Alternative Distributions for Inequality and Poverty Comparisons

John Creedy

WORKING PAPER 09/2013
June 2013

Working Papers in Public Finance

VICTORIA
UNIVERSITY OF WELLINGTON
TE WHARE WĀNANGA
O TE ŪPOKO O TE IKA A MĀUI

Chair in Public Finance
Victoria Business School

The Working Papers in Public Finance series is published by the Victoria Business School to disseminate initial research on public finance topics, from economists, accountants, finance, law and tax specialists, to a wider audience. Any opinions and views expressed in these papers are those of the author(s). They should not be attributed to Victoria University of Wellington or the sponsors of the Chair in Public Finance.

Further enquiries to:
The Administrator
Chair in Public Finance
Victoria University of Wellington
PO Box 600
Wellington 6041
New Zealand

Phone: +64-4-463-9656
Email: cpf-info@vuw.ac.nz

Papers in the series can be downloaded from the following website:
http://www.victoria.ac.nz/cpf/working-papers

Alternative Distributions for Inequality and Poverty Comparisons

John Creedy[1]

Abstract

This paper provides an introductory review of the alternative possible income distributions which can be used when making cross-sectional evaluations of the effects of taxes and transfers using a household economic survey. This paper attempts to clarify the various alternatives, both for users of data and those wishing to interpret results. Special attention is given to the choice of income unit. The need to avoid spurious comparisons is stressed. The use of adult equivalence scales and the application of an explicit sharing rule are considered. Comparisons over time, where both the tax structure and the populations differ, are also considered. Numerical examples are used to highlight the alternative approaches and distributions.

JEL CLASSIFICATION D31; D13; D63

KEYWORDS Income distribution; adult equivalence scales; income unit

[1] I have benefited from discussions with Omar Aziz and comments by Christopher Ball, Diana Cook, Margaret Galt, Nicolas Herault, Guyonne Kalb, David Law, Denis O'Brien, Grant Scobie and Justin van de Ven on an early version of this paper.

1 Introduction

Many government policies are designed to advance a redistributive objective. Many other policies may not have redistribution as their aim, but could have distributional consequences which need to be evaluated. Furthermore, a wide range of changes - including for example those affecting relative prices, or labour and asset markets, along with demographic change and the structure of households - can have substantial distributional effects. Understanding impacts on inequality and poverty is therefore likely to play a central role in evaluating policies and outcomes.

When attempting to compare distributions using measures of inequality and poverty, important decisions must first be made regarding three major elements, referred to as: 'what, when and whose'. First, a choice must be made regarding precisely what is to be measured; this is often referred to as the 'metric' or 'welfare metric'. For example, this may be pre-tax incomes, wage rates, or a measure of expenditure or consumption. In some cases the welfare metric may even attempt to allow for the value of leisure.[2]

Second the accounting period, or time period over which the selected 'welfare metric' is measured, must be chosen. This may be a week, year or even lifetime, depending on the context. A longer accounting period avoids difficulties arising from transitory changes, or variations arising from age differences which may not be regarded as relevant for inequality comparisons. For example, two individuals could have the same lifetime incomes but different time profiles of their annual incomes. However, data limitations often exist when attempting to extend the accounting period. The use of a longer period also introduces the role of systematic relative mobility.[3] Judgements about inequality may be closely related to judgements about mobility.

The third decision concerns the unit of analysis. This could be, for example, the family, the household, or the individual. Indeed, both the welfare metric and the

---

[2] The metric is usually a one-dimensional measure, although multi-dimensional approaches can also be taken. The present paper concentrates on comparisons of one-dimensional metrics.

[3] On longer accounting periods see, for example, Creedy (1997a, b).

income unit could be artificial measures, designed to allow for differences in the composition of households and involving the concept of an 'equivalent adult'.[4] Assumptions about income sharing within households and families are also often used, and these need to be treated with care.

These choices are in some ways related. For example, if a longer accounting period is used, household and family structures change over time, so that it is more appropriate to consider individuals as the basic unit. The choice of a particular welfare metric may also suggest a particular unit of analysis. The choices depend on the precise nature of the basic question motivating the analysis. A crucial point to recognise is that ultimately these choices cannot avoid the use of value judgements. The view is taken here that the role of the professional economist is to examine the implications of adopting alternative value judgements, so it is very important to make these as explicit as possible.

Even a cursory examination of publications involving inequality and poverty comparisons shows that, in reporting results, decisions regarding the welfare metric, the unit of analysis and the time period are frequently given scant attention. Often the term 'income' is used without being clearly defined. It is all too easy to make spurious comparisons between distributions. Many publications make only a limited number of comparisons, despite the fact, stressed above, that value judgements are involved at every stage.

The limited aim of this paper is therefore to provide an introductory review of the range of alternative possible distributions based on single-dimensional metrics which are variants of income and consumption concepts.[5] This paper attempts to clarify the various alternatives, both for users of data and those wishing to interpret results. Inequality comparisons nearly always give rise to technical and data difficulties, though these are outside the scope of the present paper.

---

4 This is considered in more detail in Section 2 below.

5 Hence, the use of a concept of 'money metric utility', where incomes are affected by the tax structure via labour supply variations, is not considered here as this raises a different set of problems. These are considered by, for example, Donaldson (1992) , Aaberge and Colombino (2008), Ericson and Flood (2009) and Decoster and Haan (2010) where there are heterogeneous preferences; see also Creedy and Hérault (2012).

Cross-sectional comparisons using a household economic survey are considered. Hence, the accounting period is necessarily a single period (typically a year or shorter). The contexts discussed here include analyses of the redistributive effects of direct taxes and transfers, along with the effects of indirect taxes. In addition, 'fiscal incidence' studies attempt to allocate some items of government expenditure, such as health and education, to individuals as well as considering the effects of indirect taxes, direct taxes and transfers. In allocating such expenditure, the view is therefore taken that it relates to publicly provided (tax financed) private goods.[6] Simple hypothetical numerical examples are used to highlight the alternative approaches and distributions.

