise alternative mathematical approaches or understandings for what they were. The following story illustrates how Peter's teacher inadvertently rejected a mathematically sound and legitimate method of arriving at the 'correct' answer she was seeking.

### **Peter's story: one right way**

From videotape and fieldnotes, Beach School, Mid Year 3

*Peter's group is participating in an activity where five and six-digit numbers are to be placed in order. This requires an understanding of place value. Peter's group is sent out to the school car park where each child is to collect ten different numbers from the registration plates of the parked vehicles. Peter carefully looks around at the cars, before selecting and recording his ten numbers.*

*When he returns to the classroom he finds that other children have already started on the next part of the activity – ordering the numbers. The teacher has explained to the children that the numbers are to be placed in ascending order. On a large piece of paper, Peter carefully begins to write his numbers in order. Unlike the other children who have all started with their smallest number at the left-hand side of their pages and are writing their numbers in increasing magnitude in a horizontal line towards the right, Peter begins at the top of his paper with the largest number and works downwards, the numbers decreasing in magnitude one beneath the other.*

*When he is about halfway through this process, some of the other children have finished, and Ms Summers, the teacher, comes to check their work. She then approaches Peter. All the placements of his numbers are correct at this stage demonstrating that he has a very sound understanding of the mathematical concepts the task requires. However, Ms Summers does not accept what he has done.*

Ms Summers: *(Looking at Peter's work)* No, Peter, I said you were to put them in *ascending* order.

*Peter looks bewildered. He may have been thinking that that was surely what he was doing. His method was more literal than that of the other children, because when finished, he would be able to read his numbers from the bottom of the page, to the top in ascending order.*

Ms Summers: *(Seeing the confusion on Peter's face)* Simon, could you help Peter please?

*Simon, proceeds to take Peter's pencil, another piece of paper, and rapidly write Peter's numbers from left to right from smallest to largest, while Peter looks on.*

Simon: *(To Peter)* You getting the hang of it?

Had Ms Summers taken a moment to consider Peter's approach, she may have made some discoveries: that her instructions regarding the way the numbers were to be presented on the page were ambiguous, that there was no mathematical reason why the numbers *must* be written horizontally from left to right to represent ascent, or that Peter's understandings of the mathematical ideas required by the task were very sound. Her preconceptions about this task were likely to have been based on the familiarity of convention. 'Left-to-right' was no doubt the way that she was used to seeing a written sequence of numbers. While understandable, her inflexibility when confronted with an alternative response denied Peter the opportunity either to choose a method of representation that made most sense to him, or to explain his method. In turn, this denied the other children a learning experience through seeing that there was more than one way to arrive at an appropriate arrangement of the numbers. Most importantly, it denied Ms Summers as the teacher, an opportunity to assess Peter's understanding of place value, which lay at the mathematical heart of the task.

In the three years of observation, it was noted that Peter was a child who worked at mathematics tasks methodically and systematically, exhibiting a strong sense of logic and order. These would seem to be most useful mathematical skills. He worked quite

slowly compared to classmates, seldom completing mathematics tasks before the lesson was over. His teachers remarked that he was quiet, shy, and reluctant to communicate. In all the time he was observed, he was rarely seen to voluntarily share his ideas or answer questions in classroom discussions of mathematics, consequently his teachers often had little idea of what, or how, Peter learned.

- Researcher: Do you usually put your hand up when the teacher asks questions?  
 Peter: No.  
 Researcher: Why is that?  
 Peter: I'm not sure.  
 Researcher: How do you feel when the teacher asks you a question and you haven't got your hand up?  
 Peter: Sort of bad.  
 Researcher: Why is that, Peter?  
 Peter: 'Cause I won't know it.

The place value lesson perhaps helps to explain Peter's reluctance to communicate his mathematical ideas. While Peter had developed his own ways of working things out, he had learned to have little faith in them since they were routinely unrecognised or devalued by his teachers. Other children in Peter's classes were also observed to offer mathematically sound ideas only to have them rejected by his teachers. Within the classroom climate of closed questions, all but the most confident children were put off answering questions for fear of being 'wrong'.

From videotape, Beach School, Late Year 3.

*Mr Ripley is a student teacher. He is marking a maintenance question in the daily Quick Ten. A figure has been drawn on the board as in Figure 33, A)*

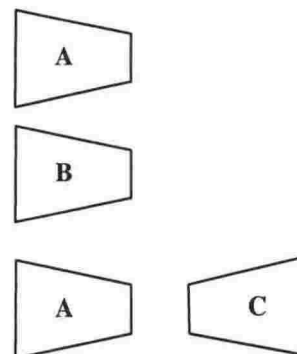
Mr Ripley: What is the reflection of this shape?

*(Nadine is chosen to draw her answer on the board. She draws another shape beneath the first one as in Figure 33, B)*

Mr Ripley: No. Someone else?

*(Peter is watching intently but keeps his hand down. Jack is chosen. He draws a shape alongside the first one as in Figure 33, C)*

Mr Ripley: Well done. That's exactly what I was looking for.  
 (Late Year 3)



**Figure 33: Reflections of a trapezium, Beach School, Late Year 3**

Mr Ripley appeared to have a fixed idea that the line of symmetry should be vertical rather than horizontal, and was therefore unable to consider alternatives. In this case, Nadine's response was likely to have been correct. While Mr Ripley may have interpreted her answer as illustrating 'translation' rather than 'reflection', had he asked Nadine to explain and justify her answer, he may have discovered that she had developed an excellent understanding of the concept of reflection. He could then have asked whether there were any other ways that the trapezium might be reflected.

From fieldnotes, Beach School, Early Year 4

Mrs Waverley: What would I see wrapped around a cylinder? (*To a child with his hand up*) Yes James?

James: A rectangle.

Mrs Waverley: (*Crossly*) Don't be so silly, James. When you can give me a sensible answer, then I will ask you!

(*Peter listens, but does not raise his hand*)

James must surely have been bewildered. His answer appeared to be a mathematically logical response to the question, but Mrs Waverley did not ask him to explain. Through a process of hinting and prompting, she elicited from another member of the class the 'correct' answer of 'Gladwrap,' a brand of plastic film used for wrapping food. Again, Peter remained silent during this episode. Like many of the other children, he had adopted a protective strategy - keeping a low profile.

Skemp (1978), in explaining children's mathematical understanding, draws a distinction between what he terms *instrumental* understanding and *relational* understanding. Instrumental understanding he describes as 'rules without reasons' (p. 9), while relational understanding is based on knowing not only what to do, but the reason why. He argues that these ways of knowing differ so fundamentally that they can almost be regarded as different kinds of mathematics. Through their teaching practices, teachers in this study were fostering instrumental rather than relational understanding, as was Ms Summers when she did not ask Peter or any of the other children, *how* they were working out the ordering of their numbers.



Skemp also believes that from a young age, children develop conceptual structures ('schemata'), or internalised representations of the world, many of which are well developed before children reach school, and continue to develop throughout their lives. These may include representations of mathematical ideas, such as the sequence of counting numbers. He argues that the more developed a child's mathematical schemata, the more able the child is to develop relational understanding. If this is so, then it is likely that where the methods of representation that are modelled or expected in the classroom, for example numbers always being represented in a sequence from left to right as in Peter's lesson with Ms Summers, do not match with or admit expression of, children's unique and personal schemata, there is potential for confusion, bewilderment or even conflict. Children might, for instance, conceive of numbers arranged in order in one of many possible ways including in a zigzag pattern such as found on a Snakes and Ladders board, from bottom to top as found on a thermometer, (Peter's way) or from foreground to background as though viewed down a street, the magnitude of house numbers increasing with distance from the viewer. Effective schemata can be seen to contribute to relational understanding and vice versa.

Dehaene (1997) has also investigated the development of individuals' mental images of spatial number representations. He describes these as 'cortical maps' and believes that they are formed at a young age. He has found that the orientations of numbers vary between individuals, appear to have a cultural link to conventions of written language, and that for some, numbers have colours and occupy very precise locations in imaginary space. It appeared that none of the teachers in the study were aware of how individuals might develop differing mental images of numbers, and of the contradictions and confusions they may have therefore been creating for the children through their narrow and conventional representations of numbers.

### **Georgina's story: the algebra lesson**

From fieldnotes, Island School, Early Year 3

Mr Solomon: I need Tapatoru [Georgina's mathematics group] to stay on the mat.

Mr Solomon brings a box of coloured blocks to the mat and asks the children to sit in a circle. He sits on the mat with them. He then explains that they are to make the number sequence 1,2,3...using the blocks. All the children take some blocks. Georgina arranges them first one block in front of her, then two blocks behind them (further away from her), then three behind them etc (Figure 34) while Mr Solomon and most of the other children have made their arrangements in front of them from left to right as in Figure 35.

Key: ☺ = Georgina/child ■ = Block [Not to scale]

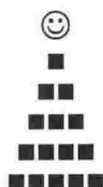


Figure 34: Georgina's representation of counting numbers



Figure 35: Mr Solomon's and the other children's representation of counting numbers

Mr Solomon: (To Georgina) You need to get yours the way we've got ours.

Georgina either fails to hear or ignores the instruction and begins to rearrange her blocks, experimenting with vertical stacking arrangements for the sequence. No other children are doing this.

Mr Solomon: How many more in this set than that? (Points to the first block in his model, then the group of two)

Georgina: (Touching her own blocks) There's one in that set, and two in that so that makes three.

Georgina now changes her blocks to standardise the colours for each member of the sequence, thereby emphasising the pattern. Mr Solomon and the other children find they are running out of room by representing the sequence in a horizontal, left-to-right line, so some of them begin to adopt Georgina's system or similar. Mr Solomon appears to give in. He stops trying to make Georgina change hers so it looks like all the others.

This episode illustrates Georgina's strong sense of patterning, clearly demonstrated through her exploration and manipulation of the coloured blocks. Her spontaneous creation of a triangular arrangement of the blocks provided the potential for rich algebraic investigation such as 'triangular' numbers. Georgina was perhaps beginning

to recognise this when she added the first two elements of her sequence instead of simply stating the difference between them, as Mr Solomon had asked. She was so captivated by her own system of pattern-making that she continued with it even when the others were working in the 'right' way – the teacher's way. Her development of a third dimension to the pattern (height), showed keen spatial awareness. Had Mr Solomon asked the children to explain their arrangements in their own words, a discussion rich in mathematical ideas might have been generated. Throughout this lesson, Georgina was experimenting, discovering, checking, justifying and communicating her mathematical ideas. These mathematical processes were largely unrecognised by Mr Solomon. During interviews he often spoke in a tone of exasperation of Georgina's lack of compliance. He saw her attitude as 'negative'.

Making sure that everyone was doing it the same way, the *right* way, was important to Mr Solomon. He was unable to recognise that individual exploration such as Georgina's, might provide vital connections for learning. So different were Georgina's style of learning and the teacher's style of teaching mathematics, that Georgina had largely lost faith in her own mathematical capabilities and often complained that mathematics was too hard. Given materials, a chance to experiment and opportunities to explain her thinking, Georgina would surely have felt differently about mathematics, which may in turn have enhanced her achievement.

Many other instances of teachers' rejection of mathematically sound alternatives in favour of one 'right' answer were noted during classroom observations.

*Toby's group has been called to the mat. They have been 'doing a worksheet' involving estimating the cost of a list of groceries.*

- Mrs Kyle: Who's got a really good way of estimating and working it out? I've got a really good way. (*Toby keeps his hand down. To a girl who has her hand up*) Yes Laila?
- Laila: You could see how many things were the same [price] then how many of those there are. [Suggesting multiplication as a strategy?]
- Mrs Kyle: (*Not responding at all to Laila's idea*) Angus?
- Angus: You could do rounding.
- Mrs Kyle: Excellent, Angus. That's what I was thinking of. (Bay School, Mid Year 4)

By failing to respond to Laila's suggestion, Mrs Kyle condemned it. She only considered the idea that was the same as her '*really good way*', thereby establishing it as the 'right' way.

The extracts from the classrooms of Dominic, Peter, Georgina and Toby, show how the teachers' interactions with children followed the I-R-F exchange structure. This style of questioning was less about finding out how the children thought, than a strategy for 'deriving', 'legitimising' and 'reinforcing' the teachers' ideas of: (a) one correct answer; (b) one correct procedure. This pattern of classroom discourse constructed mathematical knowing as dichotomous: right or wrong. Mathematical learning was in turn constructed as a process of recognition, memorisation and reproduction of socially defined correct answers and procedures.

These examples are by no means isolated. Routine neglect of children's understanding of mathematics as demonstrated through the mathematical processes of conjecturing, reasoning, and justifying advocated by *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992), was typical of the teaching observed. This served to reinforce in the children's minds, the idea that only one answer or procedure was ever possible or acceptable. The teachers were often unaware of the interpretable framing of many of their questions, and the diverse ways in which their learners might be making sense of them. The teachers were quite oblivious to the inner conflict and confusion that their classroom representations and discussions may have been producing for a proportion of the children.

Concerns about the mismatch between children's own mathematical sense-making and 'right' methods taught at school are reflected in research (e.g., Baroody & Ginsburg, 1990; Fuson et al, 2001; Jaworski, 1988). The socially constructed and taken-as-shared ways of mathematical 'seeing', embedded in mathematics textbooks (Dowling, 1999) and worksheet material, and reproduced by teachers through such practices as task design, questions (Boylan, 2002), and diagrams on the board, have been recognised as causing problems in children's learning of mathematics (Bernstein, 1990; Cooper and Dunne 1999; Goldin 2002; Mellin-Olsen, 1987; Zevenbergen 2001). Steffe (1991) sums up this view by saying 'I believe that, rather

than expecting children to learn how the teacher thinks, mathematics teachers must teach in order to learn how children think...to teach in harmony with children's approach to mathematics' (p. x).

### **Reinforcing the 'right' way**

Through everyday interactions in the classroom, *right* and *wrong* were continually defined and reinforced. Getting the *right* answer was verbally praised, rewarded, and construed as a sign of success.

Miss Fell:           Who got that all correct? Give yourselves a big clap. (Mid Year 4)

Teacher feedback for correct answers consisted of comments such as 'yes'; 'perfect'; 'correct'; 'right'; 'well done'; 'absolutely'; 'excellent'; 'spot on'; 'good girl/boy'. Incorrect answers received teacher responses such as 'wrong!'; 'no'; 'someone else?'; 'not quite'; 'nearly'; 'good try, but...'; 'who can help her?'; 'you weren't listening'; 'have another go.' During three years of observation, teachers were almost never heard to ask children how they had arrived at their answers or to explain why an incorrect answer might be incorrect.

The classroom routine of 'marking' written work 'right' or 'wrong' was found in every one of the classrooms visited. This had become a powerful act in the learning of mathematics, one which appeared to be related more to the social gains of 'success' in comparison with peers, and dependent on extrinsic signs of 'success' such as ticks, crosses and 'scores', rather than on the intrinsic satisfaction of having gained or applied mathematical understanding. Children became so habituated to this form of evaluation, that they relied almost entirely upon the teacher or textbook to verify their answers (Boylan et al, 2001; Frid, 1993).

The possibility of alternative or multiple answers to any given question, was seen to disturb both teacher and child as illustrated in the following examples:

From fieldnotes, Pukeiti School, Mid Year 5

*The children have been working through some questions on data interpretation from the NCM textbook. The teacher has called them to the mat for marking. The teacher asks a child to provide the answer to a question involving a dot plot of people's activities in the weekend.*

- Damien: Four.
- Mrs Meadows: That should be the first answer.
- Fleur: I wrote sentences.
- Mrs Meadows: Well that's good but takes time.
- Damien: I counted the dots.
- Mrs Meadows: That's what you've got to do. That's correct because one dot represents one person.
- Olivia: I thought someone could have done more than one thing.
- Mrs Meadows: (*Studying the graph for some time. Finally.*) That's correct. You can't tell.
- Nathan: Our answers are wrong then, because you can't just count the dots. So what is the answer? (*He looks frustrated and turns to the answers at the back of the book where the answer of '4' is given. Nathan marks his answer [4] as correct.*) (Early Year 5)

Without Olivia's perceptive spotting of the ambiguity of the situation on which the textbook question was based, and without the confidence she showed in challenging the answer that was so clearly validated by both teacher and textbook, the alternative and very soundly reasoned position of a number of possible answers to this question, might never have been raised. Nathan and other children became visibly agitated at the thought that the question might have no *single* correct answer, since this situation had most likely never arisen in their experience of learning mathematics. They seemed more intent on 'marking' their work than on discussing mathematical meanings.

Another case of alternative answering was observed in Jared's classroom:

From fieldnotes, Spring School, Late Year 5

*Mr Waters has written some sequences on the board which the children have been asked to continue by providing the next three elements of each sequence. He is now working through the answers with the children as they mark their work. He comes to a question where two groups of three shapes have been drawn.)*

- Mr Waters: Who picked what the pattern was doing?
- Afa: Circle, triangle, square - triangle, square, circle - circle, triangle, square - triangle, square, circle.
- Mr Waters: (*Doubtfully*) Hm. Someone else?
- Ian: Circle, triangle, square - triangle, square, circle - square circle, triangle - circle, triangle, square ...you move one each time.

Mr Waters: (*Looks pleased*) Ah, good man. Afa's right, though. You can't fault his logic because there were only two [elements of the pattern] to go on. But the others thought a little deeper. (Mid year 5)

As in the previous example, some ambiguity has been discovered in this particular question. Mr Waters implied when he said 'who picked the pattern?' that there was only one to be picked. Although Mr Waters accepted Afa's answer, he valued Ian's answer more highly by describing it as having required 'deeper' thought.

Not only were right answers reinforced, but also right procedures. The teachers were often quite insistent about the precise way things were to be done, issuing specific instructions to the children that implied that mathematics was something which must be done the *right way*.

Mrs Field: [Data representation] You must put the title 'Pictograph' on your graphs. (Hill School, Late year 3)  
(Late Year 3)

Mr Swift: [Data representation] You've got to put the 'x' and the 'y'. (River School, Late Year 4)

The children were also observed to keep one another on the 'straight and narrow'.

Emma: (*To Jessica who is creating a problem for others to solve and including the answers*) You've done a mistake. You're not supposed to write in the answer. (Mid Year 4)

Tony: (*To Georgina who is drawing a picture in her maths book in answer to a money question*) Why are you doing that? You better rub it out or you'll get in trouble. (Late Year 4)

These comments show that the children had come to believe that doing mathematics in the approved and uniform fashion as prescribed by the teacher was more important than finding their own methods or developing viable variations. Alternative approaches, regarded as transgressions, were likely to result in being 'told off'.

Teachers' endorsement of responses they considered to be superior, was seen as another taken-for-granted of the dominant teaching approach. The terms '*clever*', '*efficient*' or '*sophisticated*' are commonly used by the designers of the New

Zealand's numeracy initiative to describe the kinds of answering methods that children should be encouraged to develop. The Ministry of Education report (Ministry of Education, 2001c) quotes the project coordinators as saying 'the framework aims to support children to solve numerical problems in the *cleverest* ways', and that it is about, 'children's ability to think more cleverly and efficiently' (p. 4). As already discussed in Chapter 4, *efficiently* generally means *speedily* and for a number of the children, this expectation was unreasonable and created anxiety. *Cleverness* is modelled by the strategy stages of the framework which are arranged in a sequential progression (p. 4) from the wide bottom to the pointed peak of a *Te Maunga Tau* (The Number Mountain). *Cleverness* is defined as increasing demonstration of abstract thinking, seen as production of right answers to increasingly difficult questions without recourse to mental imaging or modelling with manipulatives. In Chapter 5, altitude metaphors and the differentiating effects on the children's mathematical identity of the hierarchical view of learning have already been discussed. *Cleverness* and *efficiency* may appear to be both sensible and admirable aims. The danger of this approach lies in privileging the teacher's view of *cleverness* over that of the child. When *cleverness* and *efficiency* are combined, there is a high chance of alienating children such as Afa, whose answers may be judged by the teacher, and therefore by classmates, as slow and/or shallow in comparison with others.

### **'Working form' – the 'right way' to calculate**

In Years 3 and 4, teachers began to introduce formal written procedures for multi-digit calculation, firstly addition, then subtraction and by Year 5, multiplication and division. Of all the mathematical procedures that the children learned, these were especially significant for the children since they were the most often cited as the mathematics skills they had been learning. Mr Ford provides an example of the way teachers taught such procedures.

