Attorney fees in repeated relationships

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Abstract

We investigate contracts between a law firm and a corporate client involved in a repeated relationship. In contrast to the previous literature pertaining to one-time interactions between clients and attorneys, we find that the contingent fee is not the best arrangement. Rather, the contingent fee is dominated by a contract which, we argue, an outside observer could not distinguish from simple hourly fee contract. This contract includes an hourly fee equal to the law firm's opportunity cost, a lump sum, and a retention function. The lump sum payment is independent of the number of hours worked by the law firm and the outcome of the case. The repeated nature of the relationship allows the client to create a contract where the desire to maintain the relationship induces the law firm to exert the optimal level of effort in the current case.
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Attorney Fees in Repeated Relationships

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1 Introduction

Contingent fee agreements are widely used in personal injury and medical malpractice litigation. These cases tend to involve clients with little experience in litigation who hire attorneys on a one-time basis. Numerous authors have considered this situation and find the contingent fee superior to other fee arrangements.[1] The typical reasoning in favor of the contingent fee stems from the familiar moral hazard problem. That is, if attorney effort is not observable, then an hourly or fixed fee arrangement does not induce the attorney to exert the effort necessary to maximize the client's expected award. The contingent fee, by giving the attorney a stake in the final award, induces the attorney to put forth a greater effort (Danzon, 1983). Contingent fees are found to be preferable to hourly or fixed fees in a number of other settings as well.[2]

However, the relationship between corporate clients and law firms has received little attention. Hadfield (2000) and Kritzer, et al. (1985) note that corporate clients and law firms typically engage in fixed or hourly fee, rather than contingent fee, contracts. One reason may be that corporate clients are better informed about the merits of their cases than the average plaintiff (Dana and Spier, 1993). Another possible explanation is that corporate plaintiffs are involved in more litigation than the average plaintiff and frequently use the same law firms for multiple cases (Coates, et al., 2011). This repeated interaction allows the law firm to establish a reputation with the corporate client that sends a clearer signal of the law firm's quality. In addition, the repeated nature of the arrangement allows the client to punish the law firm for poor performance by reducing the number of cases it sends to the law firm or by terminating the relationship. A law firm, therefore, must consider the impact of its level of effort for any single case on the continuing relationship with the client.

We show that in repeated relationships between law firms and corporate clients the contingent fee is no longer the optimal compensation arrangement. Rather, a contract combining an hourly fee, a lump sum payment, and a *retention function* can improve the client's payoff and lead to an efficient outcome.[3] The lump sum payment in our contract is independent of the outcome of the

[1] However, Schwartz and Mitchell (1970) find the hourly fee to be more desirable than the contingent fee.

[2] Contingent fees have been deemed desirable when clients are risk averse (Danzon, 1983), the attorney has better information regarding the merits of the case than the client (Dana and Spier, 1993), risk sharing is appropriate (Posner, 1986), information is asymmetric (Rubinfeld and Scotchmer, 1993), and clients cannot afford hourly or fixed fees (Rhein, 1982).

[3] We restrict our attention to legally permissible means of payments which include contingent fees, hourly wages and lump sum payments. See Santore and Viard (2001) for a discussion of restrictions on legal service payments.

case and the law firm's performance. In contrast, the retention function states the probability that the client will continue his relationship with the law firm as a function of the outcome of the current case.

We treat both the lump sum payment and the retention function as explicit elements of the contract. However, there seems to be no loss in treating the retention function as an implicit contractual element. This is made more feasible by the fact that the retention function takes a very simple and intuitive form. In particular, it is piece-wise linear and (weakly) increasing in realized payoffs. Regarding the lump sum payment, we argue in Section 5 that it may easily arise from an acknowledged distortion in reported hours. Hence, to an outside observer, the contract we suggest is indistinguishable from the hourly contracts observed in practice by Hadfield (2000) and Kritzer, et al. (1985).

The combination of the three contractual elements creates the incentives for the law firm to provide the efficient effort level. The hourly fee is set at exactly the opportunity cost of the law firm's time, and the lump sum payment is unaffected by the law firm's actions. Thus, the incentive for the law firm to exert optimal effort is not based on compensation derived from the current case but solely on the likelihood of receiving lump sum payments in future cases. Because the likelihood of receiving future cases is determined by the realized payoff in the current case, the firm acts to satisfy the client in the current case. In contrast to one-time contingent fee contracts, the repeated nature of the relationship allows the client to design a contract such that the law firm cares about the full realized payoff in the current case rather than the fraction of the payoff it receives.

The previous literature regarding attorney fees has largely found that contingent fee arrangements provide better incentives than hourly or fixed fee agreements. However, the contingent fee does not create perfect incentives for the attorney in these models.[4] Our contract provides the law firm with the incentives to choose the efficient effort level. Under mild assumptions, this leads to an outcome that the client prefers to the outcome from a one shot contract. However, our contract is likely not 'second best.' We do not set out to find such a contract, but rather to find a justification for the simple wage contracts which we observe in the market for corporate legal services.

[4]Only a 100% contingent fee completely solves the moral hazard problem in the models in the literature, but this arrangement is not legally permissible. See Danzon (1983), Halpern and Turnbull (1983), Hay (1996) and Santore and Viard (2001).

An important assumption of our model is that the client may terminate his relationship with the attorney at any time. This assumption allows us to simplify the analysis by limiting the client's decision to a choice between continuing or terminating its relationship with the law firm. Evidence suggests that corporate clients are able to reduce the intensity of a relationship with one law firm at nearly zero cost from the common use of "preferred counsel" lists that are maintained by many Fortune 500 companies.[5] Preferred counsel lists are created by corporations to inform managers about which law firms are approved for use in a particular type of case. Law firms are informed of their placement on or removal from the list and most companies list multiple law firms for each type of legal dispute (Jones, 2003). Thus, if a corporate client wishes to punish a law firm for providing insufficient effort it can simply give a larger share of its cases to one of the other approved firms. Additionally, the corporation can remove the law firm from the preferred counsel list altogether. In a recent survey of Fortune 500 corporations, Coates et al. (2011) find the most common reason for reducing the volume of cases general counsel allocated to a particular law firm was poor quality service. Furthermore, empirical evidence indicates corporate general counsel consider only very recent activity by the law firm when setting compensation and the likelihood of retention (Jones 2003).

We are not aware of other studies that provide a formal analysis of repeated legal services contracts. A few authors have considered information asymmetries related to the performance of law firms. For example, Smith and Cox (1985) find empirical evidence that firms invest in their reputations to signal to clients that they are high quality. Garoupa and Gomez-Pomar (2008) also examine contracts between clients and law firms, but in a one-shot setting. In their model, an attorney must balance the interests of the client and the law firm for which he works. They argue that the hourly fee is preferred to the contingent fee because it offers a solution to this dual agency problem. (See also Gilson and Mnookin (1985) and McChesney (1982) on agency problems within law firms.) Mixed fee arrangements have also been considered by others. For example Clermont and Currivan (1978) propose a mixture of hourly and contingent fees to solve the moral hazard problem. Likewise, Rubinfeld and Scotchmer (1993) find that a combination of fixed and contingent fees is optimal when information is asymmetric. These authors focus on one time interactions whereas we

[5]The assumption of a zero replacement cost is common in the literature on repeated contracting with moral hazard discussed below. See, e.g., Spear and Wang (1995) and Banks and Sundaram (1998).

consider repeated interactions between the attorney and client.

There are two closely related strands of literature outside of law and economics: the literature on implicit contracts, and the literature on repeated contracting under moral hazard. Our paper is distinguished from both literatures by our focus on the corporate legal services market. In particular, we take institutional constraints on legal service compensation very seriously. Our analysis is not directed towards finding a standard 'second best' contract. We do not claim to have found such a contract. Instead, we find an efficient contract that a client would chose if: he is involved in a repeated relationship, and must choose a contract that appears outwardly to be a standard legal services contract. Other distinctions arise as we discuss these literatures in the next two paragraphs.

The contract that we propose takes advantage of repetition within the relationship to improve incentives through the retention function. The literature on implicit contracts is similar (see MacLeod (2007) for a useful survey). In particular, Klein and Leffler (1981) show agents can be induced to supply high quality output when they receive a flow of rents and face the threat of losing those rents if they provide low quality output. Our model is also similar to the efficiency wage model of Shapiro and Stiglitz (1984) and the more general relational contracting models of MacLeod and Malcomson (1989) and Levin (2003). These papers, along with others such as Baker, Gibbons and Murphy (1994), allow for a variety of contract forms including bonus payments. This bonus depends upon non-contractable signals concerning the non-stochastic portion of the firm's performance. Our model contains no such bonus.

Our paper is also related to the very large literature on repeated contracting with moral hazard. Many of the papers in this literature consider long term contracts with commitment. Some papers of this sort which include the threat of termination include: Banks and Sundaram (1998), Heinkel and Stoughton (1994), Spear and Wang (2005), and Ohlendorf and Schmitz (2012). These papers take advantage of the commitment between principal and agent to allow transfers in the current period to depend upon previous periods. Such dependence allows the agent to smooth consumption, and also allows the principal to provide stronger incentives even when the agent is risk neutral. However, such explicitly multi-period contracts don't seem empirically valid in the market for legal services. By ruling out long term contracts, Olcay (2012) comes closer in spirit to our model.

However, Olcay (2012) still has many features which make it unsuitable for the current study. In particular, the agent makes a binary choice (high or low effort) in each period, and this choice determines the probability of a binary (good or bad) outcome. Finally, all the models discussed in this paragraph use contracts in which rewards depend only upon outcomes. Unlike our paper, they rule out the hourly wage contracts which seem ubiquitous for corporate clients.

The paper proceeds as follows. Section 2 presents the model. In Section 3, we present a fairly standard model of short term legal service contracts. The contract from this Section serves as a baseline for our long term contract. In Section 4, we describe our proposed contract, and present our main results. Section 5 contains a discussion of the results and concludes the paper. Proofs are relegated to the Appendix.

