Appendix 8: Flexure methodology

Flexural isostasy and modelling methodology

Isostasy is based on Archimedes' principle of hydrostatic equilibrium, where pressures produced by overlying material are everywhere the same below a specific depth, termed the compensation depth, within the Earth. Topography is locally compensated at depth by lateral crustal variations in either density (Pratt, 1855), thickness (Airy, 1855), or a combination of both (Watts, 2001). This local compensation assumes that the lithosphere has no elastic strength. However, in reality, loads are compensated over a larger area, which is dependent on load size and spatial distribution, and the elastic strength of the lithosphere (Watts, 2001).

Flexural isostasy is modelled as an elastic plate over a weak, inviscid substratum and uses the theory of elasticity, where flexural rigidity (D) of the elasitic plate is defined as such:

$$D = \frac{E{T_e}^3}{12(1-\sigma^2)} \tag{9.1}$$

Where E is Young's modulus, T_e is the elastic thickness of the plate, and σ is Poisson's ratio (subsection **Error! Reference source not found.**).

If we consider oceanic lithosphere deforming under a vertical line load $V(x)$ with no acting horizontal force, water will fill the resulting depression of the lithosphere created by the down-going plate (Watts, 2001). However, there is a restorative force ($(\rho_m - \rho_w)gw$) per unit area acting because the deforming lithosphere is not in isostatic equilibrium; i.e. a breadth of mantle ρ_m, with a thickness of w has essentially been replaced by water ρ_w. Thus:

$$D\frac{d^4w}{dx^4} = V(x) - (\rho_m - \rho_w)gw \tag{9.2}$$

The differential equation above (equation 9.2), can be solved for specific loads and boundary conditions to yield the deflection of the plate as a function of horizontal distance (Watts, 2001). Take the case of this study, where we model the load of the Australian Plate (considered to be broken and semi-infinite) in the Wellington vicinity assumed to be located at $x = 0$. Instead of having a density contrast between mantle and water ($\rho_m - \rho_w$), however, one would use the density contrast between mantle and sediment infilling the downwarp of the Whanganui Basin ($\rho_m - \rho_s$). Therefore, equation (9.2) is:

$$D\frac{d^4w}{dx^4} + (\rho_m - \rho_s)gw = 0 \qquad (9.3)$$

The solution for equation (9.3), for a line load V at $x = 0$ is:

$$w(x) = w_0 e^{-x/\alpha} \cos\left(\frac{x}{\alpha}\right) \qquad (9.4)$$

Where the deflection of the plate at $x = 0$ is:

$$w_0 = \frac{V\alpha^3}{(4D)} \qquad (9.5)$$

And the flexural parameter (e.g. Walcott, 1970), is calculated by:

$$\alpha = \left(\frac{4D}{(\rho_m - p_w)g}\right)^{1/4} \qquad (9.6)$$

Figure **Error! No text of specified style in document.**.1 displays plate deflection, determined by equation (9.4) as a function of x. The area of the plate where the deflection is positive is termed the "outer high". On the top of the outer high, $\frac{dw}{dx} = 0$; hence, by differentiating equation (9.4), we can calculate the horizontal distance from the origin ($x = 0$) to the outer high ($x = x_b$), which represents the width of the depression. Once x_b is identified, it can be used to calculate a value for the flexural parameter (α). The value for α is used to calculate the flexural rigidity (D), which is then in turn, used to calculate the elastic thickness (T_e) (equation 9.1).