Mr Ford: *(When asked whether Dominic found anything in maths difficult)* Just recently, they've done work like longer multiplications, just the structure, you know, the format of how it works, it's something I'm teaching, anyway, so the first few times that we've gone through it, um, the main problem for Dominic is just getting things out of place, you know, in the columns, and I drill that part of them every day: 'Keep



everything in line, draw the columns, the zero, you put the zero down the bottom, it goes in the 'ones' column', then they're all looking at me, and then I go, 'Shove the zero in, shove the zero in here.' But that's logical, just part of the whole thing.

(Early Year 5)

These procedures presented difficulties for a number of the children. The following extracts were typical of the children's responses when asked what things about things they had been learning in mathematics.

- Researcher: Is there anything in maths that you can do better now?  
 Toby: Yes, piling.  
 Researcher: Piling?  
 Toby: Like, you pile, um, times tables, so it's like one number here and then there's the times ...  
 Researcher: You could show me here (*giving him a piece of paper*).  
 Toby: Do an easy one...like, I don't know.  
 Researcher: Like twenty-three times six or something?  
 Toby: Yeah, it could be twenty-three times six. (*He writes this in the vertical working form he has obviously been taught*) (See Figure 36 below.) Like that. Then, well, I've got most of them wrong 'cause I don't know how to do this and now that I've learnt, six...six threes...um, eighteen.  
 Researcher: Right.  
 Toby: (*He writes 8 below the 6*) And then you put the one up here. (*He writes a very small 1 above the 2*) Six twos. (*Thinks, then writes 11 to the left of the 8*) Ah...I think it's ten and then plus one equals eleven so it'd be ...(*Points to answer 118*)

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 118 \end{array}$$

Figure 36: Toby's 'piling' (Mid Year 4)

- Researcher: One hundred and eighteen?  
 Toby: Yeah.  
 Researcher: Thanks. That's great. And does Mrs Kyle call that 'piling'?  
 Toby: She doesn't, but I do.  
 Researcher: That's your special name for it?

Toby: *(Smiling)* Yeah.

Although Toby confidently followed the procedure he had been taught, as a result of a multiplication error he did not reach the correct answer. Believing the method to be immune to error, he did not use any other method of calculation to check the accuracy of that answer. His personal description of the procedure as 'piling' and 'times tables', was no doubt to help him make sense of an otherwise context-free task. He appeared to be so focused on manipulating the numbers in the way he had learned, that the sense of the 'question' was obscured.

Hart (1986) found that children aged eight and nine years became confused and made mistakes when confronted too abruptly with the formalised use of written algorithms, without sufficient understanding. When using materials to represent the situations, they were less likely to make such errors. Other research suggests that children's learning of calculation routines must build from the personal methods of calculation that children develop based on their understandings of how numbers work (Carpenter et al, 1999; Carraher & Schliemann, 1985; Fuson, 2001; Hiebert et al 1997). New Zealand's current numeracy initiatives promote the use of mental calculation before the introduction of methods such as the one Toby had been taught, arguing against traditional approaches to the introduction of these calculation procedures. 'Many children were not ready [in the past] for the ideas, so learning the vertical setting out ended up as rote learning. Within the project the suggestion is to delay the teaching of the written form until children show the mental acuity to be able to solve [problems like]  $51 - 27$  mentally' (Ministry of Education, 2001c, p. 4). The use of concrete materials is advocated to enhance children's ability to build this facility with number.

Fleur had also learned how to calculate using written working form, and her vocalisation as she demonstrated how to subtract with renaming, shows how she made 'sense' of these procedures by manipulating the numbers following learned rules.

Fleur: I'll just do my favourite one. Twenty-four. *(Writes 24, then -46 underneath it as in Figure 37)* Oh I haven't learnt how to rename. ...*(realising the 4 in 46 is bigger than the 2 in 24), but still believing this 'sum' to be possible, according to the instruction she has been receiving in the classroom, if only she knew how to 'rename'.)*

Researcher: So put 16, say. How would you do that one?

- Fleur: *(Changes the 4 to a 1)* Well I'd put a 1 above the 2, and you'd cross out the 4, um, 14 up here, but if you had a zero you'd put a 10.
- Researcher: I see.
- Fleur: And you'd put a 1 above the 2 and you'd cross out the top numbers and you'd go 'Fourteen take away six', and you'd go 'One take away one.'

$$\begin{array}{r}
 \phantom{1} \phantom{1} 4 \\
 2 \phantom{4} \\
 - 4 \phantom{6} \\
 \hline
 \phantom{1} \phantom{1} 6
 \end{array}$$

Figure 37: Fleur's 'renaming' (Late Year 4)

The term 'my favourite one' suggested that this particular 'sum' had been memorised. Fleur had in fact been seen on an earlier occasion to use this sum to demonstrate the skill she had supposedly 'learned'. However, she made an error in her choice of 24 subtract 46, [instead of 16] which *appeared* possible to her. The only barrier she identified to successful computation of this subtraction was the need for 'renaming' in the tens column where she would have to subtract 4 from 2. For Fleur, the abstract layout and procedure were of far greater significance than the sense of what the symbols and procedure represented.

Rochelle, too, enjoyed learning written mathematical procedures, from which she seemed to gain a sense of satisfaction and security.

- Researcher: What do you like most about maths?
- Rochelle: Working form.
- Researcher: Can you show me some of that? *(Rochelle uses a piece of paper to demonstrate. She works in silence. She writes  $45 \times 5$  in working form, then moves her pencil to indicate she is multiplying  $5 \times 5$ . She records 5, on the right-hand side of the answering space, then a small 2 above the 4. She then appears to be multiplying  $5 \times 4$ , and adding two. She produces the answer of 225, as shown in Figure 38).*
- Researcher: That's pretty impressive. So what's this little '2' up here, Rochelle? What was happening there?
- Rochelle: That's the ten, you put the tens up the top... and the ones on the bottom.
- Researcher: Was it hard for you to learn that?
- Rochelle: No.

$$\begin{array}{r}
 25 \\
 \times 5 \\
 \hline
 225
 \end{array}$$

Figure 38: Rochelle's 'working form' (Early Year 5)

Like Fleur and Toby, Rochelle was very taken with such methods. Later in the year, however, Rochelle reported that she had experienced difficulty with the working form involving the multiplication of two double-digit factors, and also called this kind of multiplication 'times tables' as had Toby. For these children the everyday term *times tables*, that originally referred to the multiplication facts to  $12 \times 12$  arranged in 'table' form, was used synonymously with every other situation requiring multiplication. Significantly, Rochelle had been taught this procedure the previous year but could not remember how to do it, suggesting that her learning was based on instrumental rather than relational understanding.

Rochelle: I know how to do everything Mrs Ponting gives me. Except for there's the step thing. I can't do the three-step.

Researcher: Is that for addition, subtraction, multiplication, division?

Rochelle: Times tables. We done it last year but I forgot. (Mid Year 5)

Liam also complained of experiencing difficulty in learning this procedure:

Researcher: Is there anything you've really hated?

Liam: Double-digit times. (Late Year 5)

None of the children incorporated checking or estimating into their learned procedures, although *Mathematics in The New Zealand Curriculum* emphasises the importance of this skill in learning to compute by repeating the achievement objective: 'make sensible estimates and check the reasonableness of answers' in each of the levels 1 to 4 of the number strand (pp. 32, 36, 40, 44). The children's lack of checking indicated that their knowledge of these calculation methods was procedural rather than based on an understanding of the underlying mathematical ideas. Edwards and Mercer (1987) describe the kind of knowledge presented in this fashion as *ritual* knowledge stating that 'procedural knowledge becomes ritual where it substitutes for an understanding of underlying principles' (p. 97).

Teachers' failure to discuss computational errors as a natural part of learning, left the children at risk of losing faith in the efficacy of these methods or of their own ability when such errors arose. The fact that all errors were treated equally - that is simply as 'wrong' - indicated that teachers were abdicating responsibility when it came to the

important task of analysing children's errors for evidence of their mathematical understanding.

Such algorithms (layout and procedures) for calculating, variants of which are commonly taught in primary schools today, have been in use for many centuries. Duncan (1959) notes 'it should be remembered that a standard setting out [of a sum] is a very refined, condensed form of the process and in the same way that it took a long time for such a form to be evolved in history of arithmetic, so it may take a long time for children to understand completely the final setting out which they are expected to use ... the ultimate goal is the establishment of a clear and well-understood pattern which can become a firm, efficient, habitual response' pp. 11-12). Because the setting out procedures, as demonstrated by Toby, Fleur and Rochelle, were used exclusively in all of the sample schools, the children had no idea that these problems could be approached in other ways. It might have been useful for teachers to have introduced alternative methods. However, teachers were strongly opposed to this, as Ms Sierra's comments show:

Ms Sierra: I do renaming, but my concern is that not all teachers are doing that so ...I was talking about it this morning during morning tea with the others 'cause I want, um, a uniform way of doing it that the children won't get confused. That's all right with the strong ones, but the weak ones, once they change that they think it's a new method but it's just the other method of doing it... The parents, they have their other methods. Kids come back with 'This is how my mum told me to do it'. I feel like ringing them! (*Rolls her eyes*) (Early Year 4)

Ms Sierra seemed not to consider that her insistence on a uniform method might also be confusing and that she could profitably build understanding on alternative methods children brought from home. Her belief that only the 'strong' learners were capable of exploration of alternative methods, while the 'weak' ones were better off when taught one procedure only, indicates that she believed that deep understandings of the mathematical ideas involved in such calculations were only possible for a minority of students.

It can be seen that teachers, parents and textbooks all contributed to the propagation of the view that the currently accepted norms in written algorithms were the only right

way to perform calculations. Difficulties arose when the customary methods of one generation differed to that of the next. Uniformity of procedure appeared to be valued by the teachers for its simplicity and manageability, rather than its enhancement of mathematical understanding.

### **Right and wrong – creating high risk classroom environments**

In examining the nature and socialising function of students' experiences of everyday school mathematics, Ernest (1998a) notes the conventions of the classroom:

An analysis of the linguistic forms used and the types of mathematical activity most common in school mathematics suggests the overwhelming presence of imposed tasks in which the learner is required to carry out symbolic transformations. During most of their learning career in school, from five to sixteen years and beyond, learners work on textual or symbolically presented teacher-set tasks. They carry these out, in the main, by writing a sequence of texts ... ultimately arriving, if successful, at a terminal text - 'the answer'. (p. 223)

He estimates that, over the course of their compulsory education, students carry out tens of thousands of such individual mathematics tasks, and comments that:

the sheer repetitive nature of this activity is underaccommodated in many current accounts of mathematics learning, where the emphasis is more on the construction of meaning than on the acquisition and deployment of semiotic tools and, given that the texts are being produced and marked, the rhetorical style of school mathematics. (pp. 223-224)

For the children in the observation classrooms in which such everyday patterns presented mathematics in question-answer form, getting the right answers became not only the purpose but also the substance of mathematics.

Children were frequently observed to gain a deep sense of satisfaction from having produced 'correct' answers, including punching the air triumphantly, or exclaiming 'Yes!' or 'I got them all right!' or 'I got none wrong!' as they marked their work. This was more noticeable among certain groups of boys. At the end of a marking session, children were observed to compare their results: 'I got one wrong'; 'How many did you get?' Discussion of solutions on the other hand, was never observed.

Researcher: Do you ever talk about mathematics with your friends?

Dominic: Well, sometimes we talk about the scores, but that's pretty much it. (Early Year 5)

For some of the children, the classroom emphasis on getting answers 'right' produced feelings of anxiety, particularly when questions were asked of them in front of their classmates. In Year 4, Fleur was observed to become very red-faced and flustered when asked for the answer to 'three times four' as the class were seated on the mat. She gave the incorrect answer of 'ten', and although the teacher was patient and asked further questions to help Fleur reach the correct answer, it was clear that Fleur felt upset and embarrassed. Fleur's teacher said that Fleur rarely put up her hand to volunteer answers to questions. Consistent with the findings of Anderson (2000) and Anderson and Boylan (2000) who investigated the links between teacher questioning and pupil anxiety in mathematics classrooms, Fleur had come to view much mathematical learning as a painful and mathematics as a subject that was potentially threatening and humiliating.

Within the question-answer, right/wrong environment, many of the children were apprehensive about contributing answers or 'having a go' as their comments show, when asked about how they felt about putting up their hands to answer questions:

Fleur: Sometimes when you haven't got it right, she goes, 'Wrong!'

Researcher: Do you usually put your hand up?

Fleur: Sometimes. Sometimes, if I get it wrong, well...but if I'm, if I think that it, that my answer is right, then I'll put it up. (Mid Year 5)

Georgina: I just say the answer that's in my book. Sometimes I get them wrong and I don't like it when the teacher shouts at me.

Researcher: What does she say?

Georgina: (*Mimicking the teacher's voice*) 'Sorry! You've got it wrong!' (Mid Year 4)

Jessica: (*About being picked to answer a question*) 'Oh no, I don't know the answer and she's going to ask me'. Sometimes it gets really freaky and you know, you're going 'Don't pick me, don't pick me, please don't pick me,' and you know, you get so freaked out that she's going to pick you and oh, I'll be so embarrassed that she's going to pick me.

Researcher: Has that ever happened, that she's picked you and you don't know the answer?

Jessica: Yeah, I think it has probably happened once or twice.

Researcher: How do other people react?

Jessica: They just sit there and stare at you and stare in space waiting to hear the answer and they're thinking, 'Oh I know that, that's easy'. Sometimes I do that when she's asked someone who isn't paying attention. (Mid Year 5)

This is consistent with children's views on public exposure explained in Chapter 4. The work of Denvir et al (2001) provides accounts of the protective strategies children use when participating in whole class questioning, as advocated by the Numeracy Strategy in the UK. They argue that 'the strong 'performative' element ...prompts children to adopt classroom behaviours which mitigate [militate] against them developing good habits as learners' (p. 344). These behaviours include taking fewer risks, copying others' answers rather than relying on their own thinking, and abandoning contemplative methods in favour of speedy ones in the competitive classroom atmosphere where correct and speedy answering is valued by teachers and peers.

Some teachers noted children's feelings about right and wrong:

Ms Fell: (*Of Fleur*) She's little bit worried to take risks at the moment and, you know, wants to do it right.

Mrs Matagi: (*Of Liam*) He loves to be right, that's a real buzz for him, quite a big deal for Liam to have success, I think he feels, um, yeah, he doesn't like to get it wrong.  
(Early Year 5)

Ms Summers: They [children] have a fear of getting things wrong, so often sort of breaking through that can be a challenge. (Early Year 3)

Parents too commented on their children's feelings about being right or wrong:

Liam's mother: He's funny. He's confident, then sometimes he's not. When he gets something wrong, he thinks he's silly, he doesn't know what he's doing. When he's doing it right, he's super confident. He likes to know what he's doing. If he struggles with it, he thinks he's stupid. (Early Year 3)

No teacher in the study challenged the view that mathematical answers or procedures were either right or wrong, and no teacher commented on any problems inherent in



presenting mathematics as a right/wrong discipline despite the clear directive in *Mathematics in the New Zealand Curriculum* to teach mathematics in a variety of ways, including problem solving:

Closed problems, which follow a well-known pattern of solution, develop only a limited range of skills. They encourage memorisation of routine rather than consideration and experimentation... Real-life problems are not always closed, nor do they necessarily have only one solution. Determining the best approximation to a solution, and finding the optimum way of solving a problem when several approaches are possible, are skills frequently required in the workplace. Students need frequent opportunities to work with open-ended problems. (p. 11)

The teachers' appeared to view, and therefore teach, mathematics as a body of unassailable truths and methods that they must transfer to the children, and that this knowledge was constituted of a shared and commonly understood set of facts and procedures. Commonly, only one method was believed to be appropriate in any given situation. Teachers implied this in the ways in which they interacted with the children during the learning and practising phases of the their lessons. The widespread use of closed questions and specific instructions requiring one right response, were indicators of this. Goldin (2002) describes this kind of teaching as the *traditional* view of mathematics education, which he describes as being based on the following characteristics:

- Specific, clearly identified mathematical skills at each grade level;
- step-by-step development, abstracted or generalized in higher level mathematics;
- that much of mathematics is structured hierarchically, with more advanced techniques presupposing mastery and a certain automaticity of use of more basic ones;
- standards that are measurable;
- expository teaching methods are valued, including considerable individual drill and practice to ensure not only the correct use of efficient mathematical rules and algorithms ;
- the correctness of students' responses;

- children differ greatly in mathematical ability so that significant numbers of them may not have the capacity to succeed in higher mathematics; for these children, achieving the basics is especially important;
- class groupings should be homogeneous by ability at least after a certain grade level, to permit advanced work with high-ability students and attention to the basics with slower learners. (see pp. 199-200)

As previous chapters have shown, many characteristics of this view of mathematics were prevalent in the classrooms visited. However, such a view is contrary to the spirit and and letter of *Mathematics in the New Zealand Curriculum* which states that:

Students learn mathematical thinking most effectively through applying concepts and skills in interesting and realistic contexts which are personally meaningful to them. Thus mathematics is best taught by helping students to solve problems drawn from their own experience. (p. 11)

### **The universal language of mathematics?**

At age seven, the children were beginning to be exposed to the formal language of mathematics. This corresponds with the introduction of such language in other subjects, for instance writing and science. If mathematical 'seeing' was not always tailored to the needs of children, or built upon their experiences, nor was mathematical 'saying'. Not only were the children's unique ways of internalising mathematical ideas sometimes inconsistent with the teachers', there were also difficulties for the children with the language and context in which the tasks were embedded.

Mathematical terms were sometimes found to present a barrier to the children. It can be seen from the following statements that they sometimes found the use of formal mathematical terms confusing or off-putting, and something that mystified, rather than clarified.

Researcher: What's happening in maths for you, Fleur?

Fleur: We're doing our times tables. Factor times factor equals product.

Researcher: Oh? And how do you feel about that?

Fleur: I don't like it.

Researcher: Why not?

Fleur: It's too hard.

(Later)

Researcher: What's the best activity you've done in maths this year? Anything you have enjoyed? (*Long pause ...no reply from Fleur*) Let's look at the next one, the worst activity you have done in maths so far this year. (*On the recording sheet, Fleur writes  $F \times F = P$* ) Factor times factor equals product? (*Fleur nods sadly*)

(Later)

Researcher: Are things very different in maths this year with Mrs Heath?

Fleur: Very different.

Researcher: What are the most different things you notice?

Fleur: Well when we were in Mrs Field's class, we didn't know our times tables or anything. Or factor times factor, or what a factor was. We never had face (*pauses*) place (*pauses again*) face value, and products and factor.

Researcher: Some new words?

Fleur: Hm. (Early Year 4)

Children were sometimes found to misuse unfamiliar mathematical terminology.

Researcher: What is maths?

Rochelle: Numbers...and equations...

Researcher: Anything else?

Rochelle: Factor plus factor equals sum. (Mid Year 5)

Where mathematical terminology was unfamiliar, children would sometimes creatively replace words with something they knew.

Researcher: Are you on a hard subject [Georgina's word for 'topic'] at the moment?

Georgina: No it's really easy.

Researcher: Oh, what's the subject that you're doing?

Georgina: Cemetery. It was pretty easy because first we had to cut out a picture and copy that side, and there was this worksheet where we had to draw a pattern and on the other side we had to draw the same pattern. We were allowed to use mirrors too. (Late Year 4)

The different strands of the mathematics curriculum were something about which children were seen to be actively constructing meaning.

Jessica: I think it's called geometry, with all the shapes and everything and like, you know, those equilateral kind of things... (Late Year 5)

Dominic: We're studying algebra.

Researcher: What is algebra, Dominic?

Dominic: I don't know. All sorts of stuff.

Researcher: What sorts of things do you do for algebra?

Dominic: I think it's sort of like open equations ...I don't exactly know. (Late Year 5)

The specialised language of mathematics as used and misused by the children, seemed to define for them the nature of mathematics as a subject comprised of 'topics' such as geometry and algebra, of rules such as factor times factor equals product, of symbols, such as  $F \times F = P$ , and of specific terms such as *face value* and *symmetry*. This need not necessarily be problematic, but all the teachers seemed unaware of the potential to confuse, and rarely took sufficient time to explain new terms. *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) makes a distinction between the children's own language and formal mathematical language, and states that, among other important mathematical processes, the mathematics curriculum will provide opportunities for students to:

develop the skills and confidence to use their own language, and the language of mathematics, to express mathematical ideas. (p. 23)

The first of the achievement objectives in the *Communicating Mathematical Ideas* sub-strand of the *Mathematical Processes* strand, and an expectation of all students from Levels 2 to 8 states:

Within a range of meaningful contexts students should be able to: Use their own language, and mathematical language and diagrams to explain their mathematical ideas. (p. 28)

Zevenbergen (2001) describes the negative effects of the disjunction between the child's everyday language, and the specialised language of the classroom. Recognition of the curriculum statement's acknowledgement of the importance not

only of 'allowing' but also 'expecting' children to talk about mathematics in their own words, was rarely observed in over ninety classroom observations.