2 Model Set Up

We consider a single corporate client and an infinite number of identical law firms interacting in a perfectly competitive legal services market. In each period $t = 1, 2, ...$ the client is involved in a case requiring legal services. Whichever law firm is employed in period t is simply referred to as the law firm. The client and law firm are both risk neutral. We assume that the client is the plaintiff to ease comparisons with the existing literature on contingent fee contracts.[6]

The period t case has characteristics (i_t, λ_t, ρ_t). Standard features of the case, like the productivity of legal services, are determined by $i_t \in I$. The set I is finite, and $i_t = i$ with time invariant probability q_i. The parameters λ_t and ρ_t relate to the law firm's ability to defraud the client on the current case. In particular, λ_t determines the maximal amount by which the law firm can defraud the client, and ρ_t determines the degree to which defrauding the client biases h_t away from the efficient choice. We discuss λ_t and ρ_t more fully in the next Section when they are used.

We find two contracts. We find a benchmark *short term* contract. Discussion of the short term contract's specific features is deferred to later. The *long term* contract is designed to take advantage of the repeated interactions between client and law firm. Both contracts are designed to respect both observed institutional realities of legal service contracting and our assumption of a competitive market. To this end, we assume that the long term contract is decided at time $t = 0$

[6] Our contract is independent of the status of the client. See Section 5 for further discussion.

by a take it or leave it offer from client to all law firms. Any firm which accepts is placed within the long term contract queue. The first three features of the offered contract are the wage rate w, the contingent fee percentage α, and the lump sum payment L. These payment elements cannot be altered in period t and are independent of both the characteristics of the case and the law firm's performance in earlier cases. This modeling choice reflects the observation that wage rates and contingent fees are set by law firms in response to market pressures. Consequently, the client can't change these payments on a period by period basis as would be predicted by the literature on repeated contracting with moral hazard. We rule out $L < 0$ with a limited liability assumption.[7]

The final feature of the contract does depend upon the characteristics of the case at hand. This last feature is the Probability of Retention Function $P(\cdot)$ which determines the probability that the current firm will be retained into the following period. The function $P(\cdot)$ depends upon both the case characteristics, and the firm's performance in the current case. In principle, $P(\cdot)$ could depend upon the past performance of the law firm. However, our construction depends only upon the current case. The client is always indifferent between retaining or replacing the current law firm.

To summarize, the client's take it of leave it offer at time $t = 0$ is $(w, \alpha, L, P(\cdot))$. This contract guides all future long term interactions between the client and a firm. Each firm which accepts this offer, enters the long term contract queue. If the client wishes the period t case to be governed by the long term contract, then the first firm in the long term contract queue is given the opportunity to work on the period t case. In period $t = 0$, the client also forms a *short term contract* queue. This second queue includes all law firms.

Each period $t > 0$ proceeds as depicted in Figure 1. Subscript ts are suppressed in Figure 1. At the beginning of each period, the client learns case characteristics $i_t \in I$. If, given case characteristics i_t, the client prefers the long term contract, then he contacts the firm at the front of the long term contract queue. Upon being contacted, the firm becomes aware of information i_t and decides whether to accept or decline the case. If the case is declined, both the client and law firm receive a period t payoff of zero, and the long term contract queue remains unchanged.[8] If the

[7]Setting $L < 0$ is in any case illegal. See, e.g., Santore and Viard (2001).

[8]In practice, the client might turn to another law firm if the law firm rejected the long term contract for this particular case. We don't include any such ability because in equilibrium no case is ever rejected.

contract is accepted, the lump sum L is paid to the law firm, and the period t case begins. Once the case begins, the law firm becomes aware of its ability to defraud the client as captured by λ_t and ρ_t. The law firm then chooses: h_t, the hours worked on the case, and r_t, the hours reported to the client. The reported hours, r_t, are observable and contractable, while the actual hour worked, h_t, are neither. On the other hand, h_t stochastically determines the client's award.

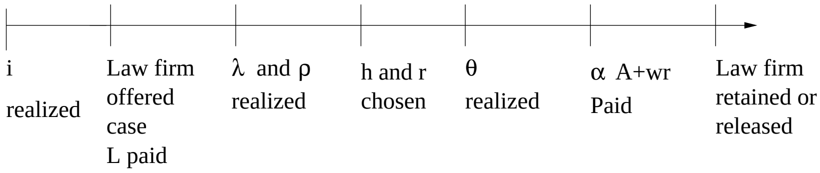

Figure 1: Timeline

Let θ_t denote the random elements of the case outside the law firm's control. The variable θ_t is i.i.d. and uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$. The damage award from the case is $A(i_t, h_t, \theta_t) = \hat{A}(i_t, h_t) + B(i_t, h_t) \cdot \xi(i_t, \theta_t)$.[9] We assume that $B(i_t, h_t) > 0$, $\xi_{\theta_t} \geq 0$ and that $E_{\theta_t}[\xi(i_t, \theta_t)] = 0$. Hence $\hat{A}(i_t, h_t) = E_{\theta_t}[A(i_t, h_t, \theta_t)]$. The convention that a hatted variable is the expected value of the variable prior to the case, but after i_t has been realized, is followed throughout. We assume the expected award increases in hours at a decreasing rate: $\hat{A}_h > 0$, and $\hat{A}_{hh} < 0$.

We denote by h_i^* the (unique) efficient number of hours given information i, as defined by $\hat{A}_h(i, h_i^*) = c$. For clarity in exposition, we define $r_i^* = h_i^*$. [10] We use starred variables to denote the value of that variable when $i_t = i$ and $r_t = h_t = h_i^*$. For example, $\hat{A}^*(i) = \hat{A}(i, h_i^*)$. Throughout a subscript θ or h denotes a partial derivative. However, a subscript i denotes that $i_t = i$, and a subscript t indicates we are referring to a variable in period t.

After h_t and r_t are set, θ_t is realized and $A(i_t, h_t, \theta_t)$ is determined. The client then makes a payment of $\alpha \cdot A(i_t, h_t, \theta_t) + w \cdot r_t$ to the law firm. Let c denote the law firm's marginal cost. Law firm profits from the current case are $\alpha A(i_t, h_t, \theta_t) + w \cdot r_t - c \cdot h_t + L$, while the client's surplus from the case is $(1 - \alpha)A(i_t, h_t, \theta_t) - w \cdot r_t - L$. In the final step of the period, the client retains the law firm with probability $P(\cdot)$. If the firm is retained, then it remains at the front of the long

[9] The C.D.F. of ξ_i, $F_i(\cdot)$, is defined by $F_i(\xi_i(\theta)) = 1/2 + \theta$. Hence, the realized reward are a function of the CDF of the random element ξ_i. This may be an unusual approach, but it allows a more uniform presentation.

[10] We look for a contract in which the total wage payment is $w \cdot r_t$. Hence, we want our contract to illicit truthful reporting. There are many other possible contracts. In particular, as long as the relationship between r_t and h_t is one to one, it is possible to set the wage payment to $w \cdot h_t$ based upon the reported r_t.

term contract queue. If the law firm is not retained, then it is removed from the long term contract queue, and the remaining firms all move up in the queue.

If the client chooses the short term contract, then things proceed in much the same manner. Again, the period begins with the client's observation of the case type i_t. The client then approaches the first firm in the short term contract queue and makes a take it or leave it offer $(\underline{w}_t, \underline{\alpha}_t, \underline{L}_t)$.[11] There is no Probability of Retention Function because the law firm in a short term contract is never retained. The law firm observes i_t, and then accepts or rejects the offer. If she rejects, then both law firm and client receive zero that period. If she accepts then things proceed as they did in the long term contract case, except that elements of the short term contract are used to determine payoffs. At the end of the case, the law firm is always removed from the queue, and the remaining law firms move up in the queue.

We are modeling a repeated game. Strategies may depend upon the entire history up to the current period. However, we severely restrict the set of strategies under consideration. To begin, in period $t = 0$ the client is committing to a particular behavior in all later periods. Furthermore, we design a probability of retention function which depends only upon the outcome in the current period. Given the contract offered by the client, the law firm has no reason to play a strategy that depends upon more than the characteristics of the current case. Hence, we do not model all of the features of repeated game strategies. We might think of the law firm as playing a Markov strategy for which the state specifies the characteristics of the case and who is at the front of the queue. That is, the law firm has a strategy which tells it what to do in a period t based purely upon (i_t, λ_t, ρ_t). For this reason, we suppress the time subscript t on h, r, and θ when discussing the long term contract.

3 One Shot Contract

In a one shot contract the client has only the elements $(\underline{w}_t, \underline{\alpha}_t, \underline{L}_t)$ with which to provide incentives. Absent some sort of penalty for misreporting hours, any $w_t > 0$ would lead the law firm to choose $h_t = 0$ and $r_t = \infty$. Accordingly, we assume that a function $\bar{r}_t(h_t)$ exists such that: if $r_t > \bar{r}_t(h_t)$,

[11] We allow the elements of the short term contract to depend on the type of case, because it makes the analysis of the short term contract simpler. This seems a costless simplification, because it leads to a stronger benchmark against which our long term contract must compete.

then the law firm will be investigated, discovered lying, and suffer a prohibitively large net loss. If $r_t \leq \bar{r}_t(h)$ the law firm is not investigated. The function $\bar{r}_t(h)$ is stochastic and determined by the random variables λ_t and ρ_t. In particular, we assume that if $i_t = i$, then $\bar{r}_t(h) = \psi(h-(h_i^*+\rho_t))+\lambda_t$. Here $\psi(\cdot)$ is a positive single peaked function which achieves its unique maximum at zero. In addition, $\lim_{x\to-\infty}\psi(x) = \lim_{x\to\infty}\psi(x) = 0$. A law firm maximizing the amount by which the client is defrauded would choose $h_t = h_i^* + \rho_t$ and $r_t = h_t + \psi(0) + \lambda_t$. Notice that $\lambda_t > 0$ lowers a defrauded client's surplus by transferring some of it to the law firm, while $\rho_t \neq 0$ lowers a defrauded client's surplus through a decrease in efficiency. We assume that λ_t is i.i.d. distributed on $[0, \Lambda]$ and ρ_t is i.i.d. distributed on $[-\Upsilon, \Upsilon]$. Both λ_t and ρ_t are distributed with full support. Let $\Gamma = \Lambda + \psi(0)$ denote the largest possible misrepresentation of hours by the law firm.