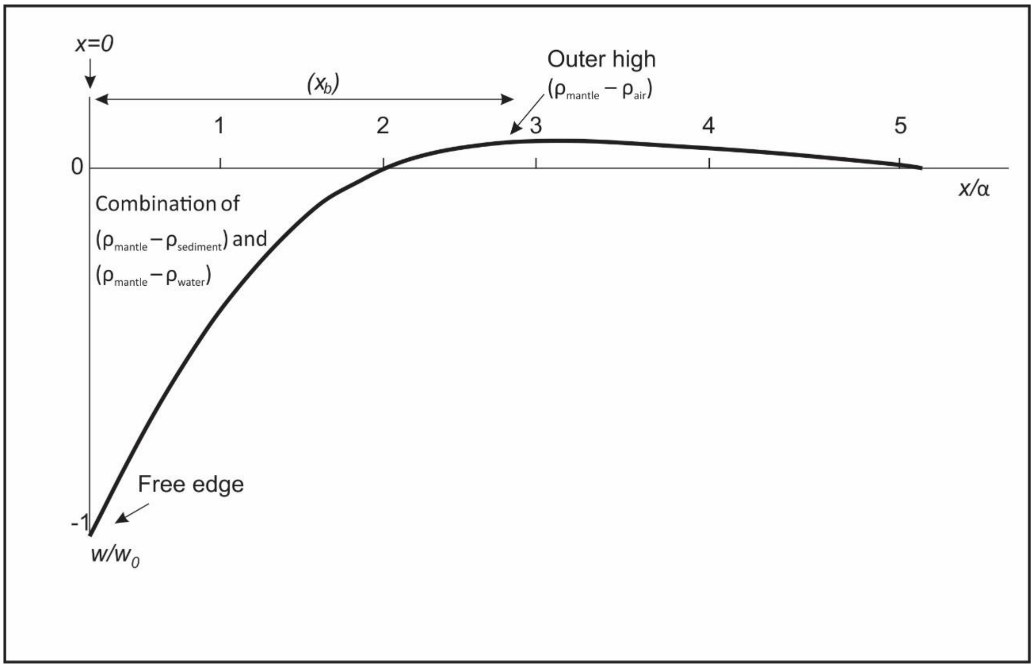

*Figure **Error! No text of specified style in document.**.1 Deflection (w_0) of a broken elastic plate with a line load at x = 0. The deflection is normalised to w_0, which in turn is determined by the load and the plates physical properties. (Figure modified from Fowler (2005).*

The flexure of oceanic lithosphere at a subduction zone can also be modelled with an additional constant horizontal compressive force per unit length (H):

$$D\frac{d^4w}{dx^4} = V(x) - H\frac{d^2w}{dx^2} \tag{9.7}$$

In this case, to obtain the deflection of the plate, we include a vertical load (V) at the free edge of the plate ($x = 0$), and we can simulate a horizontal load by applying a bending moment M per unit length:

$$w(x) = \frac{\alpha^2}{2D}\exp\left(-\frac{x}{\alpha}\right)[V\alpha + M)\cos(\frac{x}{\alpha})], \qquad x \geq 0 \tag{9.8}$$

In reality, the parameters M and V cannot be calculated accurately; however, the width and height of the outer-high can be, which can then be used to calculate the flexural parameter (α) (using equations (9.1 and 9.6), and thus the elastic thickness (T_e) can be solved.

Factors that control T_e

Both continental and oceanic lithosphere can be altered by flexure. However, the two types differ in their physical and chemical evolutions (Watts and Burov, 2003). Oceanic crust is young relative to continental crust, is characterised by a single layer rheology and thus its effective flexural rigidity (T_e) is contingent predominantly on thermal age (Watts and Burov, 2003). Continental lithosphere in contrast, has a multi-layer rheology, and effective flexural rigidity is dependent on multiple factors such as crustal thickness, the geotherm, and composition (Watts and Burov, 2003). These contrasts between oceanic and continental lithosphere is revealed in Figure **Error! No text of specified style in document.**.2 through Yield strength envelopes (YSEs), where oceanic crust has a strong elastic core, whereas continental lithosphere has weak layer in the lower crust. T_e thickness ranges from 2 – 50 km for oceanic lithosphere, and from 5 – > 100 km for continental lithosphere (Watts and Burov, 2003).

As the age of oceanic lithosphere increases, T_e also increases (Figure **Error! No text of specified style in document.**.3). However, because oceanic lithosphere is subject to high curvatures as it approaches a trench, T_e is less than what it would be otherwise based just on plate age (Watts and Burov, 2003). For example, Judge and McNutt (1991) found that subducting oceanic crust in the northern Chile trench (in a high curvature region), T_e is 22 ± 2 km. This is less than the T_e of 34 km expected based on thermal age (Judge and McNutt, 1991).

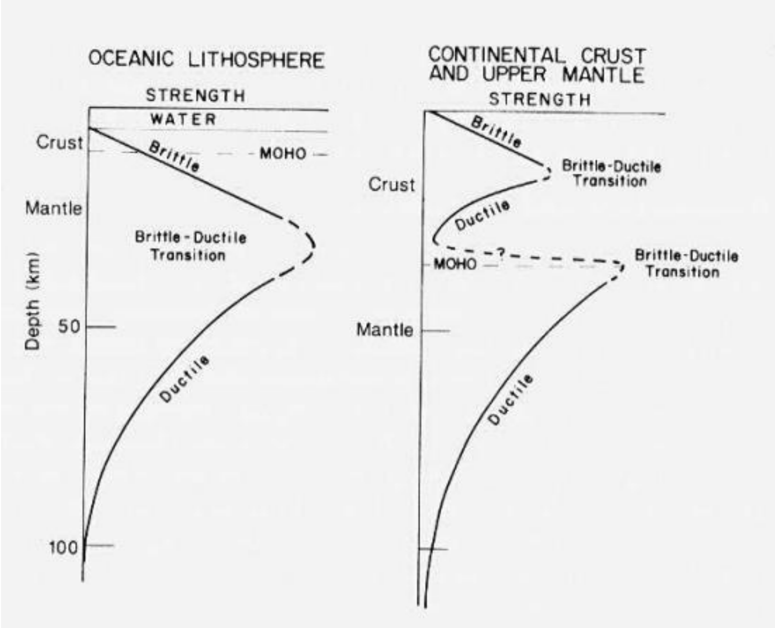

*Figure **Error! No text of specified style in document.**.2 Yield strength envelopes (YSEs) for oceanic and continental lithosphere. Oceanic lithosphere has a strong, elastic core, whereas continental lithosphere has a weak ductile layer in the lower crust. (Molnar, 1988).*