### **Teachers' views of children's learning and knowing of mathematics**

When teachers talked about the children's mathematical learning, a number of recurring phrases were used. One of these was the idea of the child 'getting', 'picking up' or 'grasping' concepts. There seemed to be an implication that this was the child's responsibility, rather than the teacher's, and that if the child failed to grasp a concept as presented by the teacher, then this was either a result of lack of attention to the teacher's explanation, lack of effort, or lack of ability on the part of the child. The teachers appeared to judge children's mathematical ability on the speed at which new concepts were 'grasped'. Where children learned mathematics more slowly, required several explanations of the same idea, needed the 'aid' of concrete materials, or failed to 'retain' what they had apparently learned, teachers seemed to regard these as signs of mathematical incompetence. Thus faster learners and those who recalled mathematical skills and knowledge were deemed to be more mathematically able.

Mr Solomon: (*Speaking of Georgina*) She takes a while to pick up a concept and run with it. She needs two or three sessions to catch on. Once she gets it she's away, but she doesn't always retain it. (Late Year 3)

Ms Seager: (*Speaking of Jessica*) She usually grasps new concepts quickly, although she found measurement harder. (Mid year 4)

Mrs Joiner: (*Speaking of Nicole*) I've got five children that are very quick and can connect ideas very easily. I don't think she'll match the top five, but I think she would be close to them. She does seem to be very quick to pick up number. (Late Year 3)

Mr Ford: (*Speaking of Dominic*) He picked up division facts quickly. (Mid Year 5)

Ms Flower: (*Speaking of Jared*) He doesn't actually listen to my instructions. He really rushes and sometimes he makes mistakes. (Mid Year 3)

Mr Waters: (*Speaking of Jared*) He's a real workhorse. He doesn't grasp it straight away, but once he's got it, it's hard to shake. (Early Year 5)

Mrs Matagi: (*Speaking of Liam*) With new work he takes a little time to think it through. He doesn't always get it first time. He needs work to consolidate. (Late Year 5)

Mrs Waverley: (*Speaking of Peter*) He's still taking time to get new concepts but not too long. (Late Year 4)

Miss Sanderson: (*Speaking of Peter*) He takes a while to grasp new things. (Mid Year 5)

Ms Firth: (*Speaking of Toby*) He's very quick to grasp concepts ...he's got a very sound base knowledge. (Late Year 3)

Mr Cove: (*Speaking of Toby*) He's one of those children that it takes one explanation and off he goes and he can do it quite confidently. (Mid Year 5)

What the teachers appeared to mean by 'picking up' or 'grasping' concepts was whether the children could readily assimilate and reproduce the prescribed ways of doing mathematics in the classroom.

When talking to, or about, the children, teachers conveyed the belief that learning mathematics was something that required considerable *brain power* as well as a specialised kind of thinking, as evidenced by the following remarks, many of which are expressed in very mechanical metaphors.

Mr Swift: (*Speaking of Dominic*) He can switch off. (Mid Year 4)

Ms Torrance: I didn't think you had your brain switched on, Matthew. It's on Mars!.. You clever cookies...Oh you switched on girl. (Mid Year 3)

Mrs Kyle: We're not thinking straight. (Early Year 4)

Mr Waters: See if you've got your brains into gear. (Mid Year 5)

Ms Summers: (*Speaking of children's learning of mathematics*) I believe children have some sort of innate...there's something there. It's quite obvious. I mean you can give them those skills that will help them but I think a child has either got it, or they haven't.

Researcher: And would you say Peter has got it?

Ms Summers: I think he hasn't really got, you know, that intuitively...lateral thought. (Early Year 3)

Since these kinds of remarks were not heard in the lessons for other subjects, it would seem that 'brain power' and brains 'in gear' were regarded as specific to mathematics, that *grasping* and *retaining* new concepts was contingent on tapping into this mathematical mode of using the brain, and that some brains were simply incapable of mathematical thinking even when the children were *given* the skills. As seen in Chapter 5 (p. 179), some of the children also used the word *brainy* to describe someone who is good at mathematics.

### Children's perspectives of learning and knowing mathematics

While some children talked of mathematics generally as being 'hard' for them, this was usually associated with particular parts of mathematics, as most were able to talk about tasks they found easy. Many children commented on the difficulty of learning mathematics, and the following comments show the alienating and marginalising effects that such difficulty was likely to produce. Their comments not only show that accessibility of tasks was crucial to their comfort during mathematics time, but also provide some insights into the coping strategies that children used when confronted with the difficulties of mathematical tasks, and the ways in which their perceived understandings of mathematical learning and knowing contributed to their mathematical identities.

Fleur: I wish I was a bit better [at maths] but I don't exactly mind that much 'cause not everybody's good at everything...Because I'm quite a bit slower, because I struggle. They [other people] know a bit more, and what they're doing, they get the point of it all. (Early Year 5)

Georgina: What do I like about maths? Oh, well, not really much about maths, but sometimes I like it and sometimes I don't... I can do some things, like one plus one, two plus two, and it's just hard to get some other things in maths like sixteen plus sixteen, that's hard! (Early Year 3)

Georgina: I don't like maths much because we do hard things and we have to do it. I just look at the paper and go 'Hmm' (*sighs deeply*) and I get told off. (Late Year 3)

Georgina: I sort of hate it when it's maths time. When we're on a hard subject. (Mid Year 4)

- Georgina: Well when we were doing fractions it was really hard for me and I didn't get it and they [other children] were just going, 'Oh yeah, I know that one!' and they got it straight away and they were correct.
- Researcher: How did that make you feel, that they could get it?
- Georgina: I just had to lie and say, like, there was this person, there's this girl, she sits next to me and she gets it right all the time, 'cause like when she goes, like, 'Seven', I just put up my hand and say, 'Seven'. (Mid Year 5)
- Researcher: What do you like most about maths, Jessica?
- Jessica: That it's sometimes hard and sometimes easy, sometimes they [teachers] give you different things to do.
- Researcher: What do you like least about maths?
- Jessica: When it's always hard. Maybe like one week or something you do something extremely hard. (Mid Year 5)
- Dominic: I'm very good at it [maths] and I learn lots of new stuff, like, I know what he [the teacher] means when he says things. Like people that aren't so good at maths don't usually understand...Although I'm real good at it, sometimes I get a bit lazy and can't be bothered.
- Researcher: Is it not very interesting for you?
- Dominic: Yeah, it's interesting, it's just sometimes I'm a bit bored of it. (Late Year 4)
- Researcher: How do you feel, Jared, when the teacher says 'It's time for maths'?
- Jared: I hate it.
- Researcher: Do you? Why?
- Jared: 'Cause it's hard.
- Researcher: So do you feel comfortable at maths time?
- Jared: Not really.
- (Later)
- Researcher: What things would make maths better for you?
- Jared: Easy work. (Late year 3)
- Liam: I like the work usually but some things are boring. Sometimes when I'm stuck and can't do it, it's boring and I go, 'What do you do?' to people and I ask for help but they don't [help] because they say they have to get on with their own work. (Early Year 5)
- Researcher: Why is the teacher giving you some work of your own?
- Mitchell: Because it's too hard for me [the other children's work]. (Mid Year 5)
- Researcher: Anything you've done in maths that you haven't liked much?
- Peter: Times tables...Because they're hard.



Researcher: Do you feel comfortable doing maths, Toby?

Toby: Yep.

Researcher: Why do you feel good?

Toby: I don't really know (*Pause*) Well, because she asks us quite a few questions, and they're good to answer, they're not, like, too easy for us or too hard for us. (Mid Year 4)

'Not knowing' caused serious difficulties for Fleur, who was absent from school for the two weeks when her class began learning the *times tables*. On her return, Fleur found herself 'behind' her classmates. The teacher suggested that she watch the others, copy what they were doing and assured her that she would soon catch on. No attempt was made to explain the missed work to Fleur. Fleur's feelings of exclusion, and her anxiety about lacking the knowledge of her peers, were so severe that this event marked a significant decline in her confidence, as both she and her teacher commented.

Fleur: (*Describing how she felt at maths time*) Well not so good because I haven't learnt my times tables 'cause I was away when they were doing the times. (Late year 3)

This lack of confidence was to continue and contributed significantly to her developing mathematical identity as a slow, and therefore marginalised, learner.

The notion of 'keeping up' with classmates and the of danger of 'falling behind' showed that mathematics was taught in a linear progression of sequenced learning events and that there was an expectation that children would and should acquire mathematical knowledge and skills at roughly the same pace and in the same order. Those who faltered like Fleur, for whatever reason, and at any point along this mathematical learning path, fell by the wayside and thus became casualties of the dominant approach to teaching mathematics as a rigid, sequential progression of learning stages.

### **The place and purpose of learning mathematics**

Mathematics is presented through national curriculum statements as an essential subject for children to learn, for example 'the need for people to be numerate, that is, to be able to calculate, estimate, and use measuring instruments, has always been

identified as a key outcome for education' (Ministry of Education, 1992, p. 7); 'the need to understand and be able to use mathematics has never been greater' (National Council of Teachers of Mathematics, 2000, p. 4), and 'numeracy is a key life skill' (Department for Education and Employment, 1999, foreword).

Through the frequency of mathematics lessons, and the pervasiveness of highly valued 'correct' answers and 'correct procedures' in the mathematics classroom, the children received strong messages about the importance of mathematics, the purpose for learning it, and its usefulness.

The children all expressed the belief that mathematics was a very important school subject. Table 6 shows the responses the children gave when asked in Years 4 and 5 which were the most important of all the subjects they learned at school. They have been recorded in the order in which they were mentioned.

**Table 6: The school subjects the children believed to be most important**

<b>Child</b>	<b>Year 4</b>	<b>Year 5</b>
<b>Fleur</b>	Handwriting , maths	Maths
<b>Georgina</b>	Maths, spelling	Maths
<b>Jessica</b>	Maths, health	English, science, maths, physical education
<b>Rochelle</b>	Maths	Maths
<b>Dominic</b>	Maths, reading	Reading, maths, science, spelling
<b>Jared</b>	Maths	Maths, handwriting
<b>Liam</b>	Language, maths	Maths, reading
<b>Mitchell</b>	Listening to the teacher	Work
<b>Peter</b>	Maths	Maths
<b>Toby</b>	Spelling, maths, handwriting	Maths, reading

In explaining why they learned mathematics, their answers suggested that mathematics was something they almost universally viewed as deriving solely from school, that was a taken-for-granted part of every child's education, and the responsibility of adults to pass on to the next generation. The following examples

show the various ways that the children responded to the question 'why do we learn maths at school?' For some, this was a difficult question.

Georgina: Um, that's a hard one. Because we need to learn maths.

Researcher: Any ideas why?

Georgina: No. (Late Year 4)

Toby: Well, just like you are sometimes learning the alphabet, you're learning numbers too.  
(Early Year 3)

Some of the children felt that the chief reason for learning maths was because, as future parents, they needed to know how to do maths in order to help their children.

Fleur: So that when we grow up and have children, the children know they just ask us, and we know the answer. (Late Year 3)

Jessica: *(Who has just rated mathematics 10 on the subject importance scale)* Like when your kid is stuck with their homework and they asked you like, 'Mum we're stuck on this' and they don't know what it is. And you really don't want them to use the calculator, or if you didn't have one, well then you'd have to help them but you won't know how to explain it 'cause you don't know how to do maths. (Early Year 4)

Others saw mathematics as useful later in life:

Rochelle: To be good at it when we're older. (Early Year 5)

Jared: I don't know.

Researcher: Is maths useful to know do you think?

Jared: Yes.

Researcher: In what ways is it useful?

Jared: For when you're older.

Researcher: How might you use it when you're older?

Jared: In school. (Late Year 5)

Mitchell: Because if they [children] get bigger they won't know anything (Late Year 5)

The older the children became, the more some of them viewed the purpose of learning mathematics as helping them to do well in tests and exams later on.

- Dominic: To help us. Just in case. For like if we have to, like, go through this big test or something, we will probably know all of them. (Late Year 4)
- Researcher: Which of the subjects you learn at school are the most important to learn do you think?
- Peter: Maths.
- Researcher: What is it about maths that's pretty important do you think?
- Peter: Um...um.
- Researcher: What would happen if you didn't learn maths?
- Peter: You wouldn't be able to, like, if at school they gave you a test and you had to get it finished, you wouldn't be able to get it finished. (Mid Year 5)

Only some of the children were able to see the wider use of mathematics.

Liam: We need to be able to add wherever we go. (Early Year 5)

Fleur: When I go to the shops I use maths. (Late Year 5)

Mathematics, as experienced by them, was something they generally felt they were obliged to learn for its usefulness in later life, either in more advanced schooling or as adults. Few children showed that mathematics had relevance and usefulness for them as children.

Parents also believed in the importance of mathematics as a subject for their children to learn. The following were typical comments:

Researcher: What are the most important subjects to learn at school do you think?

Peter's mother: My husband thinks maths is the most important subject at school. And when it comes to choice of careers, if you're good at maths, you've got a huge choice ahead of you. (Late Year 3)

Jessica's mother: I hope she takes maths right through secondary school. I mean I would hope that's going to be one of her main subjects. (Early Year 3)

However, they had difficulty when trying to think of family activities in which their child might be developing mathematical skills and understandings. Such views are

recognised by Appelbaum (1995), Dorfler & McLone (1986), and Lim & Ernest (2000), who investigate popular images of mathematics.

Because the children believed that mathematics mostly originated from, and was defined by, the enclosed school environment, the only mathematics that they recognised outside of school, was that which replicated school mathematics. When asked, 'You do lots of maths at school. At what other times do you do maths?' typical replies were:

Fleur: I sometimes do maths games on the computer. (Early Year 5)

Georgina: We only do it in the morning [at school]. (Late Year 4)

Rochelle: We've got this times tables book at home. (Early Year 3)

Peter: Only for homework. (Mid Year 4)

Dominic: We do times tables and plus at home. (Mid Year 3)

Liam: When my sister makes some questions for me. (Late Year 4)

Toby: Mum gives me basic facts to answer. (Early Year 4)

The children rarely conceived of mathematics as a body of related yet diverse skills and knowledge that all people, including children, use continually as part of their everyday lives. It might be assumed, that the adults in their lives, particularly teachers and parents, fail to either see, or make convincing links between the mathematics of the world of school, and the children's worlds outside of school, despite the fact that many of the study parents were heavy users of mathematics in their daily occupations, eg. marine scientist, engineer, insurance industry, shop assistant, banker etc.

### **Children's views of the nature of mathematics**

Of the things that the children said they had *learned* in mathematics since they had last been interviewed, number skills and knowledge featured most highly. Basic

facts, particularly 'times tables' were the most quoted, followed by working form addition, multiplication, division and fractions. When asked what they had particularly *enjoyed* in mathematics, activities involving drawing, working with equipment, co-ordinates, symmetry, and maths games were most often mentioned. It is notable that these two lists have few common elements.

Although the children could all talk about things they had done at maths times, when asked what mathematics *was*, some of the children found the subject difficult to define, as their comments show:

Georgina: That's a pretty hard question.

Researcher: What do you *think* it is?

Georgina: It's just another subject.

Researcher: What makes it different from reading?

Georgina: 'Cause you're counting. (Late Year 4)

Jessica: I think it's a subject like, when you're little you get like, one plus one, but when you're older you can still get one plus one so, like, it's just like every single maths question is like the same, but you've actually got to be good at maths to actually enjoy it. (Late Year 4)

Jared: Something we have to learn.

Researcher: What makes it different, from English or sport?

Jared: Sport is where you go outside and maths is when you're inside.

Researcher: OK, so what makes maths *maths* do you think?

Jared: When you do pluses and takeaways ... and times. (Late year 5)

Mitchell: Um...(Long pause)

Researcher: Is it the same as reading or writing?

Mitchell: No.

Researcher: What's special about maths? What's it all about?

Mitchell: You have to learn. (Late Year 5)

Other children could more clearly articulate their views about what mathematics was. They almost invariably cited number skills and knowledge as being 'mathematics'. The tendency for children to view number and the basic operations as the sum and substance of mathematics is widespread, (Cotton, 1993; Kouba & McDonald, 1991; McDonough, 1998; Perlmutter et al., 1997; Spangler, 1992).

- Researcher: What is maths, Fleur?
- Fleur: Adding, subtraction, divided by, division ...is that true?
- Researcher: Well it's just what you think it is. You do lots of it at school.
- Fleur: Well, we sometimes do fractions. (Early Year 5)
- Rochelle: Um, numbers...and equations...factor plus factor equals sum. (Mid Year 5)
- Liam: It's take away, times, dividing, plus, measuring and things like that. (Early Year 5)
- Peter: It's all these sums...and counting. Questions...you answer them. (Late Year 4)  
Maths is just like doing times tables, pluses, subtraction, division, fractions and stuff.  
(Mid Year 5)
- Toby: It's when you've got numbers and you can do all sorts of things with them. (Mid Year 5)

Dominic was the only child of the group who appeared to have developed any sense of the underlying processes of mathematics and connections between different facets of the mathematics he experienced in the classroom:

- Dominic: Um, working things out, and ...counting, and...patterns and shapes, different sizes and...yeah. (Late Year 5)

This sophisticated understanding was not acknowledged by his teachers, who interpreted Dominic's thoughtfulness as 'dreaming' (Ms Torrance) or 'switching off' (Mr Swift). Fortunately, Dominic knew better, stating that neither his teachers nor his parents really knew what he could do in mathematics.

- Dominic: Mr Swift would probably put me at about 9. (*compared with his self-rating of 10*)  
Because he usually doesn't see things. (Late Year 4)
- Dominic: (*When asked how his parents thought he was getting on in maths*) I don't think they know. (Early Year 4)

Through their definitions of mathematics, the children demonstrated that within their social worlds of home and school, they had developed personal understandings of what mathematics *was*. Because teachers and parents were largely unaware of children's views, such perceptions remained largely unchallenged or unsupported.

### The home dimension

Most of the parents became involved in their children's learning of mathematics through helping them with mathematics homework or their learning of 'times tables'. While the degree of involvement varied, the production of right answers and application of correct methods were generally reinforced by parents through these activities. Parents in helping their children with mathematics at home, would by and large replicate the I-R-F interaction patterns of teaching observed in the classrooms.

Georgina: He [Dad] tells me what to do and then I write it down, and then I tell him what the answer is and he says if it's correct or not. (Early Year 4)

Georgina: In my maths book the first week there was these really hard working forms and my dad's, like, taught me how to do the first one and then I got to do it and I got all of them right without anyone helping me. (Early Year 5)

Jessica: Well sometimes my Mum pulls out these cards and there are these questions she asks. (Mid Year 4)

Toby: Mum, she made up a big sheet of multiplication questions and I had to do every single one of them in the fastest time I could... 'cause when you know multiplication, you know division. (Late Year 5)

As already seen from Ms Sierra's comments (p. 262), teachers worried about lack of consistency in what the children were taught regarding the 'right' methods of doing mathematics. So too did parents, as this example shows:

Fleur's mother: When they did the power of ten. On the first day of that she came home with her homework and couldn't do it, and when I looked at it, I couldn't work out how they'd taught her. I got her father to look at it when he came home. He went through it with her patiently for three-quarters of an hour. With beads. And once she'd done that with two or three, after that she was right. It's the first time I've heard her say 'This is stupid. I can't do it.' If her father hadn't been able to help her, I would have had to go to the teacher and ask her to show me. I want to make sure I'm saying the same thing as the teacher. (Late Year 3)

Baker and Tomlin (2000) have explored issues arising in the relationship between home and school numeracy practices, and note that while homes contribute



significantly to the education of children, their role in mathematics education is viewed by the school as subservient.

### Teachers' views of mathematics

As the following extracts show, teachers' own experiences of learning mathematics at school appeared to have contributed significantly to their feelings about maths and to the ways they viewed and taught the subject, which in turn affected the children in their classrooms.