We don't find the optimal short term contract, but rather demonstrate that it must involve a loss in efficiency and positive profits for the law firm. Given some one shot contract, the firm acts to maximize his expected payoff

$$\max_{r,h} \underline{\alpha}_t \cdot \hat{A}(i_t, h) + \underline{w}_t \cdot r - c \cdot h + \underline{L}_t \tag{1}$$

$$\text{subject to } r \leq \bar{r}_t(h) \tag{2}$$

Either $\underline{w}_t = 0$ in which case r has no payoff implications, or $\underline{w}_t > 0$ in which case the law firm's objective is strictly increasing in r. Hence, we substitute $r = \bar{r}_t(h)$ into the objective function. This leads to the following first order condition.

$$\underline{\alpha}_t A_h + \underline{w}_t \bar{r}_h = c \tag{3}$$

We begin our analysis of Equation 3 with some simple observations. Because $\underline{L}_t$ has no impact on incentives, the client always sets $\underline{L}_t = 0$. In addition, the client always sets $\underline{w}_t \leq c$, because if $\underline{w}_t > c$ then the law firm sets $h_t = \infty$. We run through some simple cases to give a feel for the contracting environment. Throughout, we set $i_t = i$.

We consider first the case in which $\underline{\alpha}_t = 0$. In this case, if $\underline{w}_t = c$, then the law firm sets $h_t = h_i^* + \rho_t$ with $E(h_t) = h_i^*$. On the other hand, if $\underline{w}_t < c$ then the law firm sets $h_t < h_i^* + \rho_t$. If the difference $c - \underline{w}_t$ is sufficiently large, then h_t is driven to zero. Even in this simplest case with

$\underline{\alpha}_t = 0$, it is not clear that $E(h_t) = h_i^*$ is desirable. Depending on the curvature of $\hat{A}(i_t, h)$ it is quite possible that the expected value of the case is maximized with $E(h_t) < h_i^*$. In addition, the client's desire to minimize the law firm's profits will also tend to put a downward pressure on $\underline{w}_t$ which leads to a downward pressure on h_t.

If, on the other hand, $\underline{w}_t = 0$, then the first order condition becomes $\underline{\alpha}_t \hat{A}_h = c$. In this case, the law firm's choice of h_t is independent of ρ_t which makes it non-stochastic from the client's perspective. However, the client faces a tradeoff in setting $\underline{\alpha}_t$. The closer $\underline{\alpha}_t$ is to zero, the further the law firm's choice will deviate below h_i^*. On the other hand, the larger is $\underline{\alpha}_t$, the larger are the law firm's profits. Clearly then the optimal value for $\underline{\alpha}_t$ lies somewhere between $\frac{c}{\hat{A}_h(i,0)}$ and 1. In this range the law firm chooses $h_t < h_i^*$ and makes strictly positive expected profit.

The client can certainly do no worse if she has the ability to set both $\underline{\alpha}_t > 0$ and $\underline{w}_t > 0$. Clearly, increasing $\underline{\alpha}_t$ and decreasing $\underline{w}_t$ decreases both the dependence of the firm's choice on ρ_t and the cost associated with the law firm's over reporting of hours. For $\underline{\alpha}_t$ close to zero, this might both increase efficiency and decrease the law firm's profits. However, as $\underline{\alpha}_t$ approaches 1 the increase in efficiency must come with an increase in law firm profits. Consequently, $\underline{\alpha}_t < 1$ in the optimal short term contract. Let $\underline{h}(\rho_t)$ denote the dependence of the firm's optimal choice of h_t on ρ_t. The expected value of the case under the one shot contract is

$$\int_{-\Upsilon}^{\Upsilon} [\hat{A}(i_t, \underline{h}(\rho_t)) - c \cdot \underline{h}(\rho_t)] g(\rho_t) d\rho_t \tag{4}$$

If $\underline{w}_t = 0$, then $\underline{h}(\rho_t) < h_i^*$, while if $\underline{w}_t > 0$, $\underline{h}(\rho_t)$ is not constant. In either case this integral is less than $\hat{A}(i_t, h_i^*) - ch_i^*$.

Our adherence to the institutional constraints on legal service compensation rules out some possible first best one shot contracts. For example, simply removing the limited liability assumption allows a first best contract. In particular, the client could sell the case to the law firm by setting $\underline{\alpha}_t = 1$, and setting $\underline{L}_t$ to the negative of the value of the case. Such a contract is illegal.[12]

Let X_i denote the difference between $\hat{A}(i, h_i^*) - c \cdot h_i^*$ and the client's expected payoff in the best one shot contract for cases of type i. Let $\bar{X} = \min_i\{X_i\}$ denote the minimum (across types of cases) deviation from the first best payoff. Recall that Γ is the maximal amount by which a firm

[12]See Santore and Viard (2001) for a discussion of the relevant restrictions on legal service contracting.

might misrepresent hours worked. To compare Γ to $\bar{X}$, we must put Γ in dollar terms, which we do by multiplying by the marginal cost. Consider first the limit as $\Upsilon \to 0$. The optimal short term contract can be no worse than setting $\underline{\alpha}_t = 0$ and $\underline{w}_t = c$, in which case $\underline{h}(\rho_t) \to h_i^*$. Hence, as $\Upsilon \to 0$ we have $\bar{X} \leq c \cdot [\psi(0) + E(\lambda_t)] < c \cdot \Gamma$. On the other hand, if $\psi(0)$ is small and Υ is large, then in the limit as $\Lambda \to 0$ we have $\bar{X} > c \cdot \Gamma = c \cdot \psi(0)$. Clearly then $\bar{X}$ may be greater than or less than $c \cdot \Gamma$.

4 The Repeated Interactions Contract

In this Section, we construct a contract that provides the law firm with the incentives to maximize the joint value of each case. We focus on the law firm's incentives because our criteria for the contract is that it is efficient and looks like a wage contract, not that it is second best from the client's perspective. Let δ denote the discount rate. We make the following Assumption regarding the noise term $\xi(i, \theta)$.

Assumption 1 *For each i,*

(1) $\xi_{\theta,\theta}(i, \theta) \geq 0$ (≤ 0) if $\theta > 0$ $(\theta < 0,)$

(2) $\xi(i, -\theta) = -\xi(i, \theta)$ and

(3) $A_\theta(i, h_i^*, 0) = B(i, h_i^*) \cdot \xi_\theta(i, 0) < \delta \bar{X}$.

Assumptions 1.1 and 1.2 are equivalent to assuming that the distribution on $A(i, h, \theta)$ is uni-modal and symmetric.[13] Assumption 1.3 requires that values of θ such that $A_h(i, h_i^*, \theta) \approx c$ are sufficiently likely.[14] To emphasize the repeated nature of the interactions, we assume that δ is close to one. Hence, Assumption 1.3 has bite if and only if the one shot contracts are close to being first best and $\bar{X}$ is small.

The Probability of Retention Function, P, is a central feature of our contract. This function states the probability with which the client continues his relationship with the law firm past the current case. The probability of retention is (weakly) increasing in the client's payoff from the current case. Recall that the law firm cannot defraud the client by more than $w \cdot \Gamma$ without getting caught.

[13] As shown in the Appendix.

[14] In other words, this is an assumption that the implied density function for ξ is sufficiently large at $\xi = 0$.

Theorem 1 *Let Assumptions 1 hold. If* Γ *is sufficiently small, then one may construct a long term contract such that:*

(1) when $i_t = i$ *the law firm accepts the case and sets* $h = h_i^*$ *and* $r = r_i^*$*, and*

(2) the client receives an expected payoff from the current case larger than the expected payoff from any one shot contract.

In one such contract: $L > 0$, $\alpha = 0$, $w = c$, *and* P *is piecewise linear and weakly increasing in* $A(i, h, \theta) - w \cdot r$.

Conclusion (1) of Theorem 1 is that the law firm chooses the efficient number of hours, and reports this truthfully. It is worth noting that the only contingent element in the described contract is the retention probability function P. Hence, the efficient and truthful behavior predicted by Theorem 1 follows entirely from the law firm's efforts to maintain the relationship. The magnitude of Γ is important because the contract for which we solve makes $r = h = h_i^*$ a local maximum for the law firm. Defrauding the client by maximizing $w \cdot \bar{r}_t(h) - c \cdot h$ is a non-local alternative which creates an incentive compatibility constraint. [15] The larger is Γ, the harder it is to satisfy that constraint. Conclusion (2) stops short of asserting that our proposed contract is either first or second best from the client's perspective.

The contract satisfying statements (1) and (2) from Theorem 1 is not unique. To begin, even if the other elements of the contract were fixed, there would be a range of feasible values for L. However, since the market is competitive, the selected contract should have the lowest feasible value for L. In addition, there may be feasible contracts with $\alpha > 0$ and $w < c$. These issues are discussed further below.

There are broadly speaking two steps in the proof of Theorem 1. We first construct a contract with the desired properties for a fixed value of i. Because this construction illuminates how our contract functions, we provide an outline in the body below. Standard fixed point arguments demonstrate that one may do this simultaneously for each value of i.

[15] The issue here is that in defrauding the client, the law firm might push $P(\cdot)$ to zero. At this point, the client has nothing to loose by defrauding the client as much as possible.

We use a tilde to indicate that expectations have been taken over both θ and i. For example, $\tilde{P} = E_i[E_\theta[P]]$. The firm's Bellman equation is

$$\hat{V}(i) = \max_{h,r} \left\{ L + \alpha \hat{A} - c \cdot h + w \cdot r + \delta \hat{P} \tilde{V} \right\} \tag{5}$$

We assume that all actions at times $t+1$ and later are optimal, for, from the one-shot deviations principle, it is sufficient to show, as we do below, the law firm does not wish to deviate from the equilibrium strategy in the current period.

We construct the Probability of Retention Function, P, as follows

$$\tau(i,h,r,\theta) = \frac{A(i,h,\theta) - c \cdot r - b_i}{\Delta_i} \tag{6}$$

$$P(i,h,r,\theta) = \max\{0, \min\{1, \tau(i,h,r,\theta)\}\} \tag{7}$$

The parameters Δ_i and b_i are chosen by the client. Whatever values are chosen for Δ_i and b_i, τ is a linear and increasing function in the realized (and reported) joint return from the case, $A(i,h,\theta) - c \cdot r$. The transformation from τ to P assures that $0 \leq P \leq 1$.