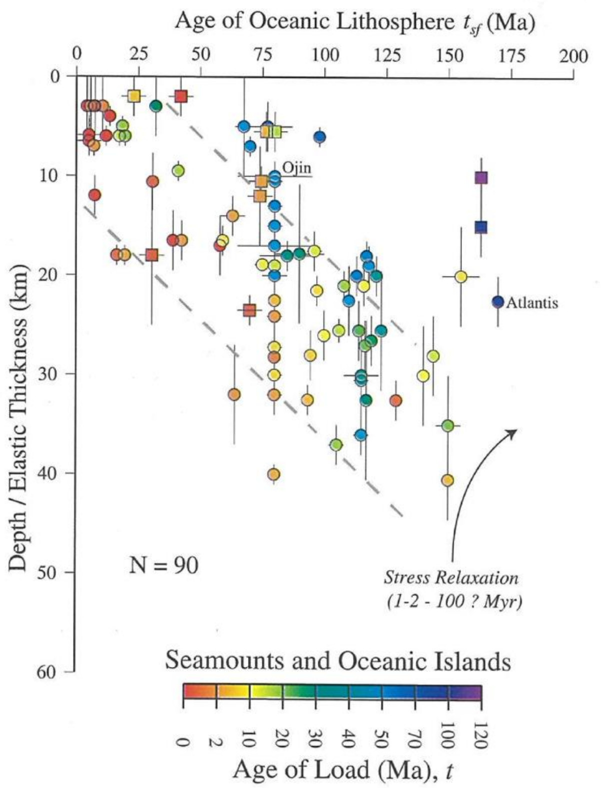

*Figure **Error! No text of specified style in document.**.3 Age vs elastic thickness (T_e) for oceanic lithosphere. Colour coding corresponds to load age (Watts, 2001).*

Modelling constraints

Listed below are the modelling constraints and their associated justifications

1) Mapped K Surface remnants

   Minimum, maximum and average K Surface heights across a ~30 km X 30 km profile (**Error! Reference source not found.**) are used to help constrain the wavelength of the model.

2) Line load (kg m^{-1}):

   The vertical line is modelled to be 1.51 X 10^{12} N m^{-1}.

3) Horizontal in-plane stress:

   Horizontal in-plane stress is modelled as 2.5 X 10^{12} N m^{-1}. This value is consistent with the total ridge push of an old oceanic plate (Bott, 1993).

4) Density contrast:

   Due to the simplicity of the model, only one density contrast can be input. In reality, the density contrast will vary vertically and laterally depending on what restorative force acts when the plate subducts. For example, the “outer high” which is used to model the surface uplift of K Surface remnants, will represent the density contrast between mantle and air (3300 kg m^{-3}) (Figure **Error! No text of specified style in document.**.1). Conversely, the area where the plate in deflected downwards, the density contrast will be a combination of $(\rho_m - \rho_s)$ and of $(\rho_m - \rho_w)$ due to Australian plate lithosphere, and Whanganui basin subsidence and sediment deposition (Figure **Error! No text of specified style in document.**.1). Therefore, for simplicity, a density contrast of 1000 kg m^{-3} is modelled.

5) Effective elastic thickness (T_e):

   As was mentioned above in subsection 0, T_e ranges 2 – 50 km for oceanic lithosphere. We use a T_e value of 27 km based on Watts and Talwani (1974), for topography profiles seaward of the Aleutian, Kuril, and Northern Bonin trenches.

Simple plate flexure model limitations

There are issues with simple flexure models. One of the major assumptions with the simple flexure model is that the computational origins are at the trench axis, and that the forces acting on and deforming the plate can be separated into simple components of vertical and horizontal force (Hetényi, 1946). In reality, it is more complicated than this. Multiple studies (Ludwig et al., 1966; Shimazaki, 1972; Stauder, 1968) have reported tensional focal mechanisms in trenches where flexure has been the proposed mechanism for the outer high. Horizontal compressive stresses as large as what were modelled for the red model in **Error! Reference source not found.**, would probably result compression in both the lower and upper parts of the plate (Watts and Talwani, 1974). Therefore horizontal in-plane stress was potentially highly overestimated (Hetényi, 1946; Watts and Talwani, 1974). It is also unlikely that deformation at the plate boundary would be perfectly elastic; rather plastic deformation would occur (Watts and Talwani, 1974). This would greatly complicate the observed (K Surface data) and calculated (elastic sheet) profiles, and thus was not considered in the model.

Notwithstanding the limitations outlined above, simple flexure models are useful as a first order approach to understanding how oceanic lithosphere reacts to different forces at subduction systems.

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