Mrs Waverley: I can make reading fun, but I can't make maths fun. ...*(Later)* I actually love maths. I think it's because I like the organisation. I take it first thing in the morning for forty minutes because I think 'Oh, I've done that now,' and I don't need to worry, so we take it religiously, and so devotedly. (Early Year 4)

Mrs Washbourne: I failed School Certificate maths twice.... Maths is not my favourite [subject to teach]. I try to appear passionate about maths for the children because it's so critical, but actually every day I think 'Ugh!', but doing this [cross-grouping system] with a really good thing to follow [Wellsford programme] is making me a lot more confident about my maths teaching. (Early Year 5)

Mr Solomon: I enjoyed maths at school, especially the straight out number problems rather than the lateral thinking, I don't know if I'm a real lateral thinker, so it's just basic number sentences and that sort of thing [that I like] ...The hardest [part of maths to teach] is teaching place value. I just sort of found I was banging my head against a wall at times and didn't know where to turn, you know, 'Where do I go from here? The kids just haven't picked up on it through this way, so...?' Yeah that's the hardest I've found. (Early Year 3)

Mrs Ponting: I enjoy teaching it [maths] but I didn't really enjoy it as a student. I went into book keeping, I suppose. Hm. Got confused with that too...I try my best to do what the objective [from the curriculum] says. Sometimes I wonder if I'm doing it the right way. (Early Year 4)

Mr Swift: I used to enjoy it [maths] at primary school but secondary school it, ah, I didn't understand it that much and I failed...Never grasped the concepts of it... I'd like to say it [classroom maths programme] is stimulating, but the proof would be in the pudding. Um, yeah, I try to get hands on, outside, make it real life situations but sometimes it's easier to photocopy out of the book. (Early Year 4)

- Mr Ford: I did a B.A. and did Statistics because I needed to, it wasn't something I chose to do...I got into other kinds of maths areas like chaos theory and fractals, how maths relates to all that kind of thing...I'm right into Science Fiction you see. And Stephen Hawking and his black hole theory, so that's why I probably enjoy maths because I like that kind of stuff.
- Researcher: So do you feel your enjoyment of maths helps you to teach it?
- Mr Ford: Sometimes what we're working on in the classroom, is just difficult to make exciting.
- Researcher: What things in particular in maths do you find hard to make exciting?
- Mr Ford: Place value. (Early Year 5)
- Mrs Isles: I do like maths. I always liked that feeling, there has to be an answer, you just have to work the process out, there is an answer there. Yeah, it is, it's logical.  
(Early Year 5)

There is a considerable body of research evidence linking teachers' experience of mathematics and their beliefs about its nature, to their competence, confidence and style in teaching the subject (Ernest 1989; Haylock, 1995; Thompson, 1992). As these reflections show, lack of enjoyment or success in mathematics in their own schooling was a common experience for many of these teachers. For some, the alienation they themselves had felt, produced long-term effects.

Irrespective of their enjoyment of teaching mathematics, the teachers in this study seemed to hold common views about the nature of the subject. The 'right answer' and the 'right way' were recurring themes that shaped their teaching approaches. While they seemed to believe that 'making maths fun' might assist the children's learning, this presented dilemmas for them when faced with certain prescribed objectives, such as place value, that appeared to be impossible to present in interesting ways. This suggests that they viewed the subject as constituting facts and rules that it was their responsibility to pass on to the children through time-honoured pedagogical practices they believed to be the 'right way'. Voigt (1995) states 'according to folk beliefs, the tasks, questions, symbols ... of mathematics lessons have definite, clear-cut meanings' (p. 167), and goes on to argue that contrary to this belief, studies of microprocesses in classrooms reveal instead, ambiguity and individual interpretation. The view of mathematics as creative, dynamic and open to multiple ways of *knowing* was not apparent in the teachers' statements, or in their teaching.

When the teachers were asked about their inclusion of mathematical processes<sup>1</sup> in their teaching and assessment of mathematics, their responses revealed a very restricted understanding of this dimension of mathematical *learning* and *knowing*.

Ms Torrance: Processes? What are they again? (Mid year 3)

Mrs Cayo: I'm limited in understanding some of the problem solving. But also linking those processes, I mean there's so many things in it. Maths is not a simple subject, is it? (Early Year 4)

Mr Waters: Problem solving is a great one. Putting equations into sentences, that's something that I keep telling my kids a big one is to read the question before they put their brains into gear. (Early Year 5)

Ms Seager: What we are going to do next term is have a number focus Monday to Wednesday and then Thursday more topic-based, hands on, then Friday is problem solving. (Early Year 4)

While most were aware of the curriculum emphasis on 'problem solving' many teachers appeared to have interpreted this to mean routine number calculations posed as 'story' problems. This narrow interpretation of problem solving contrasts with the definition provided by *Teaching Problem Solving in Mathematics: Years 1-8* (Ministry of Education, 1999b): 'a problem is a problem when there is something that stops you from immediately going to the answer...it is often unclear at the start what strategy students need to use to solve the problem...a problem should be something that interests the students and that they definitely want or need to solve' (p. 9).

Mathematical processes were not found to be integrated into the teachers' mathematics planning despite the teaching requirements of *Mathematics in the New Zealand Curriculum*. The not untypical school policy of placing of problem solving on Fridays or during special times of the year, indicated to many pupils that this was not 'proper' maths, a phenomenon described by Clark (1999). The comments above suggested strongly that teachers were uncomfortable and possibly incompetent in this area. Counter to the curriculum, their attitude did not support a multifaceted view of

<sup>1</sup> Mathematical processes is the first of the six strands of *Mathematics in the New Zealand Curriculum* and describes the 'how' of doing mathematics as compared with the 'what' of the five content strands. The curriculum states that 'The mathematical processes skills – problem solving, reasoning, and communicating mathematical ideas – are learned and assessed within the context of the more specific knowledge and skills of number, measurement, geometry, algebra, and statistics.'

mathematics, which may in turn have encouraged in pupils a similar narrow and limited idea of the subject.

### Mathematical identity

Lave and Wenger (1991) stress the link between *learning* and *knowing*, which they see as essentially social in nature, and *identity*. They state that:

as an aspect of social practice, learning involves the whole person; it implies not only a relation to specific activities, but a relation to social communities – it implies becoming a full participant, a member, a kind of person. In this view, learning only partly – and often incidentally – implies becoming able to be involved in new activities, to perform new tasks and functions, and to master new understandings. Activities, tasks, functions and understandings do not exist in isolation; they are part of broader systems of relations in which they have meaning. These systems of relations arise out of and are reproduced and developed within social communities, which are in part systems of relations among persons. The person is defined by as well as defines these relations. Learning thus implies becoming a different person with respect to the possibilities enabled by these systems of relations. To ignore this aspect of learning is to overlook the fact that learning involves the construction of identities. (p. 53)

The right/wrong of mathematics as presented and defined by teachers, textbooks, families and classmates through social interaction, contributed significantly to the children's beliefs about their own and others' mathematical ability. Along with speed and completion, as described in Chapters 4 and 6, 'rightness' and 'wrongness' was cited by the children as one of the key indicators by which they gauged their own and others' competence, and hence shaped their mathematical identities.

Fleur: *(Explaining why she has rated herself 7 on the self-rating scale for ability in mathematics)* Because I don't always get all of my times tables and take aways right. (Late Year 3)

Georgina: There's a boy in our class that's at my group and he always gets them right. When we do something like divided by, he always gets the right answer.

Researcher: Why is that do you think?

Georgina: Because he's brainy. (Late Year 4)

- Researcher: How do you know that you're not very good at it [maths]? What makes you think that you're not very good?
- Jessica: Well sometimes I get everything wrong and stuff like that. (Late Year 3)
- Researcher: How do you know they're [other children] good?
- Jessica: Because they're the ones that are always answering questions [correctly]. (Mid Year 4)
- Researcher: How do you know you're getting better at maths?
- Rochelle: Because I get most of the answers right. (Mid Year 4)
- Dominic: I'm one of the best in the class... 'cause every basic facts test I get eighty out of eighty. (Late Year 4)
- Jared: (*Reason for rating himself best in the class along with Justin*) Because we both know all the answers. (Early Year 5)
- Liam: (*Having rated himself 0 at maths on the self-rating scale*) Because I do things wrong. (Late Year 3)
- Mitchell: (*Reason for not being as good as others at maths*) Because they're doing it right and I get some of them wrong. (Late Year 4)
- Peter: (*Having rated himself 9 at maths on the self-rating scale*) I mostly get everything right. (Early Year 4)
- Toby: (*Explaining his improvement in mathematics*) We have a test each week and you get a certificate if you get them all right.
- Researcher: Have you got a few certificates then?
- Toby: Yes. (Mid Year 5)

For the children, 'rightness' or 'wrongness' were critical criteria of competence, while creativity, or effective methods used to solve difficult or non-routine problems were not considered to be measures of mathematical ability, as Frid (1993) has also observed. This impoverished view of *knowing* mathematics, seen only as the ability to reproduce correct answers and procedures, led the children to abandon intuitive or alternative thinking. Correspondingly, they came to regard *learning* in mathematics as the process of memorising these correct answers and procedures. They had learned to ignore their own and others' perspectives and methods, and to develop an almost total reliance on the external evaluations of teacher and text.

## Conclusion

Dominant images of mathematics within the social structures of school, home and popular culture contributed to the children's beliefs about mathematics, which they came to view as a subject consisting of questions requiring the right answers produced through the accepted methods. Their beliefs about *learning* and *knowing* mathematics were tied to this perception of mathematics as either right or wrong. The epistemological beliefs of teachers' who viewed mathematics as a subject consisting of non-negotiable facts and procedures which must be passed on to the children, and parents' who viewed mathematics as a discipline consisting of important life-skills that their children would need as adults, were demonstrated in the ways they interacted with the children during their learning of mathematics. For those children who had developed alternative methods of doing mathematics, such beliefs undermined their ways of perceiving, learning, and knowing.

Mathematical 'questions' were invariably posed by an exterior authority such as the teacher, parent or textbook, rather than by the children themselves, and answers were judged for 'correctness'. Accuracy was routinely rewarded, and errors routinely overlooked for their potential to assist learning. Understanding of underlying concepts or application of mathematical skills in solving meaningful and purposeful problems was routinely treated as of secondary importance to production and reproduction of correct answers and methods as defined by teachers and textbooks. This resulted in impoverished mathematical learning for the children.

Mathematics appeared to hold little relevance for children in their lives beyond the classroom, and came to be associated almost exclusively with school or school-based tasks such as homework. Learning environments dominated by the demand for 'right' answers, were threatening and intimidating for all but the most confident children. For those who learned quickly and easily, mathematics was satisfying and success produced a powerful sense of achievement. For the majority of the children the fear of 'being wrong' created strong feelings of insecurity at mathematics time and for some, a profound sense of alienation.

### CONCLUSIONS

The aim of this research was to describe the sociomathematical worlds of the children, in particular, to identify the significant interactions, objects and views of 'self' (identities) peculiar to those worlds. The research also sought to discover how the interactions *within* those worlds contributed to the children's negotiation of meaning *about* those worlds, and what aspects of those worlds appeared to enhance or inhibit the children's learning of mathematics. In light of these questions, this chapter provides a summary of the research findings, considers the implications and limitations of the study and suggests areas for future research.

#### **What does the sociomathematical world of a child 'look' like?**

The concept of the sociomathematical world of the child was developed to provide a framework for the description of the complex and dynamic interplay between the child and the objects, language and social interactions within, around, and through which each child made and remade meaning about mathematics. By dipping into those worlds (Blumer, 1969) and piecing together evidence from the direct accounts of the participants, archival material, and observations of classroom settings and typical everyday events, the research was able to provide insights into the lived experiences of the children within these worlds. It was found that there were elements of the sociomathematical worlds of the children that had a marked impact on the ways the children 'did' maths, the ways they viewed maths, and the development of their mathematical 'selves'.

Chapters 4, 5, 6 and 7 have focussed on what the research revealed as four distinctive common features of the sociomathematical worlds of the children. These chapters have documented, through evidence from a variety of sources including teachers, parents, classrooms, mathematical texts, curriculum publications and most importantly the direct accounts of children themselves, the entrenched and pervasive



nature of these widely accepted practices in the teaching and learning of mathematics. They were:

- the extensive use of mathematics speed activities;
- the identification and separation of children based on socially constructed notions of mathematical ability;
- the everyday nature of 'doing mathematics' as individual written work;
- the domination of the teacher, parent or textbook as the questioner who seeks and values the one *right* answer or procedure from the learner.

Speed activities that took the form of lists of quick questions to start the lesson, regular timed tests, timed oral performances or competitive games, the memorisation and rapid recall of basic facts, particularly 'the times tables', were found to dominate mathematics teaching and learning in Years 3, 4 and 5. The belief that without this skill, children would be denied access to all further mathematical learning, was found to be widespread within schools, teaching resources, curriculum publications and homes. While there were few indications of the facts being taught to the children at school in ways that would build understandings of the underlying principles and usefulness of these facts, there was strong evidence of frequent testing of basic facts recall. In most cases, teachers held the child responsible for memorising basic number facts and such learning was expected to occur at home.

Mathematical 'ability' as expressed by teachers, parents, textbooks and policy-makers was found to be an arbitrary notion based on beliefs about the naturally-occurring distribution of achievement in an hierarchical and often trichotomous arrangement (top, middle, bottom). Mathematical talent was widely viewed as innate and fixed. In most cases, teachers gauged mathematical ability by children's performance in written tests, in which speed and accuracy were the primary measures, rather than evidence of mathematical *understanding*. The teacher always determined group placement, promotion and demotion. As a result of the narrow range of assessment methods, misclassifications of students occurred, for example Georgina, Dominic and Mitchell, whose mathematical skills remained largely undetected and therefore underdeveloped. Commonly used assessment practices in mathematics were found to be both inaccurate, and acted as a filter that excluded those children for whom timed written



tests were not the best way to demonstrate their skills and understandings. All the children were subject to some form of ability grouping in mathematics at some time during the three years of the study. Children placed in the bottom groups tended to remain there over the three years of the study, and in spite of children's competencies in other areas of mathematics such as Georgina's spatial understandings, it was numerical calculation skills that were considered the most significant determinant of mathematical ability.

'Doing mathematics' was constructed within the daily life of the classroom as a solo pursuit, consisting predominantly of completing set written tasks on worksheets or in mathematics exercise books. Teachers or children marked this 'work' with ticks, dots or crosses. Teacher feedback was principally about presentation, or took the form of non-specific comments such as 'Good work.' The use of concrete materials was not generally encouraged during this 'work' time, except for the few who still 'needed' such aids. As the children progressed from Years 3 to 5, concrete materials disappeared, and textbook use increased. Discussion was not generally considered desirable or even necessary within this ethos of 'doing maths' as academic 'work'.

Mathematics was presented to the children as an important, discrete subject consisting of fixed facts and procedures. Teachers generally presented new concepts to the children based on a transmission model of teaching whereby they demonstrated new concepts to the children, who were then expected to practise and 'master' them. Much teacher interaction took the form of triadic dialogue (Lemke, 1990). The children were rarely seen to be encouraged to ask their own meaningful questions, and were actively discouraged from using processes that diverged from those taught in the classroom.

These features of the everyday lives of the children were found to be embedded within the 'cultural scripts' (Stigler and Hiebert, 1997) of classroom routine and parent/child interaction, and as such, were largely taken-for-granted. They were not seen as stand-alone characteristics of the children's sociomathematical worlds. Rather, they occurred as interwoven themes within many of the classroom and homework mathematics activities, and constituted the key philosophical and pedagogical structures of the children's mathematical environments.

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**How do the interactions within the sociomathematical worlds of the child contribute to the child's negotiation of meaning about that world?**

The research illustrated how children came to understand their roles as learners of mathematics through their daily experiences of mathematics both in the classroom and at home. The children developed beliefs about *learning* and *knowing* mathematics, and about their own and others' mathematical competency, and personal understandings about the nature of mathematics. These understandings were observed to evolve over the three years of the study.

The children absorbed and constructed meaning about mathematics from the familiar and regular activities of their classrooms. These social practices came to differentiate for them the learning of mathematics from the learning of other subjects, and to define for them, the nature of mathematics itself. Some of the practices that constructed mathematics as an important subject compared with other subjects were the length and placement of the daily delivery of mathematics lessons, the regularity of mathematics homework, and the testing and 'marking' of children's mathematics work as right or wrong. Practices peculiar to mathematics were the use of specialised graph paper exercise books for mathematics, mathematics work as answering questions from examples out of a textbook or on a worksheet, the 'marking' of work, and the lack of display of mathematics work compared with other kinds of classroom activity, and speed recall tests, games and competitions. In common with other forms of academic work, mathematics was presented as essentially an individual and a written form of activity with little discussion or concrete aids allowed. Apart from reading, ability grouping was rare in the teaching of primary school subjects but commonly used for mathematics.

The recurring emphasis on speed observed in all classrooms, contributed to the children's beliefs that 'real' mathematics consisted of externally-imposed questions, based primarily on basic number facts. Within the speed culture of mathematics, most of the children felt anxious or nervous. For many children in the group, learning the times tables was difficult and demoralising, and came to define for them both the nature of mathematics and their own mathematical identities. The 'fast' children were viewed as the most mathematically able, while slowness was equated with

incompetence. The children viewed failure to come up to speed as either as an innate personal shortcoming, or because mathematics was simply 'hard', while success was seen as the result of 'smartness', or lots of practice.

For many of the children, labelling and group placement by ability was another critical facet of everyday life by which they determined their own and others' mathematical competence, as evidenced by this research. They explained such differences either as an innate attribute of the learner ('it's a gift thing'; 'I'm a slow learner'; 'I'm smarter than them'), as a result of the inherent difficulty of mathematics as a subject ('maths is hard') or as a result of work habits ('They don't listen').

Through daily exposure to mathematics lessons and homework tasks based largely on questions on the board, worksheets and pages of the textbook, children came to regard doing 'proper' mathematics as an individual process of writing answers in response to externally imposed questions. They gauged their own and others' mathematical competence by how neatly and how fast they could complete these tasks and by how many ticks or stickers they received from the teacher. Many of the children identified collaborating with others and use of concrete materials as learning methods which they not only enjoyed, but viewed as important in assisting their learning, and yet they came to regard these activities as less 'proper maths' than written work.

As a result of daily immersion in the question-answer right/wrong culture of mathematics learning, the children came to view doing mathematics as the memorisation and production of correct answers and correct procedures. While the children viewed mathematics as very important, since it was highly valued by teachers and parents, they had developed an impoverished view of the subject, based on limited conceptions about its nature, and the belief that it was of little relevance to their daily lives.

While there were notable differences in the ways the children responded to these pedagogical practices, illustrating how learning and sense-making processes are highly particular and personal, their responses were also consistent with a recognised range of children's coping strategies (Pollard and Filer, 1999). In response to the discomfort experienced during mathematics time, Fleur became 'helpless' and heavily

reliant on her friends or the teacher, Georgina looked at others' answers and claimed them as her own, Jessica, Rochelle, and Peter feigned work poses rather than admit to needing help, Mitchell ran away (literally) from tasks that were too hard, Liam berated and belittled himself, and Jared wrote down any answer to 'fill the gaps' rather than seek assistance.

When the children's responses to the questions 'How do you feel when you do maths?' and 'Do you think you are good at maths?' were collated and presented graphically to show the changes over the three years of the study, significant trends and relationships were revealed (Appendix 11). Enjoyment and self-efficacy beliefs were usually linked, although not in every case. An increase or decrease in enjoyment often accompanied a corresponding movement in self-perception of mathematical competence, evidence of a significant connection between the two. Self-efficacy beliefs were in turn linked to the children's interpretations of how ability was constructed through assessment and grouping practices in the classroom.

The children's self-ratings on the enjoyment and 'ability' scales were found to be strongly influenced by the everyday practices described above. Dramatic changes were observed for some of the children, and these were invariably found to be linked to specific events or alterations in the children's social environments, such as placement in the bottom group for a fractions unit as experienced by Fleur (Mid Year 4), the move to a school with a more supportive mathematics learning environment as in Georgina's case (Early Year 5), group demotion as the result of a written pre-test as in Liam's case (Mid Year 5), or the introduction of oral speed activities resulting in demotion to the special needs group as happened in Jared's case (Late Year 3). The changing feelings and beliefs of the children point to the dynamic nature of their sociomathematical worlds and the meanings created within them.

### **What aspects of the child's sociomathematical world appear to assist or inhibit the child's learning of mathematics?**

For Rochelle, Dominic, Toby and Liam, speed activities fostered strong feelings of competence and success. While speed activities motivated these children to practise

and memorise the basic facts at home, for the others these activities were a major contributor to a sense of failure and incompetence. Because it was expected that the facts be learned at home, this presented difficulties for a number of the children for whom appropriate family support was limited. In no case were such activities seen to contribute to the development of the kinds of relational understandings of number combinations that were likely to assist memory and recall, and lead to meaningful application.