The law firm's Bellman Equation 5 contains the expectation of P. To find this expectation, we define the following.

$$\underline{\theta}_i(h,r,b_i) = \min\{\theta | A(i,h,\theta) \geq c \cdot r + b_i\} \tag{8}$$

$$\overline{\theta}_i(h,r,b_i,\Delta_i) = \max\{\theta | A(i,h,\theta) \leq \Delta_i + c \cdot r + b_i\} \tag{9}$$

$\underline{\theta}_i$ is the minimum value of θ_i needed to ensure $\tau \geq 0$. $\overline{\theta}_i$ is the maximum value of θ_i such that $\tau \leq 1$. Figure 2 illustrates two possible relationships between τ, P, and θ. All i subscripts are suppressed.

The diagram to the left is for a case in which $\tau(h,r,i,-1/2) < 0$ and $\tau(h,r,i,1/2) > 1$, while the picture to the right is a case in which $\tau(h,r,i,-1/2) = 0$ and $\tau(h,r,i,1/2) \leq 1$. These are the equilibrium cases for τ and P. With this notation we have

$$\hat{P}(i,h,r) = \int_{-1/2}^{\underline{\theta}_i} 0 d\theta + \int_{\underline{\theta}_i}^{\overline{\theta}_i} \tau d\theta + \int_{\overline{\theta}_i}^{1/2} 1 d\theta \tag{10}$$

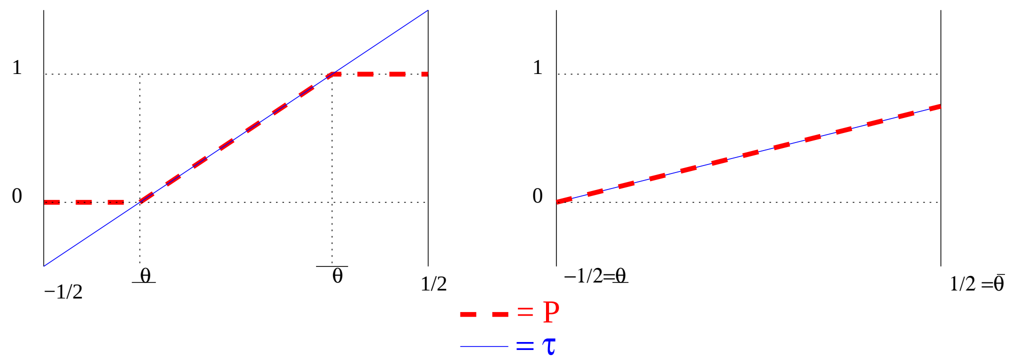

Figure 2: Graphs of Two Retention Probability Functions

$$= \int_{\underline{\theta}_i}^{\overline{\theta}_i} \tau d\theta + 1/2 - \overline{\theta}_i \tag{11}$$

With the relationship between $\hat{P}$, Δ_i and b_i spelled out, we are ready to state the following.

Proposition 1 *Let Assumptions 1 hold. Set* $\alpha = 0$*, and* $w = c$*. Fix a value* i *and then fix* b_j *and* Δ_j *for* $j \neq i$*. Further, assume that for* $j \neq i$*, the law firm must set* $h_j = r_j = h_j^*$*. If* Γ *is not too large, then there exist* L*,* Δ_i*, and* b_i *such that in any period* t *with* $i_t = i$*:*

(1) the client prefers the long term contract to any one shot contract.

(2) The long term contract induces the law firm to set hours efficiently and report hours truthfully.

Obviously the assumption that $h_j = r_j = h_j^*$ for $j \neq i$ renders Proposition 1 questionable as anything other than an intermediate result.[16] However, that is exactly what it is. The gap between Theorem 1 and Proposition 1 is bridged by removing this questionable assumption. Instead, we look for a contract which simultaneously leads the law firm to choose optimally and report truthfully for each possible value of i_t.

Proposition 1 is achieved by manipulating the first order conditions for the firm's Bellman Equation 5. Lemma 1 simplifies the first order conditions.

Lemma 1 *Let* $i_t = i$*. If* $A^*_{h,\theta} = 0$*, then a necessary condition for the firm to choose* $r = h = h_i^*$ *is* $w = (1 - \alpha) \cdot c$*.*

[16] Proposition 1 may also be taken as the final result if I is a singleton.

If the law firm cannot control volatility, then $A^*_{h,\theta} = 0$. Since we don't rule out this possibility, we need to respect the restriction that $w = (1-\alpha) \cdot c$.[17] Clearly if there is no contingent fee ($\alpha = 0$), then the wage equals the firm's opportunity cost, $w = c$. In this case, if the contract is written so that the law firm reports hours honestly, $r = h$, then the law firm's only incentive in setting hours is through the impact those hours have on the probability of being retained. On the other hand, if $\alpha > 0$ and $w = (1-\alpha)c$, then the law firm's profit (with $r = h$) on the current case is $L + \alpha(A - c \cdot h)$. Since $L \geq 0$, due to limited liability, the larger is the contingent percentage α, the harder it is to write a long term contract that the client prefers to a short term contract. Henceforth we set $\alpha = 0$ and $w = c$.

In the desired contract, the firm's first order conditions must hold when $h = r = h^*_i$. We define $\underline{\theta}^*_i(b_i) \equiv \underline{\theta}_i(h^*_i, r^*_i, b_i)$ and $\overline{\theta}^*_i(b_i, \Delta_i) \equiv \overline{\theta}_i(h^*_i, r^*_i, b_i, \Delta_i)$. The first order condition for $r = r^*_i$ is

$$1 = \left(\frac{\overline{\theta}^*_i - \underline{\theta}^*_i}{\Delta_i}\right)\left(\frac{\delta L}{1 - \delta\tilde{P}^*}\right). \tag{12}$$

The second term on the RHS of Equation 12 is $\delta\tilde{V}$. The first term on the RHS of Equation 12 is $\frac{1}{c} \cdot \frac{\partial \hat{P}_i}{\partial r}$. So multiply both sides of Equation 12 by the wage rate $w = c$. The resulting LHS is w, the marginal benefit of increasing r. The resulting RHS is the marginal cost of an increase in r, as reflected by the decreased probability of receiving the future value $\delta\tilde{V}$.

If Equation 12 holds, then the first order condition for $h = h^*_i$ is

$$\frac{1}{(\overline{\theta}^*_i - \underline{\theta}^*_i)} \int_{\underline{\theta}^*_i}^{\overline{\theta}^*_i} A^*_h d\theta = c. \tag{13}$$

From Equation 12, the fraction preceding the integral on the LHS of Equation 13 equals $\frac{\delta\tilde{V}}{\Delta}$. The integral itself is $\Delta \cdot \frac{\partial \hat{P}_i}{\partial h}$. Hence the LHS of Equation 13 is the marginal benefit of an increase in h, as reflected in an increased probability of receiving the future benefit $\delta\tilde{V}$. The RHS is the marginal cost of h.

Clearly then, the client's objective is to set Δ_i and b_i so that Equations 12 and 13 hold. By the definition of h^*_i, Equation 13 holds when $\underline{\theta}^*_i = -\frac{1}{2}$ and $\overline{\theta}^*_i = \frac{1}{2}$. However, because $\underline{\theta}^*_i$, $\overline{\theta}^*_i$ and

[17] The possibility that $A^*_{h,\theta} = 0$ is the easiest way to require $w = (1-\alpha) \cdot c$. However, there are other compelling reasons. See, for example, footnote 18.

$\tilde{P}^*$ are all determined by b_i and Δ_i, it is not immediately obvious that Equation 12 can be solved when $\underline{\theta}_i^* = -\frac{1}{2}$ and $\overline{\theta}_i^* = \frac{1}{2}$. In the Appendix, we show that we can solve 12 with $\underline{\theta}_i^* = -\frac{1}{2}$ and $\overline{\theta}_i^* = \frac{1}{2}$ so long as L is sufficiently large.[18] On the other hand, the client prefers the long term contract only if L is sufficiently small. It is here that Assumption 1.3 comes into play. Recall that $\bar{X}$ is the minimum loss in efficiency from a short term contract. In the Appendix, we show that, under Assumption 1.3, there is a value of $L < \delta\bar{X}$ for which we can solve Equations 12 and 13. Any $L < \delta\bar{X}$ will serve to satisfy the requirement that the client is better off than under a short term contract.

To use Proposition 1 to prove Theorem 1, one must find $\{(b_i, \Delta_i)\}_{i\in I}$ that satisfy Equations 12 and 13 simultaneously for each i. We do this using standard fixed point arguments. In particular, we first establish a relationship $b_i(\Delta_i)$ such that if $b_i = b_i(\Delta_i)$, then Equation 13 holds for $i_t = i$. This reduces the client's task to choosing Δ_i. We then define Δ_i^e as the solution (with $b_i = b_i(\Delta_i)$) to Equation 12 given the values of $\{\Delta_j\}_{j\neq i}$. Finally, we show that $\Delta^e = (\Delta_1^e, ...\Delta_n^e)$ satisfies the requirements for the Brouwer Fixed Point Theorem. This fixed point, is then our desired contract which simultaneously satisfies Equations 12 and 13 for each i.

A range of values for L satisfy the requirements for Theorem 1. However, we assume that the market for law firms is competitive. Hence, L should take the smallest value which both: satisfies the incentive compatibility constraint, and allows a simultaneous solution of Equations 12 and 13 for each value of i.

4.1 The Equilibrium Retention Probability

A cursory read of Appendix B reveals that the maximum possible equilibrium value for $\hat{P}$ is $1/2$. We now present a slightly more general retention function which allows the client to set the probability of retention as close to one as desired.

Let us keep the definitions of τ and P, but suppose that the probability of retention is equal to

$$\mathcal{Q}(i, h, r, \theta) = z \cdot P(i, h, r, \theta) + (1 - z)$$

[18] If $w \neq (1-\alpha)c$, then $\underline{\theta}_i^* \neq -\overline{\theta}_i^*$. Hence, one cannot set $\underline{\theta}_i^* = -\frac{1}{2}$ and $\overline{\theta}_i^* = \frac{1}{2}$. In this case, it is not clear that we can find a solution for 'large' L. This might leave us without a solution.

for some $z \in (0, 1]$. This leads to an expected probability of retention of

$$\hat{\mathcal{Q}}_i = z \cdot \hat{P}_i + (1 - z) = z \left(\int_{\underline{\theta}_i}^{\overline{\theta}_i} \tau d\theta + 1/2 - \overline{\theta}_i \right) + (1 - z).$$

Clearly, as $z \to 0$ the probability of retention goes to 1. Let us keep to the restriction that $w = c$ and $\alpha = 0$, and recall that for each i the simple retention function, P_i, is defined by b_i and Δ_i.

Proposition 2 *Let Assumptions 1 hold. Let $Q \in [1/2, 1)$ be a desired probability of retention. If Γ is sufficiently small, and δ is sufficiently close to one, then one may design a contract using $\mathcal{Q}$ as the retention function such that*

(1) The results from Theorem 1 hold, and

(2) The equilibrium expected probability of retention is at the desired level; $\tilde{\mathcal{Q}}^* = Q$.

To put it simply, when agents are patient enough (or interact frequently enough,) one can set the probability of retention as close to one as desired.

The key to the proof of Proposition 2 is the first order conditions Equations 12 and 13. Using $\mathcal{Q}$ with $z < 1$ rather than P as the retention function does have effects on incentives. On the one hand, decreasing z increases $\tilde{\mathcal{Q}}^*$. This increases $\delta\tilde{V}$, the value of having the relationship continue onto the next period. This strengthens the law firm's incentive to maintain the relationship. On the other hand, because $\partial\hat{\mathcal{Q}} = z \cdot \partial\hat{P}$, decreasing z decreases the impact that both h and r have on $\hat{\mathcal{Q}}$. This weakens the law firm's incentives. In the limit as $\delta \to 1$, these two effects exactly cancel out. In the Appendix we show that the firm's first order conditions become independent of z in the limit as $\delta \to 1$. In other words, whatever value of z is chosen, the first order conditions converge to Equations 12 and 13. Since all results are based upon these two first order conditions, Proposition 2 follows.

5 Discussion

We have constructed an efficient contract in a repeated relationship between a corporate client and a law firm consisting of three central elements: a wage rate equal to the law firm's opportunity cost, a lump sum payment, and a probability of retention function. Our results are in contrast to the generally accepted result that (in a one shot setting) contingent fee contracts create the best incentives for attorneys. We make no claim that our contract is 'second best.' Our claim is only to have found a justification for the ubiquitous presence of wage contracts between corporate clients and law firms.

Our ability to construct this contract follows from three things: the repeated nature of the relationship, the ability of a client to costlessly change law firms, and the presence of a lump sum payment. We take it as self evident that there are repeated relationships between corporate clients and law firms. We now discuss the the cost of changing law firms and the lump sum payment.

It is not far off to suggest that the client can change law firms costlessly. In creating preferred counsel lists, corporations establish contracts with many law firms. Thus, if the corporation wishes to end its relationship with one law firm it is able to redistribute the caseload to other firms on the list without the need for extensive negotiations or an expensive search for a new law firm. In addition, given the high value that law firms place on 'rainmakers,' it seems likely that any search cost associated with finding a new law firm will fall largely on the prospective law firms.

We suggest two possible interpretations of L. The simple interpretation is that L might represent the interest payment on what is known as a 'pure retainer.' A pure retainer is a payment to the law firm that is not related to any specific service but rather to ensure that the law firm will be available to the client and that it will not represent any parties adverse to the client (see Calloway and Robertson, 2002). The interpretation which appeared in the Introduction is that L appears in the wage payment. In particular, L might represent an accepted difference between r and h. If one were to replace r in τ (and hence P and $\mathcal{Q}$) with $r - L/w$, then the law firm would have the incentive to set $r = h + L/w$. The difference between r and h might arise because the law firm charges every time for administrative costs that it only bears in the first interaction. On the other hand, it may be that the law firm and client simply agree upon the difference between r and h.

In our model, the client reviews the performance of the firm following each case. One might believe that it is more likely that the firm's performance is reviewed over a longer period of time. However, a recent survey of corporate general counsel found that corporate clients are willing to change law firms the moment they feel that the firm is not putting forth enough effort (Jones, 2003). The general counsel for a Kansas City based corporation stated, "As soon as I feel that one of our national firms is taking us from granted, I'll give one of our Kansas City firms a little more work [that otherwise would have gone to the larger firm], and then issue a press release letting everyone know what I've done." Thus, it appears that firm performance is continuously monitored.

Our results are robust to the status of the client. That is, they hold whether the client is the plaintiff or defendant in the legal case. It does not matter if the expected reward from the case is positive or negative. Our results also apply to settlements and other negotiations and financial dealings that do not involve a trial but necessitate the use of lawyers. Our model is, therefore, robust to the client's position in the case and more general than contracts previously proposed.

Since we assume that the client is fully informed concerning the value of the case, frivolous lawsuits are outside the scope of our model. However, it seems likely that if our model were properly enlarged, then our suggested contract would discourage frivolous lawsuits. The law firm will not agree to represent the client in any case that has no merit because it knows that the outcome will be less than what the client expects. Representing the client in such a case would reduce the likelihood that the client would hire the law firm in future cases.

A Appendix

Before we proceed with the Proofs of Proposition 1 and Theorem 1, we verify the claims regarding Assumption 1. Let $F_i(\xi)$ and $f_i(\xi)$ denote the CDF and density of the noise term $\xi(i, \theta)$. Clearly $F_i(\xi(i, \theta)) = 1/2 + \theta$. Taking a derivative with respect to θ yields $f_i(\xi) \cdot \xi_\theta = 1$ or

$$f_i(\xi) = \frac{1}{\xi_\theta}$$

Now clearly ξ is symmetrically distributed around zero if and only if $f_i(\xi) = f_i(-\xi)$, which (from the above) holds if and only if $\xi_\theta(i, -\theta) = \xi_\theta(i, \theta)$. Hence, ξ is symmetrically distributed if and only if $\xi(i, -\theta) = -\xi(i, \theta)$, which is Assumption 1.2.

Taking a derivative of $f_i(\xi)$ yields

$$\frac{df_i}{d\xi} = \frac{-\xi_{\theta\theta}}{[\xi_\theta]^3}$$

So Assumption 1.1 is the same as assuming that the density on ξ is weakly increasing for $\xi < 0$ and weakly decreasing for $\xi > 0$. If there are no point masses, then this is a necessary and sufficient condition for the distribution to be uni-modal.

The above Equations are not defined if $\xi_\theta = 0$. If $\xi_\theta = 0$ over some range $\theta \in [a, b]$, then there is a point mass at $\xi(i, a)$, and F_i takes a discrete jump at this point. However, notices that Assumption 1.1 assures that $\xi_\theta \geq 0$ achieves its minimum at zero. Hence the only possible point mass is at $\xi(i, 0) = 0$. Again, we have a uni-modal distribution.

B Proof of Proposition 1

Let $\mathcal{Q} = zP + (1 - z)$ for $z \in (0, 1]$. Clearly $\mathcal{Q}$ is a more general retention probability function. Rather than proving Proposition 1, we prove a more general result. We use the following Assumption to make the statement of the more general result easier.

Assumption 2 Γ *is sufficiently small and one of the following holds:*

(1) $z = 1$, *or*

(2) δ *is sufficiently large.*

Proposition 1 uses Assumption 2.1.

Proposition 3 *Let $\alpha = 0$, $w = c$ and $R_i = h_i^*$. Fix $z \in (0, 1]$. Fix a value i and then fix b_j and Δ_j for $j \neq i$. Further, assume that for $j \neq i$, the law firm must set $h_j = r_j = h_j^*$. If Assumptions 1 and 2 hold, then there exist L, Δ_i, and b_i such that in any period t with $i_t = i$:*

(1) the client prefers the long term contract to any one shot contract.

(2) The long term contract induces the law firm to set hours efficiently and report hours truthfully.

The firm acts to maximize it's discounted expected stream of profits. Hence, he acts to satisfy first order conditions of his Bellman Equation

$$\hat{V}(i) = \max_{h,r} \left\{ L + \alpha \hat{A} - c \cdot h + w \cdot r + \delta \hat{\mathcal{Q}} \tilde{V} \right\} \tag{14}$$

The firm's first order condition with respect to h is

$$\frac{d\hat{\mathcal{Q}}}{dh} = \frac{c - \alpha \hat{A}_h}{\delta \tilde{V}} \tag{15}$$

while the first order condition with respect to r is

$$\frac{d\hat{\mathcal{Q}}}{dr} = \frac{-w}{\delta \tilde{V}} \tag{16}$$

Our next step is to replace the derivatives of $\hat{\mathcal{Q}}$ in Equations 15 and 16 with more explicit expressions. We recall that the expectation of P is

$$\hat{P}(i, h, r) = \textstyle\int_{\underline{\theta}_i}^{\overline{\theta}_i} \tau d\theta + 1/2 - \overline{\theta}_i \qquad \text{[Equation 11]}$$

Taking a derivative of Equation 11 with respect to h yields

$$\begin{aligned} \frac{d\hat{P}}{dh} &= \tau(\overline{\theta}_i)\frac{d\overline{\theta}_i}{dh} - \tau(\underline{\theta}_i)\frac{d\underline{\theta}_i}{dh} + \int_{\underline{\theta}_i}^{\overline{\theta}_i} \frac{d\tau}{dh} d\theta - \frac{d\overline{\theta}_i}{dh} = -\tau(\underline{\theta}_i)\frac{d\underline{\theta}_i}{dh} + [\tau(\overline{\theta}_i) - 1]\frac{d\overline{\theta}_i}{dh} + \int_{\underline{\theta}_i}^{\overline{\theta}_i} \frac{d\tau}{dh} d\theta \\ &= \int_{\underline{\theta}_i}^{\overline{\theta}_i} \frac{d\tau}{dh} d\theta. \end{aligned}$$