Ability grouping was seen to either inhibit the children's learning or to make insignificant difference. Labelling, sorting and the delivery of different curricula to different groups were disheartening for those children in 'lower' groups who appeared to lose faith in their capacity to learn as a result of grouping practices. The learning needs of Mitchell and Jared were not met through having been classified as special needs and separated from peers, and Peter gained little from his placement in the extension group. The learning of the middle and top group students was not noticeably enhanced through segregation apart from the motivation to maintain their placements through achievement in written tests. Demotion resulted in a marked decline in self-efficacy, and a subsequent loss of engagement in the learning of mathematics.

For many of the children, the compulsory academic work mode of individual completion of written questions failed to foster their learning since it required them to work in isolation without the support or appeal of concrete materials, on tasks that were irrelevant and meaningless, and where presentation and completion were more highly valued than the quality of mathematical ideas. While Rochelle, Dominic and Toby gained a sense of satisfaction from this routinised kind of activity, the others were put off by the tedium and difficulty of written tasks. The establishment of independent work habits took precedence over mathematical learning as a meaningful and purposeful social process, resulting in impoverished learning for all of the children.

Enjoyment, confidence, self-efficacy and achievement were shown to linked. In combination, the elements of speed, ability differentiation, individual written work and the dominant culture of 'right' answers created environments that inhibited rather

than nurtured mathematical learning. Speed was scarier when the right answers must be found, and producing the right answers was higher stakes when performance was to be judged. Working alone without concrete materials or interesting contexts was more threatening or off-putting when right answers were to be produced, and upon which performance was to be evaluated. In this way, classroom climates were established in which a proportion of learners were likely to 'fail' or switch off, and in so doing develop fear, uncertainty, boredom or lack of confidence. This has been amply illustrated by the children in this study.

### **Implications of the research**

Consistent with the findings of Garden (1997) who noted in his analysis of the New Zealand results of the TIMSS data, a decline beginning from a fairly young age, in an increasing proportion of children's interest and achievement in mathematics, this research has also shown that a proportion of the children had experienced loss of interest and impaired learning. It provides some insights into the social worlds of children that may redirect our thinking about why children 'fail' or lose interest in mathematics. The case study methodology of the research has exposed some of the dynamics of this process of decline.

Although the social texture of each classroom was unique, comprising a collection of individuals each with his or her own thoughts, feelings and experiences, the general processes, procedures and principles surrounding and driving mathematics teaching and learning were surprisingly consistent across the many classrooms and schools visited, indicating that among schools, teachers and parents there was a common vein of epistemological thought about the nature of mathematics, and beliefs about how mathematics is 'learned' and 'known', and how it should be taught. Everyday practices revealed the common beliefs that teachers held about what mathematics *was*, how children learn it, how it should be taught, what mathematics skills and knowledge it was important for them to learn and at what ages, and how and why they assessed children's achievement.



The powerful position of mathematics in our society, including popular beliefs about its mystique and inaccessibility must also be considered central in the debate surrounding its teaching. The National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000) state that the 'pervasive belief in North America that only some students are capable of learning mathematics' is 'in stark contrast to the equally pervasive belief that all students can and should learn to read and write in English.'(p.12). This research suggests that such a belief appears to be echoed in this country.

As Pollard and Filer (1999) have asserted, pupils are often treated as though education were something that is done *to*, rather than *with*, them. This research strongly supported that view. Within the sociomathematical worlds of the children, it was found that adults decided what mathematics children should learn, and how they should learn it. Adults defined and measured children's mathematical 'needs', 'ability' and 'achievement.' Children's feelings, opinions, and preferences, and their particular and unique perspectives of the world based on their personal experiences and sense-making, were found to be routinely disregarded in the implementation of the mathematics curriculum. In the words of a participant (unknown) at the 24<sup>th</sup> Annual Conference of the Mathematics Education research Group of Australasia, Sydney, July 3, 2001, who commented after the delivery of a paper based on the direct accounts of children in this study (Walls, 2001), 'it seems as though we [teachers] are fitting children to mathematics rather than mathematics to children.'

As this study has demonstrated, children are not passive or inanimate objects upon which curricula or teaching formulae can be applied. Adults' beliefs about mathematics - its importance, its nature, and the methods by which it should be taught and learned - disregard many of the fundamental learning needs of children. *Mathematics in the New Zealand Curriculum* promotes mathematics education that aims to 'help students develop a belief in the value of mathematics and its usefulness to them, to nurture confidence in their own mathematical ability, to foster a sense of personal achievement, and to encourage a continuing and creative interest in mathematics' (p. 8). In addition, *Health and Physical Education in the New Zealand Curriculum* (Ministry of Education, 2000c) outlines the responsibility of adults in meeting children's learning needs:

All adult members of the school community should recognise the powerful influence they have as role models since their values and attitudes are continuously demonstrated to students by their actions...All members of the school community should work together to:

- respond sensitively to students' needs;
- value the unique contributions of students from various cultural backgrounds;
- provide experiences that support the development of positive attitudes, trust, and mutual respect;
- use teaching and learning approaches that reinforce the development of personal and social responsibility;
- provide a 'safe physical and emotional environment for students', as required by National Administration Guideline 5 (i). (p. 54)

This study has shown that many widely used mathematics teaching practices failed to create a positive learning atmosphere, as defined above, for the children. Classroom mathematics environments were far from 'emotionally safe' for many of the children in the study, as evidenced by their vivid descriptions of the humiliation, embarrassment, nervousness, shame, sadness, frustration, despair and anger they experienced as a result of mathematics teaching practices in their classrooms, so much so for some, that mathematics time had become their least favourite part of the school day.

The study concludes that deeply entrenched traditional approaches to mathematics teaching, embedded within views of mathematics as hierarchical, fixed and rule-bound, and learners as passive, must be questioned since they demonstrably alienate and marginalise a significant proportion of children, and result in restricted learning. As already discussed, a growing number of reports now exist that expose the negative social effects of current mathematics pedagogical practices (e.g., Boaler, 2000; Anderson & Boylan, 2000). In light of these findings, compelling alternative models are needed to replace traditional methods. It could be argued that *New Zealand in the Mathematics Curriculum* and subsequent Ministry of Education support material has failed to provide such models, and that this lack has left teachers to interpret and eventually fall back on 'tried-and-true' mathematics teaching methods. As the literature has shown, 'back to basics' rhetoric of the 1990s in the wake of radical economic and public sector restructuring may also have encouraged and legitimised teachers' use of traditional teaching methods in 'core' subjects such as mathematics.



Current curriculum reform in primary mathematics such as the Numeracy Projects in New Zealand and elsewhere, overlook many of the concerns that this research has raised.

A number of mathematics educators (e.g., Carpenter et al, 1996; Yackel & Cobb, 1996), have demonstrated that with support, individual teachers can successfully create within their classrooms, cultures of 'doing mathematics' that are based on alternative everyday norms. The need for *all* students to feel comfortable and capable, and to believe that learning is personally meaningful and purposeful, should be the guiding principles of every teaching programme, mathematical or otherwise. Changes in mathematics teaching practice need to attend to both to the emotional welfare of young students as well as the quality of their mathematical learning experiences.

By the age of seven, Georgina had become disaffected with mathematics. Indications were that Georgina would continue to experience the debilitating and long-term exclusion produced by alienating, impoverishing and marginalising mathematics teaching practices. The final interview with ten-year-old Georgina was telling:

- Researcher: Can you see yourself enjoying maths when you get to high school?  
 Georgina: No.  
 Researcher: Do you think you'll be good at it at high school?  
 Georgina: (*Frowning*) Yeah ... sort of.  
 Researcher: What sort of job do you think you might be doing [as an adult]?  
 Georgina: Um ... well, I really want to be a designer, but that's got to do with lots of maths. (*Pause*) But I'm really good at, like, designing things, like how long it needs to be and stuff ... Mum and Dad think I'm a really good designer and drawer.  
 (Late Year 5)

While Georgina was generally quite negative about her future participation in learning mathematics, she was able to recognise some of her mathematical interests and strengths. However, she was also aware of the limitations that mathematical learning and knowing, as presented and defined by school and reinforced at home, was likely to have on her future choices. This research suggests that the potential of Georgina and of the nine other unique children in this study, is unlikely to be fully realised through current teaching practices of mathematics.

## **Limitations of the research and suggestions for further investigation**

### *Sample size*

Because this research focussed on the lives of only ten children within a relatively confined geographical area – at least to begin with – and was conducted by a single researcher, issues of manageability inevitably limited the range and depth of data gathered. A larger sample would certainly have generated more generalisable results. The restricted sample contained insufficient subcategories of children with which to meaningfully investigate additional dimensions of the children's sociomathematical worlds such as gender, ethnicity, or socioeconomic circumstance. The findings of other studies suggest that these variables are likely to be highly significant. There were indications in this research that the differences noted in the experiences of the children from disadvantaged socioeconomic backgrounds and of differing sex are worthy of further investigation. Future research involving a larger sample of case studies and extending over a longer period to include early childhood experiences, and key transition points through compulsory schooling would also contribute much-needed information about the long-term effects of children's mathematical experiences in primary school. Comparative studies of this nature describing the sociomathematical worlds of children from regions of differing cultural and historical heritage could also prove enlightening.

### *Data collection*

Through careful triangulation, this research revealed that some of the lessons observed were atypical. This suggested that teachers had deviated from their everyday routines in order to give a more positive impression of their teaching practices than would normally have been seen. The testimonies of some of the children suggest that negative teaching practices, for instance being 'told off' and being 'shouted at', were significantly reduced when lessons were being observed. While this is entirely understandable, it shows that an 'unedited' version of life in the classroom is difficult to obtain without the establishment of a high degree of familiarity and trust between teacher and observer. An increase in the number of annual observation visits may have helped to build such relationships.

The intrusiveness of the video camera was also a problem. The data yielded through this method were extremely useful, however. Greater frequency of its use may have reduced both the children's excitement and the teachers' nervousness, and replaying taped classroom scenes for the children to reflect upon might also have added a revealing dimension to the research.

Some of the children, especially as seven-year-olds, were difficult to interview either because of their shyness or limited oral skills. The inclusion of group interviews may have helped to overcome this problem, as well as providing deeper insights into the peer dimension of the sociomathematical world.

#### *Analysis and scope*

Because of the sheer volume and depth of the data, several facets of the children's sociomathematical worlds were either included peripherally or omitted altogether. These bear further investigation, analysis and reporting. They include the principals' views and school mathematics policy, parents' schooling and experiences of mathematics, the construction and influence of popular images of 'mathematics' and mathematics education through the media, and the effect of children's out-of-school experiences on their learning of mathematics.

#### **Contributions of the research**

This study has created and developed the concept of the *sociomathematical world* of the child in an attempt to examine and explain the complex and dynamic social environments within which children experience, internalise and reflect socially constructed meanings about mathematics, about learning and knowing mathematics, and about their mathematical 'selves'. The research places the child as the learner at the centre of the investigation, and employs methodological tools that provide a much-needed 'voice' for the child's lived experience of mathematical learning. Such an approach regards 'cognitive' and 'affective' dimensions of learning as inseparable. This concept of *sociomathematical world* offers many possibilities for further investigation.

## REFERENCES

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- Abelson, R. (1976). Script processing in attitude formation and decision making. In J. Carroll & J. Payne (Eds.), *Cognition and social behaviour* (pp. 33-45). New Jersey: Lawrence Erlbaum Associates.
- Abreu, G. de. (2002). Mathematics learning in out-of-school contexts: A cultural psychology perspective. In L. English (Ed.), *A handbook of international research in mathematics education* (pp. 323-354). Mahwah, NJ: Lawrence Erlbaum Associates.
- Abreu, G. de, Bishop, A., & Pompeu, G. (1997). What children and teachers count as mathematics. In T. Nunes, & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 233-264). East Sussex: Psychology Press.
- Adams, G. (1996). *Topic Based Maths Programme, Levels 2 -3*. Wellsford, New Zealand: Pinada Publications.
- Aitken, L. (1963). Personality Correlates of Attitude Toward Mathematics. *Journal of Educational Research*, 56, 476-480.
- Aitken, L. (1976). Update on attitudes and other affective variables in learning mathematics. *Review of Educational Research*, 46(2), 293-311.
- Aitken, L., & Dreger, R. (1961). The effect of attitudes on performance in mathematics. *Journal of Educational Psychology*, 52(1), 19-24.
- Allport, G. (1935). Attitudes. In C. Murchison (Ed.), *A Handbook of Social Psychology* (pp. 789-844). Worcester, Massachusetts: Clark University Press.
- Alton-Lee, A., Nuthall, G., & Patrick, J. (1993). Reframing classroom research: A lesson from the private world of children. *Harvard Educational Review*, 63(1), 50-84.
- Alton-Lee, A., & Pratt, A. (2000). *Explaining and addressing gender differences in the New Zealand school sector: A literature review*. Wellington: Ministry of Education.
- Anderson, A., Weiner, B., & Prawat, R. (1984). *Affective experience in a classroom*. Paper presented at the annual meeting of American Education Research Association, New Orleans, April 1984.
- Anderson, G. (1990). *Fundamentals of educational research*. Hampshire: Falmer Press.
- Anderson, J. (2000). *Teacher questioning and pupil anxiety in the primary classroom*. Paper presented at British Educational Research Association Conference, Cardiff University, 7-10 September, 2000.

- Anderson, J. & Boylan, M. (2000). *The national numeracy strategy, teacher questioning and pupil anxiety*. British Society for Research into the Learning of Mathematics, Loughborough, 2000.
- Anderson, S. (1997). Worldmath curriculum: Fighting Eurocentrism in mathematics. In A. Powell & M. Frankenstein, (Eds.), *Ethnomathematics: Challenging Eurocentrism in mathematics education* (pp. 291-306). New York: State University of New York Press.
- Anthony, G. & Knight, G. (1999). *Teaching for understanding and memory in mathematics in Year 4-5 mathematics*. Wellington: Ministry of Education Research Division.
- Anttonen, R. (1969). A Longitudinal Study in Mathematics. *Attitude The Journal of Education Research*, 62 (10), 467 – 471.
- Anttonen, R. & Deighan, W. (1971). An exploration into teacher and student arithmetic attitude at grades 3,5,and 6. *Paper presented at Annual Educational Research Association meeting*. New York: February 1971.
- Appelbaum, P. (1995). *Popular culture, educational discourse and mathematics*. Albany: State University of New York Press.
- Atkinson, S. (1996). *Developing a scheme of work for primary mathematics*. Kent: Hodder and Stoughton.
- Atweh, B., Forgasz, H., & Nebres, B. (Eds.), (2001). *Sociocultural research on mathematics education: An international perspective*. Mahwah, NJ. Lawrence Erlbaum Associates.
- Australian Education Council, (1990). *A national statement on mathematics for Australian schools: A joint project of the states, territories and the Commonwealth of Australia initiated by the Australian Education*. Carlton, Victoria: Curriculum Corporation.
- Baker, A., & Baker, J. (1990). *Mathematics in Process*. South Yarra: Eleanor Curtin Publishing.
- Baker, D., & Tomlin, A. (2000). Schooled and community numeracies; Understanding social factors and 'under-achievement' in numeracy. *Proceedings of the 2<sup>nd</sup> International Mathematics and Education in Society Conference*, Portugal, 2000.
- Baroody, A., & Ginsburg, H. (1990). *Children's Mathematical Learning: A cognitive view*. *Journal For Research in Mathematics Education, Monograph Number 4 Constructivist Views on the Teaching and Learning of Mathematics*. Virginia: National Council of Mathematics Teachers.
- Barton, B. (1993). Ethnomathematics and its place in the classroom. In McKinley, (Eds.), *SAME papers* (pp 46-67). Hamilton: Centre for Science and Mathematics Education.

- Bassham, H., Murphy, M., & Murphy, K. (1964). Attitude and achievement in arithmetic. *The Arithmetic Teacher*, 11, 66-72.
- Becker, H. (1963). *Outsiders*. Oxford: Free Press.
- Beesey, C. & Davie, L. (1991). Melbourne: *Macmillan Mathematics* series, Macmillan Company of Australia.
- Bennie, N., Henry, E., & Ratcliff, B. (1990). *Beginning School Mathematics: A study of implementation*. Wellington: Ministry of Education.
- Berger, P. & Berger, B. (1976). *Sociology: A biographical approach*. London: Penguin.
- Berger, P., & Luckmann, T. (1966). *The social construction of reality: A treatise in the sociology of knowledge*. London: Penguin Books, 1991 Reprinted Edition.
- Berger, T. & Keynes, H. (1995). Everybody Counts/Everybody Else. In N. Fisher H. Keynes, & P. Wagreich (Eds.), *Changing the culture: Mathematics education in the research community. Issues in Mathematics Education, Vol. 5* (pp. 89 – 110). Washington, D.C.: Conference Board of the Mathematical Sciences.
- Bernstein, B. (1990). *The structuring of pedagogic discourse: Vol 4. Class codes and control*. London: Routledge.
- Bibby, T. (2002). Primary school mathematics: An inside view. In P. Valero & O. Skovsmose (Eds.), *Proceedings of the Third International Mathematics Education and Society Conference* (Part 1, pp. 165-172). Copenhagen: Centre for Research in Learning Mathematics.
- Bierhoff, H. (1996). *Laying the Foundations for Numeracy*. London: ESRC.
- Bishop, A. (1991). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht: Kluwer.
- Blumer, H. (1969). *Symbolic interactionism: Perspective and method*. Englewood Cliffs, NJ: Prentice-Hall.
- Boaler, J. (1997a). *Experiencing school mathematics: Teaching styles, sex and setting*. Buckingham: Open University Press.
- Boaler, J. (1997b). Setting, social class and the survival of the quickest. *British Educational Journal*, 23(5) 575-595.
- Boaler, J. (1997c). When even the winners are losers: evaluating the experiences of top set students. *Journal of Curriculum Studies*, 29(2), 165-182.
- Boaler, J. (2000). Mathematics from another world: Traditional communities and the alienation of learners. *Journal of Mathematical Behavior*, 18(4), 379-397.

- Boaler, J., Wiliam, D., & Brown, M. (2000). Students' experience of ability grouping, polarisation and the construction of failure. *British Educational Research Journal*, 26(5), 631-648.
- Boaler, J. & Wiliam, D. (2001). 'We've still got to learn!' Students' perspectives on ability grouping and mathematics achievement. In P. Gates (Ed.), *Issues in Mathematics Teaching* (pp. 77-92). London: RoutledgeFalmer
- Boylan, M. (2002). Teacher questioning in communities of political practice. In P. Valero & O. Skovsmose (Eds.), *Proceedings of the Third International Mathematics Education and Society Conference* (Part 1, pp. 182-191). Copenhagen: Centre for Research in Learning Mathematics.
- Boylan, M., Lawton, P., & Povey, H. (2001). 'I'd be more likely to speak in class if...': Some students' ideas about increasing participation in whole class interactions. *Proceedings of the 25<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 201-208). Utrecht, Netherlands.
- Brousseau, G., Davis, R. & Werner, T. (1986). Observing students at work. In B. Christiansen, A. Howson & M. Otte (Eds.), *Perspectives in mathematics education* (pp. 205-241). Dordrecht: Reidel.
- Brown, T. (1997). *Mathematics education and language: Interpreting hermeneutics and post-structuralism*. Dordrecht: Kluwer.
- Bruner, J. (1964). Some Theorems on instruction illustrated with reference to mathematics. In NSSE Yearbook, *Theories of Learning and Instruction*. Chicago: NSSE.
- Brush, T. (1997). The effects on student achievement and attitudes when using integrated learning systems with cooperative pairs. *Educational Technology Research and Development*, 45(1), 51-64.
- Brush, L. (1981). Some thoughts for teachers on mathematics anxiety. *Arithmetic Teacher*, 24(4), 37-39.
- Burn, M. (1968). *Numbers and Shapes – Quick Book 1*. London: Ginn & Company.
- Burton, L. (1988). Images of mathematics. In Ernest, P. (Ed.), *The social context of mathematics teaching. Perspectives* 37(p. 7-16). Exeter: University of Exeter.
- Bussi, M. (1998). Joint activity in mathematics classrooms: A Vygotskian analysis. In F. Seeger, J. Voigt & U. Waschescio (Eds.), *The culture of the mathematics classroom* (pp. 13-49). Cambridge: Cambridge University Press.
- Campbell, P. (1981). What do children see in textbook pictures? *Arithmetic Teacher*, 59(2), 12 –16.