The final equality follows from the relationship between τ and $\underline{\theta}_i$ and $\overline{\theta}_i$. In particular, if $\tau(\overline{\theta}_i) < 1$, then $\overline{\theta}_i = 1/2$ and $\frac{d\overline{\theta}_i}{dh} = 0$. Likewise, if $\tau(\underline{\theta}_i) > 0$, then $\underline{\theta}_i = -1/2$ and $\frac{d\underline{\theta}_i}{dh} = 0$. Since $\frac{d\hat{\mathcal{Q}}}{dh} = \frac{d\hat{\mathcal{Q}}}{d\hat{P}} \cdot \frac{d\hat{P}}{dh}$

and $\tau(i,h,r,\theta) = \frac{A-c\cdot r-b_i}{\Delta_i}$ we have

$$\frac{d\hat{\mathcal{Q}}}{dh} = z\int_{\underline{\theta}_i}^{\overline{\theta}_i} \frac{A_h}{\Delta_i} d\theta \tag{17}$$

Combining Equations 15 and 17 yields Equation 18.

$$\frac{c-\alpha\hat{A}_h}{\delta\tilde{V}} = z\int_{\underline{\theta}_i}^{\overline{\theta}_i} \frac{A_h}{\Delta_i} d\theta \tag{18}$$

By arguments analogous to those used in the derivation of Equation 17, the derivative of $\hat{\mathcal{Q}}$ with respect to r is

$$\frac{d\hat{\mathcal{Q}}}{dr} = z\int_{\underline{\theta}_i}^{\overline{\theta}_i} \frac{-c}{\Delta_i} d\theta = \frac{-c(\overline{\theta}_i - \underline{\theta}_i)z}{\Delta_i} \tag{19}$$

Combining Equations 16 and 19 yields Equation 20 below.

$$\frac{\Delta_i}{z} = \left(\frac{c}{w}\right)(\overline{\theta}_i - \underline{\theta}_i)\delta\tilde{V} \tag{20}$$

Equations 18 and 20 both include $\tilde{V}$ which is endogenous. This motivates us to divide Equation 18 by Equation 20 to remove common terms including $\tilde{V}$. Doing this yields Equation 21.

$$\int_{\underline{\theta}_i}^{\overline{\theta}_i} A_h d\theta = \left(\frac{c}{w}\right)(c-\alpha\hat{A}_h)\cdot\left(\overline{\theta}_i - \underline{\theta}_i\right) \tag{21}$$

Equation 21 is a necessary condition for both first order conditions to hold.

Proof of Lemma 1:

By definition $\hat{A}_h^* = c$. If $A_{h,\theta}^* = 0$, then A_h^* is constant in θ. That is to say $A_h^* = \hat{A}_h^* = c$ within the integral in Equation 21. This reduces Equation 21 to $w = (1-\alpha)c$.♣

As in the body, we set $\alpha = 0$ and $w = c$ henceforth. Doing this, and requiring that Equation 21 holds when $h = r = h_i^*$ yields the first order condition for h presented in the body

$$\tfrac{1}{(\overline{\theta}_i^* - \underline{\theta}_i^*)}\textstyle\int_{\underline{\theta}_i^*}^{\overline{\theta}_i^*} A_h^* d\theta = c. \qquad \text{[Equation 13]}$$

If $\alpha = 0$, $w = c$, and Equation 21 holds, then Equations 18 and 20 both reduce to

$$\frac{\Delta_i}{z} = (\overline{\theta}_i - \underline{\theta}_i)\delta\tilde{V} \tag{22}$$

We convert Equation 22 into Equation 24 by replacing $\tilde{V}$ with an endogenously determined value. If $\alpha = 0$, $w = c$, and the first order conditions hold so that $h = r = h_i^*$, then the firm's Bellman Equation 5 becomes

$$\hat{V}(i) = L + \delta \hat{\mathcal{Q}}_i^* \tilde{V}$$

Taking expectations over i, and recalling that, e.g., $\tilde{\mathcal{Q}}^* = E_i(\hat{\mathcal{Q}}_i^*)$, we have

$$\tilde{V} = \frac{L}{1 - \delta \tilde{\mathcal{Q}}^*} \tag{23}$$

Plugging Equation 23 into Equation 22 and setting $r = h = h_i^*$ yields

$$1 = \left(\frac{\overline{\theta}_i^* - \underline{\theta}_i^*}{\Delta_i} \right) \left(\frac{z\delta L}{1 - \delta \tilde{\mathcal{Q}}^*} \right) \tag{24}$$

Setting $z = 1$ makes $\tilde{\mathcal{Q}}^* = \tilde{P}^*$, and turns Equation 24 into Equation 12.

We now argue that it is possible to solve Equations 24 and 13 simultaneously. The first step to this is to show that we have some freedom in choosing pairs (b_i, Δ_i) to solve Equation 13. There are two simple cases which we dispense with first. If $A_{h,\theta}^* = B_h^* \cdot \xi_\theta = 0$, then Equation 13 holds automatically. Likewise, if b_i and Δ_i are such that $-1/2 = \underline{\theta}_i^*$ and $\overline{\theta}_i^* = 1/2$, then Equation 13 holds by the definition of h_i^*.

Equations 8 and 9 imply that if $-1/2 = \underline{\theta}_i^*$ and $\overline{\theta}_i^* = 1/2$, then $\Delta_i \geq A(i, h_i^*, \overline{\theta}_i^*) - A(i, h_i^*, \underline{\theta}_i^*)$, while if $-1/2 < \underline{\theta}_i^* < \overline{\theta}_i^* < 1/2$, then

$$\Delta_i = A(i, h_i^*, \overline{\theta}_i^*) - A(i, h_i^*, \underline{\theta}_i^*). \tag{25}$$

Lemma B.1 *Set $\alpha = 0$ and $w = c$. Let Assumption 1 hold, fix i and assume that $B_h(i, h_i^*) \neq 0$. If $0 < \Delta_i \leq A(i, h_i^*, 1/2) - A(i, h_i^*, -1/2)$, then $\exists! b_i(\Delta_i)$ with $b_i + c \cdot h_i^* \in [A(i, h_i^*, -1/2), A(i, h_i^*, 0))$ such that Equations 13 and 25 hold for $\underline{\theta}_i^*(b_i)$ and $\overline{\theta}_i^*(b_i, \Delta_i)$.*
Furthermore, $b_i(\Delta_i)$ is strictly decreasing in Δ_i, and $\underline{\theta}_i^*(b_i) = -\overline{\theta}_i^*(b_i, \Delta_i)$.

Before we prove Lemma B.1, there are some preliminaries to attend. We consider the comparative statics from changing b_i and Δ_i. Let $\underline{A}^* = A(i, h_i^*, \underline{\theta}_i^*)$ and $\overline{A}^* = A(i, h_i^*, \overline{\theta}_i^*)$. Since it is the case for Lemma B.1, assume that $B_h^* \neq 0$ and $-1/2 < \underline{\theta}_i^* < \overline{\theta}_i^* < 1/2$. In this case $\underline{\theta}_i^*$ is defined by

$\underline{A}^* - ch_i^* - b_i = 0$. It follows that

$$\frac{d\underline{\theta}_i^*}{db_i} = \frac{1}{\underline{A}_\theta^*} > 0$$

Given $\underline{\theta}_i^*$, we determine $\overline{\theta}_i^*$ by $\int_{\underline{\theta}_i^*}^{\overline{\theta}_i^*} A_h d\theta = c(\overline{\theta}_i^* - \underline{\theta}_i^*)$. It follows that

$$\frac{d\overline{\theta}_i^*}{d\underline{\theta}_i^*} = \frac{\underline{A}_h^* - c}{\overline{A}_h^* - c} < 0$$

Using the functional form $A = \hat{A} + B \cdot \xi$ and Assumption 1, we have

$$\frac{d\overline{\theta}_i^*}{d\underline{\theta}_i^*} = \frac{\hat{A}_h^* - B_h^* \cdot \xi(\underline{\theta}_i^*) - c}{\hat{A}_h^* - B_h^* \cdot \xi(\overline{\theta}_i^*) - c} = \frac{\xi(\underline{\theta}_i^*)}{\xi(\overline{\theta}_i^*)} = -1 \tag{26}$$

The final equality derives from the following. We know that when $\underline{\theta}_i^* = -\frac{1}{2}$, then $\overline{\theta}_i^* = \frac{1}{2} = -\underline{\theta}_i^*$. By Assumption 1.2, the final equality must then hold at $\underline{\theta}_i^* = -\frac{1}{2}$. Furthermore, as long as $\underline{\theta}_i^* = -\overline{\theta}_i^*$, the equality will hold, and as long as the equality holds along the path from $-\frac{1}{2}$ to $\underline{\theta}_i^*$, then $\underline{\theta}_i^* = -\overline{\theta}_i^*$. Hence, Equation 26 holds, as does the following Lemma.

Lemma B.2 *If $\alpha = 0$, $w = c$ and Assumption 1 holds, then $\overline{\theta}_i^* = -\underline{\theta}_i^*$.*

Given the negative symmetry of ξ it follows that $\xi_\theta(-\theta) = \xi_\theta(\theta)$ so that $\underline{A}_\theta^* = \overline{A}_\theta^*$. Using the above results, we have that

$$\frac{d\overline{\theta}_i^*}{db_i} = \frac{d\overline{\theta}_i^*}{d\underline{\theta}_i^*} \cdot \frac{d\underline{\theta}_i^*}{db_i} = \frac{-1}{\underline{A}_\theta^*} < 0$$

Finally, $\Delta_i = \overline{A}^* - \underline{A}^*$, which leads us to

$$\frac{d\Delta_i}{d\underline{\theta}_i^*} = \overline{A}_\theta^* \cdot \frac{d\overline{\theta}_i^*}{d\underline{\theta}_i^*} - \underline{A}_\theta^* = -2\underline{A}_\theta^*$$

and

$$\frac{d\Delta_i}{db_i} = \frac{d\Delta_i}{d\underline{\theta}_i^*} \cdot \frac{d\underline{\theta}_i^*}{db_i} = -2$$

Proof of Lemma B.1: Given the above, we can define $b_i(\Delta_i) = \hat{A}_i^* - ch_i^* - \frac{1}{2}\Delta_i$ for $0 < \Delta_i \leq A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$. It remains only to verify uniqueness. To this end we first consider if, given $\underline{\theta}_i^*$ it is possible to choose $\overline{\theta}_i^* \neq -\underline{\theta}_i^*$. Equation 13 can be written as

$$\int_{\underline{\theta}_i^*}^{\overline{\theta}_i^*} A_h^* d\theta - c(\overline{\theta}_i^* - \underline{\theta}_i^*) = 0.$$