- Carpenter, T., Fennema, E., Loef Framke, M., Levi, L., & Empson, S. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.
- Carraher, T. & Schliemann, A. (1985). Computation routines prescribed by schools: Help or hindrance? *Journal for Research in Mathematics Education*, 16(1), 37-44.
- Cesar, M. (1995). Pupils ideas about mathematics. In L. Meira & D. Carraher (Eds.), *Proceedings of the 19<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education*.
- Chamberlain, M. (1997). Introduction. In R. Garden (Ed.), *Mathematics and science performance in the middle primary school: Results from New Zealand's participation in the Third International Mathematics and Science Study* (pp. 3-15). Wellington: Ministry of Education, Research and International Section.
- Cheung, K. (1988). Outcomes of Schooling: Mathematics achievement and attitudes towards mathematics learning in Hong Kong. *Educational Studies in Mathematics*, 19, 209 – 219.
- Clark, M. (1999). *School-wide Assessment: Mathematics*. New Zealand Council for Educational Research, Wellington: New Zealand.
- Cobb, P. (1995). Introduction: the coordination of psychological and sociological perspectives in mathematics education. In P. Cobb, & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 1-16). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Cobb, P., Wood, T., Yackel, E., & Perlwitz, M. (1992). A follow-up assessment of a second-grade problem-centred mathematics project. *Educational Studies in Mathematics*, 23, 483-504.
- Cohen, A. (1985). *The Symbolic construction of community*. Chichester, Sussex: Ellis Horwood.
- Cohen, L. & Manion, L. (1994). *Research methods in education*. Fourth Edition. London: Routledge.
- Connolly, P. (1997). In search of authenticity: Researching young children's perspectives. In A. Pollard, D. Thiessen, & A. Filer (Eds.), *Children and their curriculum: The perspectives of primary and elementary school children* (pp. 162-183). London: The Falmer Press.
- Cooper, B., & Dunne, M. (1999). *Assessing children's mathematical knowledge: Social class, sex and problem solving*, London: Open University Press.
- Cotton, T. (1993). Children's impressions of mathematics. *Mathematics Teaching*, 143, 14-17.



- Cross, L. & Hynes, M. (1994). Assessing mathematics learning for students with learning differences. *Arithmetic Teacher*, March 1994, 371-377.
- D'Ambrosio, U. (1997). Foreword. In A. Powell & M. Frankenstein, (Eds.), *Ethnomathematics: Challenging Eurocentrism in mathematics education* (pp. xv - xxi). New York: State University of New York Press.
- Davidson, N. (1985). Small group learning and teaching in mathematics: A selective review of the research. In R. Slavin, S. Sharan, S. Kagan, R. Hertz-Lazarowitz, C. Webb & R. Schmuck (Eds.), *Learning to Cooperate, Cooperating to Learn* (pp. 211-230). New York: Plenum Press.
- Davies, B. (1982). *Life in the classroom and playground*. London: Routledge.
- Davies, J. & Brember, I. (1999). Reading and mathematics attainments and self-esteem in years 2 and 6: An eight-year cross-sectional study. *Educational Studies (UK)*, 25(2), 145-158.
- Davis, B. (1996). *Teaching mathematics: toward a sound alternative*. New York: Garland.
- Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.
- Denvir, H., Askew, M., Brown, M., & Rhodes, V. (2001). Pupil participation in interactive whole class teaching. *Proceedings of the Twenty-fifth Conference of the International Group for the Psychology of Mathematics Education* (pp. 337-344). Utrecht, Netherlands.
- Denzin, N. (1989). *Interpretive interactionism*. Newbury Park, CA: Sage.
- Department for Education and Employment (1999). *The National Numeracy Strategy: Framework for teaching mathematics from reception to year 6*. Cambridge: Cambridge University Press.
- Department of Education (1980). *Sex-role stereotyping in mathematics textbooks*. Research Report, Wellington: Department of Education.
- Department of Education (1983). *School Mathematics Books 2 & 3, and Booklets B & C*. Wellington: School Publications Branch, Department of Education.
- Department of Education (1985). *Mathematics: Junior classes to standard four*. Wellington: Department of Education.
- Department of Education (1987). *Mathematics achievement in New Zealand secondary schools: A report on the conduct in New Zealand of the Second International Mathematics Study within the International Association for the Evaluation of Educational Achievement*. Wellington: Department of Education.

- Dillon, J. (1985). Using questions to foil discussion. *Teaching and Teacher Education*, 1(2), 109-121.
- Donlan, C. & Hutt, E. (1991). Teaching maths to young children with language disorders. In K. Durkin & B. Shire (Eds.), *Language in mathematical education: Research and practice* (pp. 198-207). Buckingham: Open University Press.
- Dorfler, W. & McClone, R.R. (1986). Mathematics as a School Subject. In B. Christiansen, A. Howson, & M. Otte, (Eds.), *Perspectives on Mathematics Education: Papers submitted by members of the Bacomet Group* (pp. 49-97). Dordrecht: Reidel.
- Douglas, J. (1970). *Understanding everyday life* (Ltd. Edition, 1971). London: Aldine Publishing Company.
- Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogic texts*. London: The Falmer Press
- Downie, D., Slesnick, T. & Stenmark, J. (1981). *Math for girls and other problem solvers*. Berkeley; CA: Lawrence Hall of Science, University of California.
- Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist*, 23(2) 167 – 180, Lawrence Erlbaum Associates.
- Doyle, W. (1983). Academic work. *Review of Education Research*, 53(2), 159-199.
- Drew, D. (1996). *Aptitude Revisited Rethinking math and science education for America's next century* Baltimore: The John Hopkins University Press.
- Duncan, E. (1959). *Suggestions for the teaching of arithmetic in the junior and middle school (Books 2,3, & 4)*. Wellington: Department of Education.
- Dungan, J. & Thurlow, G. (1989). Students' attitudes to mathematics: A review of the literature. *Australian Mathematics Teacher*, 45(3), 1989.
- Dutton, W. (1951). Attitude of prospective teachers toward arithmetic. *Elementary School Journal*, 51, 84-90.
- Dutton, W. (1954). Measuring attitudes toward arithmetic. *Elementary School Journal*, 55, 24-31.
- Eagly, A., & Himmelfarb, S. (1974). Current trends in attitude theory and research. In S. Himmelfarb & A. Eagly, (Eds.), *Readings in attitude change* (pp. 594-609). New York: John Wiley & Sons.
- Education Review Office, (2000). *In time for the future: A comparative study of mathematics and science education*. New Zealand: Crown.

- Edwards, D. & Mercer, N. (1987). *Common knowledge: The development of understanding in the classroom*. London: Methuen.
- Eisenhart, M. (1988). The ethnographic research tradition and mathematics education research. *Journal for Research in Mathematics Education*, 19(2), 99-114.
- English, L. (Ed.) (2002). *Handbook of international research in mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Erickson, T. (1989). *Get it Together: Maths problems for groups – Grades 4-12*. California: Equals, Lawrence Hall of Science, Regents of University of California.
- Ernest, P. (1988). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics Teaching: The State of the Art* (pp. 249-254). Sussex: The Falmer Press.
- Ernest, P. (1998a). *Social Constructivism as a philosophy of mathematics*. Albany: State University of New York Press.
- Ernest, P. (1998b). Changing views of 'the gender problem' in mathematics. In V. Walkerdine, *Counting girls out: Girls and mathematics* (pp. 1-14). *New Edition*. London: Falmer Press.
- Even, R. & Tirosh, D. (2002). Teacher knowledge and understanding of students' mathematical learning. In L. English (Ed.), *A handbook of international research in mathematics education* (pp. 219-240). Mahwah, NJ: Lawrence Erlbaum Associates.
- Fedon, J. (1958). The role of attitude in learning arithmetic. *Arithmetic Teacher* V, 304-310
- Fennema, E. (1989). The study of affect and mathematics: A proposed generic model for research. In D. McLeod & V. Adams, (Eds.), *Affect and problem solving: a new perspective* (205-219). New York: Springer-Verlag.
- Fennema, E., & Peterson, P. (1985). Autonomous learning behaviour: A possible explanation of gender-related differences in mathematics. In L. Wilkinson & C. Marrett (Eds.), *Gender-related differences in classroom interactions* (p. 17 – 35). New York: Teacher's College Press.
- Fishbein, M. & Azjen, I., (1975). *Belief, attitude, intention and behavior: An introduction to theory and research*. Reading, MA: Addison-Wesley.
- Flockton, L., & Crooks, T. (1998). *Mathematics: Assessment results 1997. National Education Monitoring Report 9*. Dunedin: Educational Assessment Research Unit, University of Otago.
- Flournoy, F. (1964). *Elementary school mathematics*. Washington D.C.: The Centre for Applied Research in Education.

- Forgasz, H. (1995). *Learning mathematics: Affect, gender, and classroom factors*. Unpublished Doctoral Thesis, Clayton, Victoria, Australia: Monash University.
- Forgasz, H., & Leder, G. (1996). Mathematics classrooms, gender and affect. *Mathematics Education Research Journal*, 8(1), 153-173.
- Frid, S. (1993). Communicating Mathematics: a social sharing of language and decisions pertaining to truth and validity. In M. Stephens, A. Wywood, D. Clarke, & J. Izard (Eds.), *Communicating Mathematics: perspectives from classroom practice and current research* (pp. 271-278). Victoria, Australia: Australian Council of Educational Research.
- Fuson, K. C., Grandau, L., & Sugiyama, P. (2001). Achievable numerical understandings for all young children. *Teaching Children Mathematics*, 7(9), 522-526.
- Garden, R. (Ed.) (1997). *Mathematics and science performance in the middle primary school: Results from New Zealand's participation in the Third International Mathematics and Science Study*. Wellington: Ministry of Education, Research and International Section.
- Gilbert, C., & Cooper, D. (1976). The relationship between teacher/student attitudes and the competency levels of sixth grade students. *School Science and Mathematics*, 76(7), 469 – 476.
- Gierl, M. & Bisanz, J. (1995). Anxieties and attitudes related to mathematics in grades 3 and 6. *Journal of Experimental Education*, 63(2), 139-158.
- Goffman, E. (1959). *The presentation of self in everyday life*. Harmondsworth: Penguin Books (1971 Edition, Pelican Books)
- Goldin, G. (2000). Representation in mathematical learning and problem solving. In L. English (Ed.), *A handbook of international research in mathematics education* (pp. 197-218). Mahwah, NJ: Lawrence Erlbaum Associates.
- Goldman, S. & Hasselbring, T. (1997). Achieving meaningful mathematics literacy for students with learning disabilities. *Journal of Learning Disabilities*, 30(2), 198-208.
- Gooding, A. & Stacey, K. (1993). How children help each other learn in groups. In M. Stephens, A. Waywood, D. Clarke & J. Izard (Eds.), *Communicating mathematics: Perspectives from classroom practice and current research* (pp. 41-50). Hawthorn: Australian Council for Educational Research.
- Goodnow, J & Burns, A. (1985). *Home and school: A child's-eye view*. Sydney: Allen & Unwin.
- Gottfried, A.E., Fleming, J., & Gottfried, A.W. (1998). Role of cognitively stimulating home environment in children's academic intrinsic motivation: A longitudinal study. *Child Development*, 69(5), 1448 – 1460.

- Gough, M. (1954). *Mathemaphobia: its causes and treatments* Clearing House 28, 290-291.
- Hadfield, M. & Haw, K. (2000). *The 'voice' of young people: Hearing, listening, responding. A booklet for those working with young people.* Nottingham: University of Nottingham.
- Haladyna, T., Shaughnessy, J., & Shaughnessy, J.M. (1983). A causal analysis of attitude toward arithmetic. *Journal for Research in Mathematics Education*, 14(1), 19-29.
- Hallam, S., & Toutounji, I. (1996). *What do we know about the grouping of pupils by ability?: A research review.* London: Institute of Education, University of London.
- Harlen, W. & Malcolm, H. (1999). *Setting and Streaming: A research review.* Edinburgh: Scottish Council of Educational Research.
- Harries, T. & Spooner, M. (2000). *Mental Mathematics for the Numeracy Hour* London: David Fulton Publishers.
- Hart, K. (1986). The step to formalisation. *Proceedings of the 10<sup>th</sup> International Conference for the Psychology of Mathematics Education* (pp. 159-64). London: Institute of Education, University of London.
- Hart, L. (1989). Describing the affective domain: Saying what we mean. In D. McLeod and V. Adams, (Eds.), *Affect and problem solving: a new perspective* (pp. 37 - 45). New York: Springer-Verlag.
- Hart, L. and Alleksaht-Snyder, M. (1996). Sociocultural and motivational contexts of mathematics learning for diverse students. In M. Carr (Ed.), *Motivation in Mathematics* (pp. 1-23). Cresskill, New Jersey: Hampton Press.
- Haylock, D. (1995). *Mathematics explained for primary teachers.* London: Paul Chapman Publishing.
- Heibert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.
- Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, 21(1), 33-46.
- Herrington, T. (1998). Thoughts about learning: The subject matters. In A. McIntosh and N. Ellerton (Eds.), *Research in Education: A contemporary perspective* (pp. 134-143). Perth: Edith Cowan University.
- Higgins, J. (1998). *Learning and Teaching Mathematics in the First Two Years at School.* Unpublished PhD Thesis, Wellington: Victoria University of Wellington, New Zealand.

- Higgins, J. (2001). *Exploring issues in mathematics education: An evaluation of the Year 4-6 Numeracy Exploratory Study*. Wellington, NZ: Ministry of Education.
- Higgins, K. (1997). The effect of year-long instruction in mathematical problem solving on middle-school students' attitudes, beliefs, and abilities. *Journal of Experimental Education*, 66(1), 5-28.
- Himmelfarb, S, & Eagly, A. (Eds.) (1974). *Readings in attitude change*. New York: Wiley.
- Hoffer, T. (1992). Middle school ability grouping and student achievement in science and mathematics. *Educational Evaluation and Policy Analysis*, 14(3), 205-227.
- Hungwe, G., & Nyikahadzoi, M. (2002). Marking pupils' written exercises in mathematics: what are the benefits? In P. Valero & O. Skovsmose (Eds.). *Proceedings of the Third International Mathematics Education and Society Conference* (Part 2, pp. 322-329). Copenhagen: Centre for Research in Learning Mathematics.
- Ireson, J. & Hallam, S. (1999). Raising standards: Is ability grouping the answer? *Oxford Review of Education*, 25(3), 343-358.
- Ivey, K. (1994). *World views in the mathematics classroom: Students translating beliefs into actions*. Unpublished PhD dissertation. Washington: Washington State University.
- Jacobs, D. Watson, T. & Sutton, J. (1996). Effects of a cooperative learning method on mathematics achievement and affective outcomes of students in a private elementary school. *Journal of Research and Development in Education*, 29(4), 195-202.
- Jaworski, B. (1988). 'Is' versus 'seeing as': constructivism and the mathematics classroom. In D. Pimm (Ed.), *Mathematics, Teachers and Children* (pp. 287 -296). London: Hodder and Stoughton.
- Johnson, D. (1957). Attitudes in the mathematics classroom. *School Science and Mathematics*. 57, 113-120.
- Kloosterman, P. & Cougan, M. (1994). Students' beliefs about learning school mathematics. *The Elementary School Journal*, 94(4), 375-388.
- Kouba, V. & MacDonald, J. (1991). What is mathematics to children? *Journal of Mathematical Behaviour*, 10, 105-113.
- Kulm, G. (1980). Research on mathematics attitude. In R.J. Shumway (Ed.), *Research in mathematics education*, (pp. 356-387). Reston, VA: National Council of Teachers of Mathematics.



Lakoff, G. & Johnson, M. (1980). *Metaphors we live by*. Chicago: The University of Chicago Press.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Leder, G. (1992). Guest Editorial: Attitudes to mathematics. *Mathematics Education Research Journal*, 4(3), 1-7.

Leder, G. (1993). Reconciling affective and cognitive aspects of mathematics learning: Reality or a pious hope? *Proceedings of the 17<sup>th</sup> conference of Psychology of Mathematics Education, 1993*. Tsukuba: Japan.

Leikin, R., & Zaslavsky, O. (1997). Facilitating student interactions in mathematics in a cooperative learning setting. *Journal for Research in Mathematics Education*, 28, 331-354.

Lemert, E. (1951). *Social pathology*. Maidenhead: McGraw-Hill.

Lemke, J. (1990). *Talking Science: Language, Learning and Values*. Norwood: Ablex Publishing Company.

Leron, U., & Hazzan, O. (1997). The world according to Johnny; A coping perspective in mathematics education. *Educational Studies in Mathematics*, 32, 265-292.

Likert, R. (1932). A technique for the measurement of attitudes. *Archives of Psychology*, 22(140), 44-53.

Lim, Chap Sam & Ernest, P. (2000). A survey of public images of mathematics. In T. Rowland & C. Morgan (Eds.), *Research in Mathematics Education: Papers of the British Society for Research in Learning Mathematics* (Vol. 2, 193-206). London: British Society for Research into learning Mathematics.

Linchevski, L. & Kutscher, B. (1998). Tell me with whom you are learning, and I'll tell you how much you've learned: Mixed-ability versus same-ability grouping in mathematics. *Journal for Research in Mathematics Education*, 29(5), 533-554.

Loughran, J. & Northfield, J. (1996). *Opening the classroom door: Teacher, researcher, learner*. London: The Falmer Press.

Ma, X. (1997). Reciprocal relationships between attitude toward mathematics and achievement in mathematics. *Journal of Educational Research*, 90(4), 221-229.

Ma, X. & Kishor, N. (1997a). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education*, 28(1), 26-47.

- Ma, X. & Kishor, N. (1997b). Attitude toward self, social factors, and achievement in mathematics: A meta-analytic review. *Educational Psychology*, 9(2), 89-120.
- Maier, H. & Voigt, J. (1992). Teaching styles in mathematics education. In H. Schupp et al (Eds.), *Mathematics education in the Federal Republic of Germany. Zentralblatt fuer Didaktik der Mathematik*, 24(7), 248-252.
- Mandler, G. (1989). Affect and Learning: Causes and consequences of emotional interactions. In D. McLeod & M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 3-19). New York: Springer-Verlag.
- Martin, V. (1996). The students. In R. Garden. (Ed.), *Mathematics performance of New Zealand form 2 and form 3 students: National results from New Zealand's participation in the Third International Mathematics and Science Study* (pp. 111-147). Wellington: Ministry of Education, Research and International Section.
- Martin, V. (1997). The students. In R. Garden. (Ed.), *Mathematics and science performance in middle primary school: Results from New Zealand's participation in the Third International Mathematics and Science Study* (pp. 137-173). Wellington: Ministry of Education, Research and International Section.
- Mayo, E. (1994). On power and equity: Let's not blame the victim. In J. Neyland, (Ed.), *Mathematics education: a handbook for teachers Vol. 1* (pp. 291 -306). Wellington: Wellington College of Education.
- McDonald, G. (1993). Ages, stages and evaluation: The demography of the classroom. *Evaluation and Research in Education*, 7(3), 143-154.
- McDonough, A., & Clarke, D. (1994). Accessing student affect: Strategies and concerns. *Proceedings of the Seventeenth Annual Conference of the Mathematics Education Research Group of Australasia, 1994*. Vol. 2, pp.391-401. Stellenbosch: University of Stellenbosch.
- McDonough, A. (1998). Young children's beliefs about the nature of mathematics. *Proceedings of the Twenty-second Conference of the International Group for the Psychology of Mathematics Education* (pp. 263-270).
- McGuire, W. (1986). The Vicissitudes of attitudes and similar representational constructs in twentieth century psychology. *European Journal of Social Psychology*, 16, 89 - 130.
- McKay, J. (1991). *No pain, no gain?: Sport and Australian culture*. Sydney: Prentice Hall.
- McLaren, P. (1994). *Life in Schools: An introduction to critical pedagogy in the foundations of education*. White Plains, NY: Longman.
- McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of Research on Mathematics teaching and learning* (pp. 575-596). New York: Macmillan.



- Mead, G. H. (1934). *Mind, self and society*. Chicago: University of Chicago Press.
- Meighan, R., & Siraj-Blatchford, I. (1998). *A sociology of educating. Third Edition*. London: Cassell.
- Mellin-Olsen, S. (1987). *The Politics of Mathematics Education*. Dordrecht: Reidel.
- Mendick, H. (2002). Narratives of gender and maths. In P. Valero & O. Skovsmose (Eds.), *Proceedings of the Third International Mathematics Education and Society Conference* (Part 2, pp. 373-384). Copenhagen: Centre for Research in Learning Mathematics.
- Ministry of Education (1992). *Mathematics in the New Zealand Curriculum*. Wellington: Learning Media.
- Ministry of Education (1993). *Beginning School Mathematics, Cycle 12: Modules 1, 2, 3*. Wellington: Learning Media.
- Ministry of Education (1994a). *Beginning school mathematics: A guide to the resource*. Wellington: Learning Media.
- Ministry of Education (1994b). *English in the New Zealand curriculum*, Wellington: Learning Media.
- Ministry of Education (1995). *Implementing mathematical processes*. Wellington: Learning Media.
- Ministry of Education (1996). *Development band mathematics*. Wellington: Learning Media.
- Ministry of Education (1997). *Developing mathematics programmes*. Wellington: Learning Media.
- Ministry of Education (1999a). *Figure it Out, (Levels 2/3)*. Wellington: Learning Media.
- Ministry of Education (1999b). *Teaching problem solving in mathematics: Years 1-8*. Wellington: Learning Media.
- Ministry of Education (2000a). *Figure it Out, (Level 3)*. Wellington: Learning Media.
- Ministry of Education, (2000b). *A repeat of the Third International Mathematics Science Study, 1998-1999: Final results for Year 9 students*. Wellington: Research Division, Ministry of Education.
- Ministry of Education (2000c). *Health and physical education in the New Zealand Curriculum*. Wellington: Learning Media.

- Ministry of Education (2001a). *Early Numeracy Project (Draft)*, Ministry of Education Curriculum Division, Wellington: New Zealand.
- Ministry of Education (2001b). *Curriculum Update 45*, February 2001. Wellington: Learning Media.
- Ministry of Education (2001c). What is the learning framework for number? *Education Gazette* 80(3) 4, March, 2001. Wellington, New Zealand: APN Educational Media and Copeland Wilson and Associates.
- Moore, G. (2000) *New Wave Mental*. Balcatta, Western Australia: R.I.C. Publications
- National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Neale, D. (1969). The role of attitudes in learning mathematics. *The Arithmetic Teacher*, 16(8), 631-640.
- Newstead, K. (1998). Aspects of children's mathematics anxiety. *Educational Studies in Mathematics*, 36, 53-71.
- Newman R. & Goldin, L. (1990). Children's reluctance to seek help with schoolwork. *Journal of Educational Psychology*, 82(1), 92-100.
- Nickson, (1992). *The culture of mathematics classrooms: An unknown quantity?* In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 101-114). New York: Macmillan.
- Niss, M. (2000). Key issues and trends in research on mathematical education. *Abstracts of Plenary Lectures and Regular Lectures, The 9<sup>th</sup> International Congress on Mathematical Education, July 31-August 6, 2000, Tokyo/Makuhari, Japan* (pp. 2-5).
- Northam, (1982). Girls and boys in primary maths books. *Education 3-13*, 10(1), 11-14.
- Nunes, T., Schliemann, A., & Carraher, D. (1993). *Street mathematics and school mathematics*. New York: Cambridge University Press.
- Nuthall, G. (2000). *How children remember what they learn in school*. Wellington: New Zealand Council for Research in Education.
- Oakes, J., Wells, A., Jones, M., & Datnow, A. (1997). Detracking: The social construction of ability, cultural politics, and resistance to reform. *Teachers College Record*, 98(3), 482- 510. Columbia University: Teachers College.

- Osgood, C., Suci, G., & Tannenbaum, P. (1957). *The measurement of meaning*. Urbana, IL: University of Illinois Press.
- Parker, A. & McSeveny, A. (1994 -). Signpost Mathematics Series, Pascal Press.
- Patilla, P., Montague-Smith, A., & Broadbent, P. (1996). *Longman primary maths programme*. Hong Kong: Longman.
- Pearce, K., Lungren, M., & Wince, A. (1998). The effect of curriculum practices on first graders' attitudes, activity preference, and achievements on mathematics. *Education*, 119(1), 82-90.
- Perlmutter, J., Bloom, L., Rose, T., & Rogers, A. (1997). Who uses maths? Primary children's perceptions of the uses of mathematics. *Journal of Research in Childhood Education*, 12(1), 58-70.
- Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London: Routledge.
- Pinder, D & Adams, G. (1996). *Speed, skill and success with basic facts: A monthly monitoring programme*. Wellsford, New Zealand: Pinada Publications.
- Poffenberger, T. & Norton, D. (1959). Factors in the formation of attitudes toward mathematics. *Journal of Educational Research*, 52(5) 171-176.
- Pollard, A. (1985). *The social world of the primary school*. London: Cassell.
- Pollard, A. & Filer, A. (1996). *The social world of children's learning*. London: Cassell.
- Pollard, A., & Filer, A. (1999). *The social world of pupil career*. London: Cassell.
- Pollard, A., & Triggs, P. with P. Broadfoot, E. McNess & M. Osborn, (2000). *What pupils say: Changing policy and practice in primary education*. London: Continuum.
- Postman, N. (1996). *The end of education: Redefining the value of school*. New York: Vintage Books.
- Prus, R. (1996). *Symbolic interaction and ethnographic research*. New York: State University of New York Press.
- Reay, D., & Wiliam, D. (1999). 'I'll be a nothing': Structure, agency and the construction of identity through assessment. *British Educational Research Journal*, 25(3), 343-354.
- Reyes, L. H. (1984). Affective variables and mathematics education. *The Elementary School Journal*, 84(5), 558-581.
- Rosenburg, M. & Hovland, C. (1960). Cognitive, affective and behavioral components of attitude. In M. Rosenburg, W. McGuire, R. Abelson, & J. Brehm (Eds.), *Attitude, organization and change: An analysis of consistency among attitude components* (pp. 1-14). New Haven: Yale University Press.

- Roth, W-M., Boutonne, S., McRobbie, C., & Lucas, K.(1999). One class, many worlds. *International Journal for Science Education*, 21(1), 59-75.
- Rufell, M., Mason, J. & Allen, B. (1998). Studying attitude to mathematics. *Educational Studies in Mathematics* 35, 1-18.
- Ruthven, K. (1987). Ability stereotyping in mathematics. *Educational Studies in Mathematics*, 18, 243-253.
- Schutz, A. (1970). *On phenomenology and social relations*. Chicago: The University of Chicago Press.
- Seaman, C. (1999). I've got a secret: Maths anxiety. *Philosophy of Mathematics Education Journal*, 11, 56-59.
- Sierpinska, A. (1998). Three epistemologies, three views of the classroom: Constructivism, sociocultural approaches, interactionism. In H. Steinbring, M. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 30 –62). Virginia: National Council of Teachers of Mathematics.
- Silverman, H. (1973). Design and evaluation of a unit about measurement a vehicle for changing attitude toward mathematics and self-concept of low achievers in the intermediate grades. Unpublished Ph.D. Dissertation, Fordham University.
- Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher*, 36(3), 9-15.
- Slavin, R. (1988). *Cooperative learning and student achievement* Educational Leadership, 54, 31-33.
- Slavin, R. (1990). Achievement effects of ability grouping in secondary schools: A best evidence synthesis. *Review of Educational Research*, 60(3), 471-499.
- Smith, A. and Glynn, T. (1990). Context for girls and boys learning mathematics: Teacher interactions and student behaviour in two classrooms. *New Zealand Journal of Psychology*, 19, 9-16.
- Smith, L. (1994). Biographical Method. In N. Denzin, & Y. Lincoln (Eds.), *Handbook of Qualitative Research* (pp. 286-305). London: Sage Publications.
- Spangler, D. (1992). Assessing students' beliefs about mathematics. *Arithmetic Teacher*, 40(3), 148-152.
- Stark, A.W. (1997 –2000). *AWS Mathematics Teacher Resources*. Christchurch: AWS Teacher Resources.
- Steffe, L. (1991). About the Authors. In E. von Glaserfeld (Ed.), *Radical Constructivism in Mathematics Education* (p. vii-xi). Dordrecht: Kluwer.
- Stigler, J. & Hiebert, J. (1997). Understanding and improving classroom mathematics instruction: An overview of the TIMSS video study. *Phi Kappan Delta*, September 1997, 14 –21.

- Strauss, A. & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. California: Sage Publications.
- Stright, V. (1960). A study of attitudes toward arithmetic of students and teachers in the third, fourth and sixth grades. *Arithmetic Teacher*, 7, 280-286.
- Sukhnandan, L. & Lee, B. (1998). *Streaming, setting and grouping by ability: A review of the literature*. Berkshire: National Foundation for Educational Research.
- Suydam, M. & Weaver, J. (1975). Using Research: A key to elementary school mathematics. *Mathematics Education Report*. Clearing House
- Swetman, D. (1995). Rural elementary students' attitude toward mathematics. *Rural Educator*, 16(3) 20-22.
- Swingewood, A. (2000). *A Short History of Sociological Thought*, (Third Edition). Hampshire: Macmillan Press
- Thomas, G. (1994). *Discussion in junior mathematics: helping one another learn?* Unpublished PhD Thesis, University of Otago, Dunedin, New Zealand.
- Thomas, G., & Ward, J. (2001). *Exploring issues in mathematics education: An evaluation of the Count Me In Too pilot project*. Wellington, NZ: Ministry of Education.
- Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 127-145). New York: Macmillan Publishing Company.
- Thorndike, E. (1922). *The psychology of arithmetic*. New York: The Macmillan Company.
- Thurstone, L. (1931). The measurement of social attitudes. *Journal of Abnormal and Social Psychology*, 26, 249-269.
- Tipler, J. & Catley, K. (1998 - 2001). *National Curriculum Mathematics*, Level 3 Books 1 & 2, Level 2 Books 1 & 2. Christchurch: Caxton Educational Publishers Ltd.
- Tulock, M.K. (1957). Emotional Blocks in Mathematics. *Mathematics Teacher*, 50, 572 - 576.
- Utsumi, M. & Mendes, C. (2000). Researching the attitudes towards mathematics in basic education. *Educational Psychology*, 20(2), 237-243.
- Vanyan, M., White, N., Yuen, P. & Teper, M. (1997). Beliefs and attitudes toward mathematics among third and fifth-grade students: A descriptive study. *School Science and Mathematics*, 97(7), 345-351.
- Vaughan, G., & Hogg, M. (1998). *Introduction to social psychology: Second edition*. Sydney: Prentice Hall.
- Verma, G., & Mallick, K. (1999). *Researching education*. London: Falmer Press.

- Visser, H. & Bennie, N. (1996). *An evaluation of the resource: Beginning School Mathematics*. Wellington: Research Section, Ministry of Education.
- Voigt, J. (1995). Thematic patterns of interaction and sociomathematical norms. In P. Cobb, & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 163-201). Hillsdale, NJ: Lawrence Erlbaum.
- Voigt, J. (1998). The culture of the mathematics classroom: Negotiating mathematical meaning of empirical phenomena. In F. Seeger, J. Voigt, & U. Waschescio (Eds.), *The culture of the mathematics classroom* (pp. 191-220). Cambridge: Cambridge University Press.
- Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Edited by M. Cole, V. John-Steiner, S. Scribner and E. Souberman, Cambridge, Massachusetts: Harvard University Press.
- Wagner, B. (1980). *The effect of a numeration learning hierarchy on mathematic attitudes in kindergarten children*. Unpublished Ph.D. dissertation, University of Houston.
- Walkerline, V. (1998). *Counting girls out: Girls and mathematics. New Edition*. London: Falmer Press.
- Walls, F. (2001). Children talk about their mathematics lives. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Proceedings of the Twenty-fourth Annual Conference of the Mathematics Education Research Group of Australasia*. 30 June – 4 July, 2001. Vol. 2, pp. 499-506. Sydney: Mathematics Education Research Group of Australasia.
- Walshaw, M. (1999). *Paradox, partiality and promise: A politics for girls in school mathematics*. Unpublished PhD. Dissertation, Massey University.
- Webb, N. (1982). Group Composition, group interaction and achievement in cooperative small groups. *Journal of Educational Psychology*, 74, 475-484.
- Weigert, A. J. (1981). *Sociology of Everyday Life*. New York: Longman Inc.
- Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press.
- Wheat, H. (1937). *The psychology and teaching of arithmetic*. USA: D.C. Heath and Company.
- Wigfield, A., & Meece, J. (1988). Math anxiety in elementary school students. *Journal of Educational Psychology*, 80(2), 210-216.
- Winbourne, P. (1999). Mathematical Becoming: The place of mathematics in the unfolding stories of learners' identities. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*. Vol 1, Haifa, Israel.
- Winbourne, P., & Watson, A. (1998). Learning mathematics in local communities of practice. In A. Watson, (Ed.), *Situated cognition in the learning of mathematics*.



Centre for Mathematics Education, University of Oxford Department of Educational Studies.

Winter, R. (1992). Mathophobia, Pythagoras and roller-skating. In M. Nickson & S. Lerman. (Eds.), *The social context of mathematics education: Theory and practice* (pp. 81- 93). Proceedings of the Group for Research into Social Perspectives of Mathematics Education. London: South Bank University Press.

Woods, P. (1983). *Sociology and the School: An interactionist viewpoint*. London: Routledge & Kegan Paul.

Wood, T & Yackel, E. 1990. The development of collaborative dialogue within small group interactions. In L. Steffe & T. Wood (Eds.), *Transforming children's mathematics education* (pp. 244-252). Hillsdale, NJ: Lawrence Erlbaum Associates.

Wylie, C. & Smith, L. (1993). *Learning to learn: Children's progress through the first three years of school*. Wellington: New Zealand Council for Educational Research.

Wylie, C. & Thompson, J. (1998). *Competent children at 6: Families, early education, and schools*. Wellington: New Zealand Council for Educational Research.

Yackel, E. (2000). *Creating a Mathematics Classroom Environment that Fosters the Development of Mathematical Argumentation*. Paper prepared for Working Group for Action 1, International Congress of Mathematical Education, July 31 – August 6 2000, Tokyo/Makuhari, Japan.

Yackel, E. & Cobb, B. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4) 458-477.

Young-Loveridge, J. (1991). *The Development of Children's Number Concepts from Ages Five to Nine: Early Mathematics Learning Project: Phase II*. Hamilton, NZ: University of Waikato. (Volume I: Report of Findings. Vol II: Appendices).

Zevenbergen, R. (2001). Language, social class and underachievement in school mathematics. In P. Gates (Ed.). *Issues in mathematics teaching* (pp. 38- 50). London: RoutledgeFalmer.

Zevenbergen, R. (2002). Streaming in school mathematics: a Bourdieuan analysis *Proceedings of the Third International Mathematics and Education Conference*, Part 2, pp. 512- 521. Helsingor, Denmark: Centre for Research in Learning Mathematics, The Danish University of Education.

Zevenbergen, R. & Lerman, S. (2001). Communicative competence in school mathematics: On being able to do school mathematics. In Bobis, J., Perry, B., & Mitchelmore, M. (Eds.) *Proceedings of the Twenty-fourth Annual Conference of the Mathematics Education Research Group of Australasia*. 30 June – 4 July, 2001. Vol 2, pp.571-578. Sydney: Mathematics Education Research Group of Australasia.

## APPENDIX 1: Schools' information letter

Fiona Walls  
Mathematics Education Department  
Wellington College of Education  
PO Box 17-310  
Karori  
Wellington  
26 January 1998

Ph: 476 8699 Extension 8925

Dear Principal

I am currently a lecturer in mathematics education at Wellington College of Education. I am about to undertake research into children's learning in mathematics. This research will form the basis of my doctorate thesis at Victoria University of Wellington.

### Purpose of the research:

The TIMMS report (Third International Science and Mathematics Study) released in July this year, noted that ' while a majority of students have positive attitudes to learning mathematics, beginning from a fairly young age there is an increasing proportion of students having *lost interest in the subject*, with a concomitant decline in their achievement.'

My research is intended to help us understand how children in the middle primary school, develop their attitudes to mathematics. It is hoped that if we can identify some of the major reasons why children become switched on or off mathematics, we may be able to develop strategies to increase the proportion of students who are interested in and achieving well in mathematics.

### Process of the research:

Through random selection, your school has been chosen as a possible study site for this research. I hope very much that you will agree to becoming a participant school. I must stress at this point that the identity of the school and all connected participants (principal, teacher, parents, child) will be concealed at all times and will be known only to myself as the researcher.

The research will take place over a period of **3 years** and will involve:

- Random selection of **one** 7 year-old child (year 3).
- Gaining permission from the parent/guardian to study their child over a three-year period
- Interviews with the child 2 or 3 times each year.



- Observation of the child in the classroom once each term. This will include the child's verbal interactions during mathematics and other learning sessions.
- Observation of the child's classroom environment and teacher during mathematics and other learning sessions.
- Interviews with the child's parents/caregivers, twice each year.
- Interviews with the child's teacher/s, three times each year.
- Collection of child's mathematics work samples including regular written reflection. Interviews with principal at the beginning of each year.

It is my intention to minimise the disruption to the lives of the participants and to the normal running of the participant child's/teacher's classroom. I will fit my visits and interviews around the needs of the participants.

The research at your school will only go ahead on the condition that full consent has been granted by all parties involved, and any party has the right to withdraw at any time. The research proposal has been approved by Victoria University's Human Ethics Committee.

Feedback on the research findings will be provided to participating schools, parent/caregivers and teachers regularly throughout the study, so that all of us will benefit from the research.

Are you and your 1998 teachers of Year 3 students agreeable to becoming part of this project? I would like to begin the research early in the first term of 1998 so I would be grateful if you could let me know as soon as possible whether you are willing to become involved. If you need more information, please call me or use the envelope provided.

Yours sincerely

Fiona Walls

## **APPENDIX 2: Summary letter for participants – Second Year of Study**

Fiona Walls  
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Research Report

### **HOW CHILDREN FORM THEIR ATTITUDES TO MATHS**

December 1999

Dear [Parent/teacher/principal name]

The second year of my research into children's formation of attitudes to mathematics has now been successfully completed, once again thanks to the exceptional co-operation of all participating principals, teachers, parents and of course, the children. The 10 randomly-chosen children with whom I began the study early in 1998, have by now become quite used to my visits and I greatly enjoy my contact with them. Several of these children have changed schools, or even, in the case of one family, moved out of the Wellington region. So far, all have remained in the study. It has been a privilege to observe the amazing growth in learning that these children have all experienced.

Following the same pattern as last year, I have made 3 visits to the classrooms of each child. These were spaced evenly through the year in order to give me a broad overview of each child's progress in maths. Observing and videoing in classrooms, and talking with teachers, parents, principals and the children themselves, has provided a wealth of information. I have been deeply impressed at the level of commitment shown by all participants. Their friendliness, kindness and generosity have been most heart-warming. Everyone has gone out of their way to accommodate my visits, and to share their thoughts and observations, and this has provided me with valuable insights into the lives of these very special and delightful children.

The research is proving most interesting, with some issues that began to emerge last year, now becoming quite significant. Year 4 seems to be a very important year for children's learning of mathematics. It is during this year that teachers focus on consolidating learning of Level 2 objectives of the New Zealand mathematics curriculum, and preparing children for level 3.

There has been a very noticeable increase in the expectation that children know the multiplication basic facts this year. While instant recall of all multiplication facts (commonly referred to as 'Times Tables') is not expected of the children until the end of Level 3 (year 6), many children in the study have been making progress on these this year. For most of the children in the study, it is their perceived success or failure in basic facts recall which is now becoming the gauge they use to measure how good or not, they are at mathematics. All the children in the study are now quite aware of where they stand compared with others in the class and for some, they have begun to view maths as quite competitive. For some children in the group, their ability to do well in the basic facts speed tests has given them a real boost in confidence. For others, the pressure of the tests is having a marked detrimental effect. Even for those who are succeeding at these tests, there are comments that the time constraints are making them feel 'nervous' or 'scared' or 'uncomfortable'. Several children commented that they felt they would perform better given more time.

Another change from year 3, is the disappearance of apparatus and equipment in the teaching of mathematics. Children are increasingly expected to work alone at their desks in their books or using a worksheet. While a small number of the children said they preferred to work this way, most of them said that talking maths through with others and using equipment, were their preferred and most helpful methods of learning.

When asked about something they had done in maths that they really enjoyed, they often told me about an activity that involved drawing or using interesting equipment such as mirrors. When asked about something they could do in maths now that they couldn't do before, knowing basic facts, and being able to add, subtract or multiply with two-digit numbers were the most common responses.

Once again, the children rarely showed an understanding of why they were learning maths at school. They mostly felt that they needed to know maths so that when they were older they would be able to do it. None of them said they already used maths on a daily basis as part of their lives. None of the children selected maths as their most enjoyed school subject, even those that felt very successful at it.

For several children in the study, their feelings about mathematics have markedly improved over this year, and this seems to have been mainly because of the ways in which their teachers and/or parents have been supportive and encouraging.

I would like to thank you all very much for your involvement this year. Your assistance has been greatly appreciated. I wish you and your families a wonderful holiday season and I am greatly looking forward to working with you again next year, the final year of the project.