Now if $B_h^* > 0$ (resp. < 0) then we know that $A_h^* < c$ (resp. $> c$) if and only if $\theta < 0$. Hence, $\underline{\theta}_i^* < 0 < \overline{\theta}_i^*$. The derivative of the LHS of the above equation w.r.t. $\overline{\theta}_i^*$ is $\overline{A}_h^* - c = \hat{A}_h^* + B_h^* \cdot \xi(\overline{\theta}_i^*) - c = B_h^* \cdot \xi(\overline{\theta}_i^*)$ which does not change sign for $\overline{\theta}_i^* > 0$. Hence as one moves away from $\overline{\theta}_i^* = -\underline{\theta}_i^*$ the difference between the LHS and 0 only gets larger. This verifies that $\overline{\theta}_i^* = -\underline{\theta}_i^*$ is the only possibility. Now $\Delta_i = A(i, h_i^*, \overline{\theta}_i^*) - A(i, h_i^*, \underline{\theta}_i^*) = A(i, h_i^*, -\underline{\theta}_i^*) - A(i, h_i^*, \underline{\theta}_i^*)$. The RHS is monotonically decreasing in $\underline{\theta}_i^*$. Hence there is a unique value of $\underline{\theta}_i^*$ for a given Δ_i. It is trivial from the definition of $\underline{\theta}_i^*$ that there is a unique b_i for each value of $\underline{\theta}_i^*$ which establishes uniqueness. ♣

Lemma B.1 establishes the relationship $b_i(\Delta_i)$ for $0 < \Delta_i \leq A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$ and $B_h^* \neq 0$. It is convenient to have $b_i(\Delta_i)$ defined in all cases. To that end, if $\Delta_i > A(i, h_i^*, 1/2) - A(i, h_i^*, -1/2)$, then we set $b_i(\Delta_i) = A(i, h_i^*, -1/2) - c \cdot h_i^*$. Finally, if $0 < \Delta_i \leq A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$ and $B_h^* = 0$, then we set $b_i(\Delta_i)$ so that $\underline{\theta}_i^*(b_i) = -\overline{\theta}_i^*(b_i, \Delta_i)$.

So long as $b_i = b_i(\Delta_i)$, then we know that (b_i, Δ_i) solves Equation 13. Now in trying to choose Δ_i to satisfy Equation 24, we see that both $(\overline{\theta}_i^* - \underline{\theta}_i^*)$ and $\tilde{\mathcal{Q}}^* = z \cdot \sum_j q_j \cdot P_j^* + (1 - z)$ depends upon Δ_i. However, this dependence is well behaved.

Lemma B.3 *If $\alpha = 0$, $w = c$, and Assumption 1 holds, then $\frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*}$ is weakly increasing in Δ_i and unbounded above.*

Proof: Let $y = \frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*}$. If $\Delta_i > A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$, then $y = \Delta_i$. This establishes both that the derivative is positive in this range and that the function is unbounded above. Now consider $\Delta_i < A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$. Let $x = (\overline{\theta}_i^* - \underline{\theta}_i^*)^2 \cdot \frac{dy}{d\Delta_i}$. Clearly x and $\frac{dy}{d\Delta_i}$ have the same sign. $x = (\overline{\theta}_i^* - \underline{\theta}_i^*) - \Delta_i \left(\frac{d\overline{\theta}_i^*}{d\Delta_i} - \frac{d\underline{\theta}_i^*}{d\Delta_i} \right) = (\overline{\theta}_i^* - \underline{\theta}_i^*) - \Delta_i \left(\frac{d\overline{\theta}_i^*}{db_i} - \frac{d\underline{\theta}_i^*}{db_i} \right) \cdot \frac{db_i}{d\Delta_i} = (\overline{\theta}_i^* - \underline{\theta}_i^*) - \frac{\Delta_i}{\underline{A}_\theta^*}$. We observe that $\frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} \leq \underline{A}_\theta^*$ because the first term is the average slope which by Assumption 1.1 is weakly less than the slope at the edge, $\underline{A}_\theta^*$. Hence $x > 0$. Finally, there is a kink at $\Delta_i = A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$. However, at a kink a function is inarguably increasing if both its left and right hand derivatives are positive.♣

Lemma B.4 *Let $\alpha = 0$, $w = c$, and $b_i = b_i(\Delta_i)$ as defined above. If Assumption 1 holds, then $\hat{P}_i^* = \frac{1}{2}$ for $\Delta_i \leq A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$ and is decreasing in Δ_i for $\Delta_i > A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$.*

Proof: We first consider the case in which $\Delta_i \leq A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$. In this case $b_i = A^*(i, \underline{\theta}_i^*) - c \cdot h_i^*$ so that

$$A^* - c \cdot h_i^* - b_i = A^* - \underline{A}^* = [\hat{A}^* + B^* \cdot \xi(\theta)] - [\hat{A}^* + B^* \cdot \xi(\underline{\theta}_i^*)] = B^*[\xi(\theta) - \xi(\underline{\theta}_i^*)]$$

Also

$$\Delta_i = \overline{A}^* - \underline{A}^* = [\hat{A}^* + B^* \cdot \xi(i, \overline{\theta}_i^*)] - [\hat{A}^* + B^* \cdot \xi(i, \underline{\theta}_i^*)] = B^*[\xi(i, \overline{\theta}_i^*) - \xi(i, \underline{\theta}_i^*)] = 2B^* \cdot \xi(\overline{\theta}_i^*)$$

Hence we have that

$$\begin{aligned}
\hat{P}_i^* &= \int_{\underline{\theta}_i^*}^{\overline{\theta}_i^*} \frac{B^*[\xi(\theta) - \xi(\underline{\theta}_i^*)]}{2B^* \cdot \xi(\overline{\theta}_i^*)} d\theta + (\frac{1}{2} - \overline{\theta}_i^*) = \int_{\underline{\theta}_i^*}^{\overline{\theta}_i^*} \frac{\xi(\theta)}{2 \cdot \xi(\overline{\theta}_i^*)} d\theta + \int_{\underline{\theta}_i^*}^{\overline{\theta}_i^*} \frac{\xi(\overline{\theta}_i^*)}{2 \cdot \xi(\overline{\theta}_i^*)} d\theta + (\frac{1}{2} - \overline{\theta}_i^*) \\
&= 0 + \frac{1}{2}(\overline{\theta}_i^* - \underline{\theta}_i^*) + \frac{1}{2} - \overline{\theta}_i^* = \frac{1}{2}
\end{aligned}$$

On the other hand, if $\Delta_i > A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$ then $b_i = A(i, h_i^*, -\frac{1}{2}) - c \cdot h_i^*$, $\underline{\theta}_i^* = -\frac{1}{2}$ and $\overline{\theta}_i^* = \frac{1}{2}$. That is, none of these parameters depend upon Δ_i. Hence

$$\frac{d\hat{P}_i^*}{d\Delta_i} = -\left(\frac{1}{\Delta_i}\right)^2 \int_{-\frac{1}{2}}^{\frac{1}{2}} (A^* - c \cdot h_i^* - b_i) d\theta < 0$$

♣

Lemma B.5 *Let Assumption 1 hold, and set $\alpha = 0$, $w = c$ and $b_i = b_i(\Delta_i)$ as defined above. In this case $\frac{\Delta_i(1-\delta\tilde{Q}^*)}{\overline{\theta}_i^* - \underline{\theta}_i^*}$ is weakly increasing in Δ_i and unbounded above.*

Proof: We note that if $j \neq i$, then P_j^* does not depend upon Δ_i, hence by Lemma B.4 $\tilde{Q}^* = z\sum_j P_j^* + (1 - z)$ is weakly decreasing in Δ_i. The result then follows from an application of Lemma B.3. ♣

With Lemma B.5 in hand, there are no difficulties if δL is large enough. In particular, if δL is large enough that the LHS of Equation 24 is larger than the RHS when $\Delta_i = A(i, h_i^*, 1/2) -$

$A(i, h_i^*, -1/2)$, then we can simply keep increasing Δ_i until Equation 24 holds. We now address the case in which $\Delta_i < A(i, h_i^*, 1/2) - A(i, h_i^*, -1/2)$.

Lemma B.6 *Let $b_i = b_i(\Delta_i)$ as defined above.* $\lim_{\Delta_i \to 0} \frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} = A_\theta(i, h_i^*, 0)$.

Proof: $\lim_{\Delta_i \to 0} \frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} = \lim_{\Delta_i \to 0} \frac{A(i, h_i^*, \overline{\theta}_i^*) - A(i, h_i^*, \underline{\theta}_i^*)}{\overline{\theta}_i^* - \underline{\theta}_i^*} = A_\theta(i, h_i^*, 0)$. The first equality follows from Equation 25 which holds for Δ_i sufficiently small. The second equality is just the definition of a derivative. ♣

We rewrite Equation 24 as

$$z\delta L = \left(\frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} \right) \left(1 - \delta \tilde{\mathcal{Q}}^* \right) \tag{27}$$

We use $1 - \delta \tilde{\mathcal{Q}}^* = 1 - \delta(1 - z) - z\delta \tilde{P}^*$ and $\tilde{P}^* = q_i \cdot \hat{P}_i^* + \sum_{j \neq i} q_j \cdot \hat{P}_j^*$ to transform Equation 27 into

$$z\delta L = \left(\frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} \right) \left(1 - \delta(1 - z) - z\delta \left(\sum_{j \neq i} q_j \cdot \hat{P}_j^* \right) - z\delta \cdot q_i \cdot \hat{P}_i^* \right) \tag{28}$$

Let $K_i = 1 - \delta(1 - z) - \delta z \left(\sum_{j \neq i} q_j \hat{P}_j^* \right) - \frac{q_i z \delta}{2}$. We note that $K_i \in (0, 1)$ and is constant in Δ_i and b_i. We use Equation 11 to transform Equation 28 to

$$z\delta L = \left(\frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} \right) \left(K_i + z\delta \cdot q_i \cdot \overline{\theta}_i^* - \delta z \cdot q_i \int_{\underline{\theta}_i^*}^{\overline{\theta}_i^*} \left(\frac{A^* - c \cdot h^* - b}{\Delta_i} \right) d\theta \right) \tag{29}$$