Kind regards

Fiona Walls

## **APPENDIX 3 Principal Interview Questions**

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### **Questions for Principals**

1. How would you describe the climate of your school and its community?
2. What decile rating does the school have?
3. How strong do you think maths is at the school? (Teachers, children, community)
4. How are teachers inserviced in maths at your school?
5. How many of your current staff underwent inservicing in implementing *MiNZC*?
6. What are the curriculum development priorities at the school at the moment?  
When will maths next be a priority?
7. What maths resources does the school have? Who is responsible for them?
8. Are all the staff familiar with the ministry handbooks? (1995, 1996, 1997)
9. How do you feel about your school maths policy? Is it new? Does it need updating?
10. Does your school participate in any kinds of maths competition?
11. How does your school communicate with parents about its maths programmes?
12. What feedback do you have from parents about maths?

## **APPENDIX 4 Parent Interview Questions**

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### **Parent Interview Questions**

#### **Home 'Culture'**

1. Have you got full/part time paid employment at the moment?
2. How many children in the family? Where is your child's place in the family? (eldest..etc)
3. Are you the main person who looks after them? (Principal or only caregiver?)
4. What languages do you speak at home?
5. What activities do you do regularly as a family? Daily? Less regularly?
6. How many books do you estimate you have in the house? What kinds?
7. Do you have a computer? Who uses it? Are you on the internet?

#### **Child at Home**

1. Does your child share a room?
2. Does s/he have a fixed time and place for homework?
3. Does s/he go to after school care? If yes, where? What do they do there?  
Homework?
4. What does your child like doing in her/his spare time? Who with?
5. If your child watches TV, what programmes does s/he enjoy most?
6. If your child plays on the computer, what does s/he do on it?
7. What are your child's favourite toys?
8. Does your child have any regular out-of-school activities? Eg dance, sport, music
9. Does your child enjoy games like cards, noughts and crosses, board games  
puzzles?
10. What would you rate as the most important of all your child's activities? Why?

**Parents' Education and Mathematics Backgrounds**

1. What was your favourite subject at school?
2. What qualifications do you have now?
3. How good were you at maths at school?
4. What is the highest qualification you have in mathematics?
5. How do you feel about mathematics now?
6. Where do use maths in your everyday life? How comfortable do you feel about doing this?
7. Are you good at estimating, do you think? Eg paint, fabric quantities etc
8. Do you balance a cheque book?
9. Do you fill in your tax return?
10. Do you use a calculator when you go shopping?

**Parents' Views About Mathematics**

1. How important do you think it is to be good at maths? Why?
2. What is the most important part of maths that people need to know do you think?
3. Do you think New Zealanders are generally good at mathematics?
4. Do you feel you understand the way maths is taught at school now?
5. Does it differ from the way you were taught do you think? How?
6. Would you like to see the teaching of maths change? How?

**Parents' Own Child and Mathematics**

1. What do you see as your child's special strengths or talents, generally?
2. What pre-school experience did your child have? Kindergarten, kohanga reo etc
3. Do you think your child enjoys school? Has this been so since s/he started school?
4. What are your child's strengths at school, do you think?

5. Are you happy in general with her/his progress?
6. In general, do you think your child's teacher understands your child well?
7. How do you feel about your child's progress in mathematics since starting school? (Are you anxious about your child's progress?) Why?
8. Does your child know how you feel about her/his progress in maths? How?
9. Do you think your child's teacher is over or underestimating your child's ability? Why?
10. How much time each week do you estimate your child spends on maths homework? Do you think this is too little or too much?
11. How do you help your child with her/his mathematics? Is there anyone else who helps her/him? Eg at after school care/babysitter/older sibling/grandparent?
12. Would you ever consider enrolling your child in an out-of-school maths programme such as number works?
13. If you already have, what affect is it having do you think? How do you know?
14. What do you think is the most important part of maths that your child needs to know? Why?
15. How long do you expect your child will learn mathematics for?
16. How long do you expect your child will stay at this school?
17. At what age do you expect your child to leave school?
18. What are your hopes, generally, for your child?
19. Do you think it likely that your child will end up in a job that involves maths?

### **Mathematics in Family Activities**

1. Do you get your child to help you when: baking, laying out patterns, building, running errands to the shop, at the T.A.B., poker/Housie.
2. Do you have Duplo/lego, computer, calculator, desk/study area. Does the child use these?
3. Have you ever been to Laser Strike with your family? Did you discuss the end game print-out with your child?
4. Does your child have a savings account? How does it work?

5. Do you have a pocket money system? How does it work?
6. Does your child buy things on her/his own eg ordering school lunch, buying things from the shop etc. How well can your child recognise and count different coins/notes?
7. Does your child have a watch? Can s/he tell the time?
8. Can you think of any other times when you do activities with your child which involve maths ideas or skills? Eg orienteering, sailing, gardening



## **APPENDIX 5: Child Interview Questions**

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### **Child Interview Questions** (\* additions for subsequent interviews)

#### **Child's Aspirations and Preferred Activities at Home**

1. What do you usually do in the weekends and after school now? What games do you like?
2. Do you have a favourite toy at the moment?
3. What toy haven't you got, that you would really like?
4. Do you use the computer much? What do you like doing on the computer?
5. What TV programmes do you like at the moment?

#### **Child at School**

6. Do you [\*still] like/dislike school? Why?
7. Have things ever happened to you at school [\*since we talked last] that have made you feel bad? Good? (Explain)
8. What parts of the school day do you like best? Why?
9. What different things [subjects] do you to learn at school? Which of these things [subjects] do you like best? Which do you like least? Why? [Music, PE, sport, reading, writing, swimming, Topic, science, mathematics, art, Maori, other?
10. Which subjects that you learn at school are you best /worst at, do you think?
11. Which of these things do you think are the most important to learn? Why?
12. What subjects do your parents want you to be good at? How do you know?

#### **Child and Maths at School**

13. What is maths, do you think? How do you feel these days when the teacher says 'it's time for mathematics?' Do you feel comfortable doing mathematics? Why?
14. \*Do you like maths more or less than you used to? Why?

15. (ask this only if it didn't come out in question 14) Is maths getting easier/harder for you or is it about the same?
16. \*Do you feel you've got better or worse at mathematics this year? Why?
17. Now we're going to fill in this worksheet. [\*Do you remember this worksheet that we did together last time I talked with you? Now we're going to fill it in again]. [Selection of tasks and questions will be recorded on the sheet]. Just try to tell me exactly how you think and feel. Remember there is no right or wrong answer.
18. How do feel about being X [quote the number they chose on the scale] at maths?
19. \*Where do you think the teacher would put you on the scale? What make you think that? What about your parents?
20. Why do you think you are X at maths? [prompt with 'and not W or Y?' if necessary].
21. Are some people in the class better/worse than you at maths? How do you know? Why are they better/worse do you think?
22. Are there some bits of maths that boys are better at than girls do you think? Are there some bits of maths that girls are better at than boys, do you think?
23. Are you in a maths group? Why are you in that group, do you think? How do you feel about that?
24. What do you usually do at mathematics time? What do you usually start with? Then what? How do you usually finish the mathematics time?
25. Which is the most important part of maths time do you think?
26. \*How do you feel about the things you do in maths like ....(include things that are part of the child's class routine eg):
  - Quick 10, Round the World, Buzz (or similar)
  - mathematics tests
  - worksheets
  - working in groups
  - on the mat with the teacher ?
  - being asked a question by the teacher
27. Do you like it when you write your mathematics, when you do maths using equipment [if necessary, give examples e.g. blocks, measuring instruments] or when you are talking about it? Which way/s do you like best? Why?
28. Would you rather do mathematics with other people or by yourself?
29. What can you remember doing in mathematics [\*since I saw you last]?

30. Is there something that you have done in mathematics this year that you really liked? Why? Really hated? Why?
31. How could mathematics time be made better for you?
32. \*Can you show me some of the things you can do in maths?
33. How useful is it to learn mathematics do you think? How is it useful now? How might it be useful when you're older?
34. Do you ever talk about mathematics with your friends?

### **Maths at Home**

35. Do you do ever do any maths at home? When? Who with?
36. What do you usually get for homework? What maths do you get? Can you do it by yourself? What do you do if you get stuck?
37. How do your parents feel about how you're getting on in maths? How do you know? Are you bothered by that?
38. Are you looking forward to maths next year [\*at intermediate/high school]? Why?

# APPENDIX 6: Children's Questionnaire Sheet<sup>1</sup>

Sample: Jared, Late Year 3

Date \_\_\_\_\_

## How I feel about maths



Finish this sentence. Maths is ....OK.....	
How do you feel when you do maths? Record on the scale. unhappy  ----- ----- ----- ----- ----- ----- ----- ----- -----  happy 0 1 2 3 4 5 6 7 8 9 10 <input checked="" type="radio"/> at 1	
Do you think you are good at maths? Record on the scale. no  ----- ----- ----- ----- ----- ----- ----- ----- -----  yes 0 1 2 3 4 5 6 7 8 9 10 <input checked="" type="radio"/> at 10	
Why do we learn maths at school? To get good at writing.	
What do you like most about maths? Times Tables.	What don't you like about maths? Tests.
What do you do when you don't understand something in maths? ask the teacher.	
You do lots of maths at school. At what other times do you do maths? no.	

<sup>1</sup> From Beesey & Davie, 1991: Macmillan Mathematics, Level 2b, Children's Recording Book, p.3.

## **APPENDIX 7: Teacher Interview Questions**

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### **Teacher Interview Questions**

#### **The Classroom Culture**

1. How would you describe this class generally?
2. What do you see generally as the children's strengths and needs in this class?
3. How much parental involvement is there in your classroom? Do you think this is too little or too much?
4. What would you say are your priorities as a teacher?
5. How would you describe your relationship with this class?
6. What do you see as your teaching strengths?
7. Which subject do you most enjoy teaching?

#### **The Child in the Classroom**

1. What is the child's attitude to school in general, do you think?
2. What do you see as her/his special strengths?
3. How does the child relate to others in the class?
4. What does s/he enjoy doing most during play/lunch times? Who with?

#### **Teacher's Education and Mathematics Backgrounds**

1. How long have you been teaching and where?
2. Where did you train? (NZ or overseas?)
3. What was your favourite subject at school?
4. What qualifications do you currently have? Are you continuing to study, intending to? In what areas?
5. What is the highest level you reached in mathematics?
6. How did you feel about mathematics when you were at school?

7. How did you learn to use the new maths curriculum?
8. How well do you feel you are able to use it?
9. Are you familiar with, and use, the associated ministry handbooks? [*Implementing Mathematical Processes, Development Band Mathematics, Developing Mathematics Programmes?*]
10. Which curriculum committees are you on at the school?
11. How do you yourself feel about mathematics? What is easiest/hardest for you in mathematics? What do you most/least enjoy teaching in mathematics?
12. What opportunities have you had to upskill in mathematics?
13. Have you had any contact with the maths advisors this year? Was this helpful? How?
14. How do you feel about the possibility of having specialist maths teachers in primary schools?

### **Class Mathematics Programme**

1. How would you describe your classroom mathematics programme?
2. How do you do your planning? (Cross-curricular? Syndicate? Focus on processes? Etc) Where do you get most of your resources?
3. What proportion of the timetable is mathematics? (Length of sessions - days on which maths is taught)
4. What would a typical maths lesson look like in your class? (Basic components of lesson, typical learning experiences.
5. What kinds of groupings do you have in the class? Is there a development band group?
6. What do you see as your role as a teacher of mathematics?
7. How would you describe the abilities of the children in your class? How do you know?
8. Are you satisfied with your class' general level of/progress with, their maths learning? How do you know?
9. How do you think the children feel generally about maths in this class?
10. What do they seem to find easiest/hardest? How do you know which kids have cottoned on?

11. What maths homework do you give the class? Is giving homework school policy or not? How do parents feel about homework?
12. What would you say most gets in the way of successful teaching of maths in your class? (Resources, parents, kids' behaviour, kids' lack of ability, previous teaching, *MiNZC*, own confidence etc.)
12. Is the computer used for maths in the class? How? How confident are you on the computer?
14. How are calculators used in your class? Do you use calculators yourself? For percentages/using memory/finding means/repeated addition function?
15. How do you think the year 4 maths programme differs from year 3 maths?
16. How much time do you feel you spend managing the children during maths times?

### **Child's Mathematics**

1. Were you satisfied with (study child's) level of prior knowledge on arrival in your class?
2. Are you satisfied with her/his progress with her/his maths learning so far this year?
3. How do you think s/he feels about maths?
4. What do you think s/he finds easiest/hardest in maths?
5. Do **you** think attitude matters in learning maths?

**APPENDIX 8: Child self assessment sheet for classroom use**

Sample: Fleur, Late Year 3

My Maths This week					
Week	Topic	How I felt about maths this week	How I found maths this week	One new thing I learned in maths this week	
9	number		0 1 2 3 4  ----- ----- ----- -----  Hard Easy	place value	
10	Statistics		0 1 2 3 4  ----- ----- ----- -----  Hard Easy	to make choices	
4	Money		0 1 2 3 4  ----- ----- ----- -----  Hard Easy	nothing	
7	Multiplication		0 1 2 3 4  ----- ----- ----- -----  Hard Easy	<del>math</del> 2 5 10 time tables	



# APPENDIX 9: Speed Test Example<sup>1</sup>

Sample: Peter, Early Year 5

## BASIC FACTS SPEED TEST

DATE: 4.12.00

NAME:

TOTAL: 83 / 100

9 + 5 = 15 ✓	15 - 8 = 7 ✓	2 + 3 = 5 ✓	6 × 3 =	9 × 9 = 81 ✓
7 - 4 = 3 ✓	8 + 4 = 12 ✓	7 × 7 =	10 - 4 = 6 ✓	40 + 8 =
4 × 2 = 8 ✓	4 × 6 =	13 - 5 = 8 ✓	3 + 7 = 10 ✓	11 - 3 = 8 ✓
6 + 3 = 2 ✓	28 + 4 =	6 + 9 = 15 ✓	64 + 8 =	8 + 8 = 14 ✓
2 × 5 = 10 ✓	16 - 7 = 9 ✓	14 + 7 =	2 × 6 = 12 ✓	6 × 5 = 30 ✓
9 + 8 = 17 ✓	6 + 4 = 10 ✓	3 × 8 =	5 + 6 = 11 ✓	13 - 9 = 4 ✓
5 × 0 = 0 ✓	7 × 9 = 63 ✓	9 + 3 = 3 ✓	16 + 4 =	14 + 2 = 7 ✓
18 + 2 = 20 ✓	16 + 8 = 24 ✓	8 + 3 = 12 ✓	8 × 3 =	9 + 9 = 18 ✓
3 + 4 = 7 ✓	10 - 2 = 8 ✓	5 × 3 = 15 ✓	8 - 3 = 5 ✓	54 + 6 = 60 ✓
5 - 2 = 3 ✓	4 + 2 = 6 ✓	8 - 0 = 8 ✓	3 + 6 = 9 ✓	6 × 8 =

5 - 3 = 2 ✓	6 + 6 = 12 ✓	12 - 9 = 3 ✓	7 + 7 = 14 ✓	5 × 9 = 45 ✓
4 + 8 = 12 ✓	17 - 9 = 8 ✓	7 + 2 = 9 ✓	6 - 4 = 2 ✓	11 - 7 = 4 ✓
2 × 2 = 4 ✓	9 × 7 = 63 ✓	8 - 1 = 7 ✓	2 × 9 = 18 ✓	7 + 6 =
63 + 9 = 72 ✓	9 - 5 = 4 ✓	15 + 3 = 5 ✓	72 + 8 = 80 ✓	35 + 7 = 5 ✓
9 - 4 = 5 ✓	21 + 3 =	15 - 9 = 6 ✓	25 + 5 = 5 ✓	7 × 6 =
5 + 7 = 12 ✓	9 × 8 = 72 ✓	48 + 8 =	3 + 5 = 8 ✓	12 + 2 = 6 ✓
18 + 9 = 27 ✓	3 + 3 = 6 ✓	2 + 6 = 8 ✓	4 × 8 =	7 + 9 = 16 ✓
8 × 4 =	10 × 10 = 100 ✓	5 × 6 = 30 ✓	54 + 9 = 6 ✓	10 - 8 = 2 ✓
40 + 5 = 45 ✓	13 - 6 = 7 ✓	10 + 5 = 2 ✓	7 × 4 =	36 + 9 = 4 ✓
10 - 6 = 4 ✓	8 + 7 = 15 ✓	9 - 6 = 3 ✓	11 - 6 = 5 ✓	2 + 9 = 11 ✓

Years 7 - 8 = 5 minutes

Years 5 - 6 = 6 minutes 30 seconds

Years 3 - 4 = 8 minutes

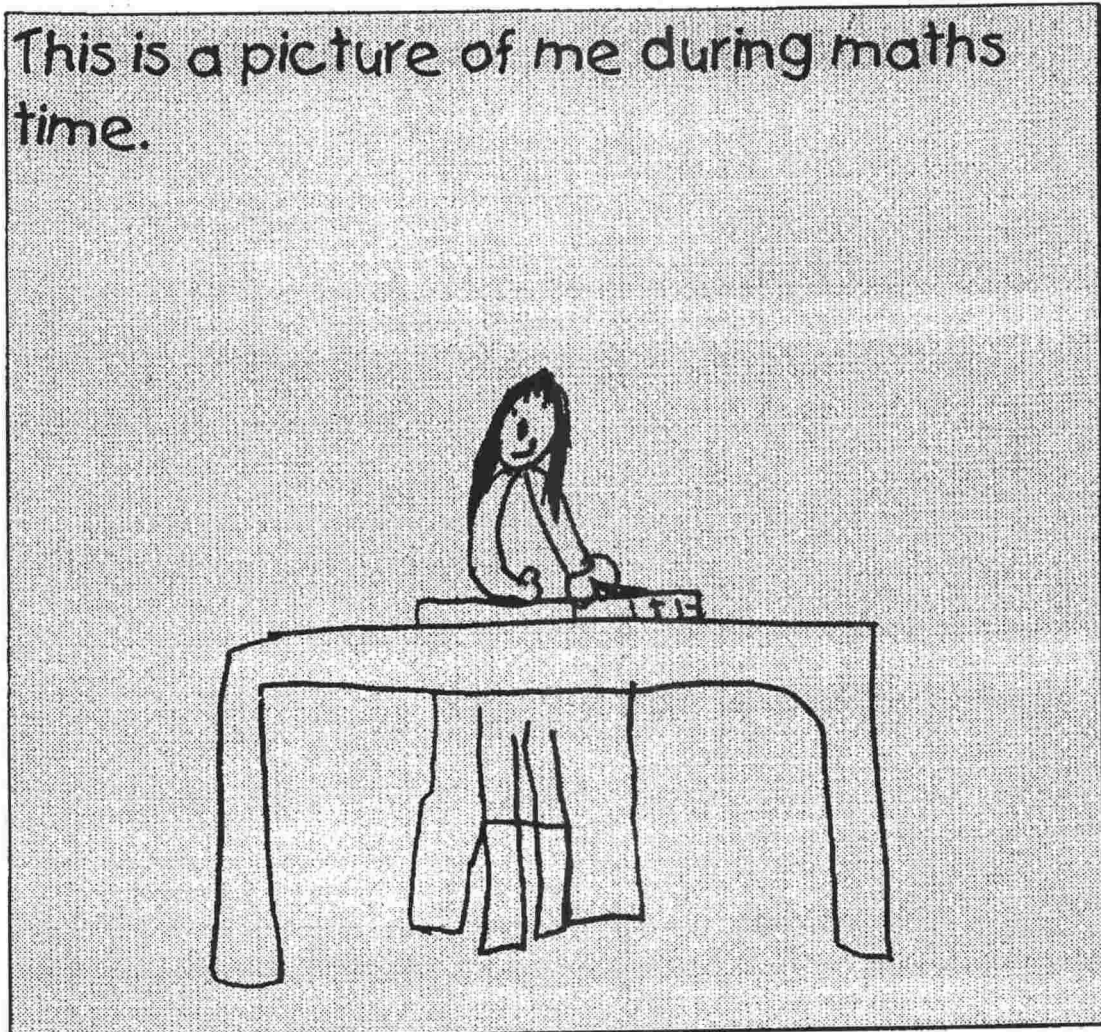
# 11 +, -, x, ÷

<sup>1</sup> From: *Speed, skill and success using basic facts: A monthly monitoring programme*. (Pinder & Adams, 1996)

**APPENDIX 10: Children's drawing sheet**

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Sample: Georgina's drawing, Early Year 3.



## APPENDIX 11

Children's self rating for **enjoyment** and **ability** over Years 3, 4, and 5