We break Equation 29 into two pieces, divide by z, and factor $\frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*}$ out of the second piece to arrive at

$$\delta L = \left(\frac{\Delta_i}{\overline{\theta}^* - \underline{\theta}^*} \right) \left(\frac{K_i + \delta z q_i \overline{\theta}_i^*}{z} \right) - \delta q_i \int_{\underline{\theta}^*}^{\overline{\theta}^*} \frac{A * - c h^* - b_i}{\overline{\theta}^* - \underline{\theta}^*} d\theta \tag{30}$$

Recall that $\bar{X}$ is the minimum (over i) efficiency loss from a one shot contract. We notice that $\lim_{\Delta_i \to 0} b_i(\Delta_i) = \beta_i \equiv A(i, h_i^*, 0) - c \cdot h_i^*$. We have that

$$\lim_{\Delta_i \to 0} \left(\frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} \right) \left(\frac{K_i + \delta z q_i \overline{\theta}_i^*}{z} \right) = A_\theta(i, h_i^*, 0) \left(\frac{K_i + \delta z q_i / 2}{z} \right) < \delta \bar{X} \left(\frac{K_i + \delta z q_i / 2}{z} \right) \tag{31}$$

$$\lim_{\Delta_i \to 0} \int_{\underline{\theta}_i^*}^{\overline{\theta}_i^*} \left(\frac{A^* - c \cdot h_i^* - b_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} \right) d\theta = A(i, h_i^*, 0) - c \cdot h_i^* - \beta_i = 0 \tag{32}$$

We must now consider which part of Assumption 2 holds. If $z = 1$, then we have

$$\frac{K_i + \delta z q_i/2}{z} = 1 - \delta \left(\sum_{j \neq i} q_j \cdot \hat{P}_j^* \right) < 1.$$

Consequently, the limit as $\Delta_i \to 0$ of the RHS of Equation 30 is less than $\delta \bar{X}(K_i + \delta q_i \overline{\theta}_i^*) < \delta \bar{X}$. Hence, $\exists \epsilon > 0$ such that if $L = \bar{X} - \epsilon$, then the contract is feasible and the client strictly prefers the current expected payoff from this contract to any one shot contract.

We next turn to the case in which δ is sufficiently close to one. We can see that

$$\lim_{\delta \to 1} \frac{K_i + \delta z q_i/2}{z} = lim_{\delta \to 1} \frac{1 - \delta(1-z) - \delta z \left(\sum_{j \neq i} q_j \hat{P}_j^* \right)}{z} = \frac{z - z \left(\sum_{j \neq i} q_j \hat{P}_j^* \right)}{z} = 1 - \sum_{j \neq i} q_j \hat{P}_j^* < 1$$

Hence for δ sufficiently close to 1, it is again the case that the limit as $\Delta_i \to 0$ of the RHS of Equation 30 is less than $\delta \bar{X}$.

It remains only to establish that the firm cannot make itself better off by attempting to defraud the client. The choice of $r = h = h_i^*$ is locally optimal. However, choosing h and r so as to defraud the client is a non-local alternative. Being a non-local alternative, it must lead to a discrete drop in the probability of being retained. Since being retained has a strictly positive value, this leads to discrete drop in the expected present value of future payoffs. For Γ sufficiently small, this lowers the payoff to the law firm. This concludes the proof of Proposition 1.

C Proof of Theorem 1 and Proposition 2

We note that the difference between these results is that in Theorem 1 Assumption 2.1 holds, while in Proposition 2 Assumption 2.2 hold.

We proceed with Δ_i determining b_i, $\underline{\theta}_i^*$, and $\overline{\theta}_i^*$. This assures that Equation 13 holds for each i. It remains to show that we can also simultaneously satisfy Equation 24 for each i. Let $\rho_i(\Delta_1, ...\Delta_n) = \left(\frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} \right) \left(1 - \delta \tilde{\underline{Q}}^* \right)$. Equation 24 can be written as $z \delta L = \rho_i$. Let Δ_i^e denote the solution to Equation 24 given $\Delta = (\Delta_1 ... \Delta_n)$.

Lemma C.1 *Let $\alpha = 0$, $w = c$, and $b_i = b_i(\Delta_i)$ as defined above. Δ_i^e is continuous in and weakly decreasing in Δ_j.*

Proof: We notice that Δ_i^e acts to set $z\delta L = \rho_i$. The function ρ_i is: continuous in both Δ_i and Δ_j, strictly monotonically increasing in Δ_i, and weakly monotonically increasing in Δ_j. Hence an infinitesimal increase in Δ_j must be met by no more than an infinitesimal decrease in Δ_i^e.♣

Let $\Delta^e \equiv (\Delta_1^e, ...\Delta_n^e)$. The Brouwer Fixed Point Theorem states that there is a fixed point if Δ^e is a continuous function on a compact and convex domain. Lemma C.1 states that Δ^e is continuous. However, the domain of Δ is $\Re^n_{++}$ which is not compact. We now demonstrate that there is a compact and convex sub-domain of $\Re^n_{++}$ which Δ^e maps into itself. This will suffice, since there must be a fixed point on this sub-domain.

We use monotonicity to establish an upper bound for Δ_i^e which we denote as $\bar{\Delta}_i$. Let us assume for the moment that $\bar{\Delta}_i > A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2})$. In this case $\overline{\theta}_i^* - \underline{\theta}_i^* = 1$, and

$$\Delta_i^e = \frac{z\delta L}{(1 - \delta\tilde{\mathcal{Q}}^*)} \leq \frac{\delta L}{(1 - \delta)}$$

Hence $\Delta_i^e \leq \bar{\Delta}_i \equiv \max\{\frac{\delta L}{(1-\delta)}, A(i, h_i^*, \frac{1}{2}) - A(i, h_i^*, -\frac{1}{2}), \}$.

We now establish a lower bound for Δ_i^e which we denote as $\underline{\Delta}_i$. Clearly 0 is a lower bound for Δ_i^e. However, neither τ nor $\mathcal{Q}$ are defined for $\Delta_i = 0$. Hence, what we need is to establish a lower bound $\underline{\Delta}_i > 0$. We work with $\Delta_j \leq \bar{\Delta}_j$ in which case we have

$$\frac{\Delta_i^e}{\overline{\theta}_i^* - \underline{\theta}_i^*} = \frac{z\delta L}{1 - \tilde{\mathcal{Q}}^*} > z\delta L \tag{33}$$

The strict inequality follows since $\hat{\mathcal{Q}}_j^* = 0$ only if $\Delta_j = \infty$ and $z = 1$. We know from Lemmas B.3 and B.6 that we may set the LHS of Equation 33 to any value strictly greater than $A_\theta(i, h_i^*, 0)$ which itself is strictly less than $\delta\bar{X}$. Hence if $z = 1$, then we can fix ϵ with $0 < \epsilon < X - \frac{A_\theta(i, h_i^*, 0)}{\delta}$. Set $L = \bar{X} - \epsilon$. Now since $\delta L > A_\theta(i, h_i^*, 0)$ and $\frac{\Delta_i}{\overline{\theta}_i^* - \underline{\theta}_i^*}$ is increasing, it follows that there exists a $\underline{\Delta}_i > 0$ such that $\frac{\underline{\Delta}_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} = \delta L$. From Equation 33 it follows that $\Delta_i^e > \underline{\Delta}_i$ as long as $\Delta_j \leq \bar{\Delta}_j$.

Now on the other hand, suppose that $z < 1$. In this case we note that

$$\lim_{\delta \to 1} \frac{z\delta L}{1 - \tilde{\mathcal{Q}}^*} = \lim_{\delta \to 1} \frac{z\delta L}{1 - \delta(1 - z) - \tilde{P}^*} = \frac{L}{1 - \tilde{P}^*} > L$$

Hence, for δ sufficiently close to one, we can for identical reasons find a $\underline{\Delta}_i > 0$ such that $\frac{\underline{\Delta}_i}{\overline{\theta}_i^* - \underline{\theta}_i^*} = \delta L$.

Again, from Equation 33 it follows that $\Delta_i^e > \underline{\Delta}_i$ as long as $\Delta_j \leq \bar{\Delta}_j$.

With this lower bound established, we may apply the Brouwer Fixed Point Theorem. A fixed point to Δ^e is a simultaneous solution to Equation 24 for each value of i. Hence, the firm voluntarily sets $h_j = r_j = h_j^*$ for each value of j. This renders moot the fact that Δ^e was defined as the solution for Equation 24 with $h_j = r_j = h_j^*$ for $j \neq i$. That is, define $\bar{\Delta}^e$ as we defined Δ^e, but with the requirement that each h_j and r_j are chosen to maximize the discounted present value of payments to the firm. The fixed point to Δ^e must also be a fixed point for $\bar{\Delta}^e$. Hence, at this fixed point the law firm is choosing $h_i = r_i = h_i^*$ for each value of i absent any assumptions concerning how other h_j and r_j are chosen. Finally, we note that L was set less than $\bar{X}$. Hence, the client prefers the long term contract for each value of i.

As in the proof of Proposition3, the law firm has no desire to defraud the client. This follows for the exact same reasons. This concludes the proof of Theorem 1.

To complete the proof of Proposition 2 we need to show that we can choose z to set $\tilde{\mathcal{Q}}^*$ to any value in $[1/2, 1)$. We first note that $\tilde{\mathcal{Q}}^* = \tilde{P}^* \leq 1/2$ when $z = 1$ and $\tilde{\mathcal{Q}}^* \to 1$ as $z \to 0$. Hence, it remains only to show that $\tilde{\mathcal{Q}}^*$ is continuous. We note that an infinitesimal change in z creates an infinitesimal change in the law firm's incentives which can be rebalanced with an infinitesimal change in L, and $\{\Delta_i\}_i$. Hence, if we think of L and $\{\Delta_i\}_i$ as functions of z, then $\tilde{\mathcal{Q}}^*$ is continuous in z. This completes the proof of Proposition 2.

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TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

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