It should also be remembered that any measures of redistribution, measured in terms of a move from pre-tax to post-tax (and transfer) incomes, need to be interpreted with caution because the structure of taxes and benefits, and of government expenditures, itself influences the pre-tax distribution.

The structure of the paper is as follows. Section 2 introduces a hypothetical population consisting of just four households, chosen to illustrate the range of distributions which can be obtained. It describes the role of adult equivalence scales, designed to deal with the fact that individuals are not homogeneous, and of explicit sharing rules whereby total household income is allocated among all members of that household. Section 3 reports various measures of redistribution and progressivity for a range of comparisons. Section 4 examines the additional difficulties involved in making inequality comparisons over time. In particular, both the tax and transfer system and the population structure (for example the age distribution of the population) can change over time. It is therefore useful to be able to disentangle the separate effects of tax and demographic changes. Brief conclusions are in Section 5.

[6] This is of course debatable. It may, for example, be thought that such expenditure gives rise to considerable externalities. Furthermore, some people may argue that health expenditure devoted to children could instead be added to the welfare metric of parents, who would otherwise need to pay for the care.

2 A Hypothetical Population

To make the various ideas and comparisons concrete, it is useful to construct a small hypothetical dataset. Suppose there are just four households, which is a sufficient number for present purposes. In order to reduce the number of possible comparisons here, households and families are considered to be synonymous; the term 'household' is used throughout to refer to an income unit, consisting of one or more individuals, within which resources may be shared.

The left-hand block of Table 1 shows the hypothetical market incomes of members of the households in the relevant period. The term 'market income' here refers to an individual's income from all sources, such as labour income, self-employment income and the ownership of assets (including rental and interest income, and so on). In some contexts, emphasis may of course be on a particular source, such as wage and salary income from employment. Market income is necessarily assigned to individuals.

In Table 1, a dash indicates that there is no one in the particular cell, while zero indicates that the person has no market income. Thus household 1 consists of one adult (A1) with a market income of 100, while household 2 consists of two adults (A1 and A2) with market incomes of 60 and 40 respectively. Household 3 has one child (C1) and household 4 has two children (C1 and C2). The incomes of the adult individuals have been deliberately chosen so that total household market income is 100 for each household. Non-income differences, other than the demographic structure of households, are not considered relevant here. However, in practice judgements may well depend on other features of individuals and income sources.

The right-hand block of Table 1 shows the hypothetical disposable incomes of each individual, after the application of the income tax and transfer (or benefit) system. This system, which need not be specified in detail for present purposes, is progressive in form. The worker (adult A1) in household 4 is assumed to have a higher disposable income than the single individual in household 1, despite having the same market income. This can be considered to result from some kind of in-work payment related to children. Adult A2 in household 3 has a higher disposable income than market income, also reflecting the nature of the benefit system.

Table 1 Individual Gross and Disposable Incomes

	Market income				Disposable Income			
HH	A1	A2	C1	C2	A1	A2	C1	C2
1	100	-	-	-	80	-	-	-
2	60	40	-	-	45	35	-	-
3	75	25	0	-	70	30	0	-
4	100	0	0	0	90	0	0	0

A range of income distributions may be considered based on the market and disposable incomes of Table 1. If the focus of attention is on individual market incomes, the distribution has six non-zero observations and is:

[100, 60, 40, 75, 25, 100],

where elements are arranged by taking each household, and adults within households from Table 1, in turn. Here the zero incomes of relevant adults and children are necessarily excluded from the population of market incomes. This distribution may be compared with that of the six individual disposables incomes, given by:

[80, 45, 35, 70, 30, 90].

If concern is primarily with how the market distributes the flow of income, and the way this is altered by the tax and transfer system, these distributions containing six observations are the focus of analysis. However, there is clearly a complex relationship between the distribution of market incomes and inequality in the distribution of resources, as more widely perceived, over all individuals in the population. This relationship clearly depends on the incomes of partners and the treatment of those without market incomes.

If concern is with comparisons of total market income among households, rather than individuals, the distribution has just four observations of:

[100, 100, 100, 100],

where elements are arranged by taking each household in turn. Inequality is in this case clearly zero. There is a corresponding distribution of household disposable income, with the household as unit of analysis, of:

[80, 80, 90, 100].

In this case, because the benefits available in the tax and transfer system depend on the existence of children, the use of the household as unit of analysis suggests that the structure introduces inequality. In practice the distributions are likely to show a reduction in inequality, but this example shows why the comparison between these two distributions, having the household as the unit of analysis, may not be very instructive.

Nevertheless, it should be recognised that – given the ubiquitous role of value judgements - some people may actually take the view that household structure is irrelevant in making comparisons. They would object to the special treatment of household composition by the tax and benefit structure and would judge that taxes and transfers (in this example) have indeed increased inequality. Those who take this view may, for example, object to treating children in terms merely of a cost or burden faced by parents, rather than as a desired benefit or advantage. They may consider household structure, fertility decisions, household production and market income as jointly determined to a considerable extent.

2.1 Adult Equivalence Scales

This subsection considers a method of dealing with the heterogeneity of households. A common approach, given only the market and disposable incomes, is to make comparisons on the basis, not of observed actual income either of households or individuals, but of an artificial income construct which reflects the differences in the demographic structure of the households. The simplest way to convert household income into a measure of income per person is clearly to divide total income by the number of individuals in the household. But the view is widely taken that not all members of the household have the same consumption needs. Furthermore, there may be economies of scale within a household. The latter can arise because some goods (including some durables and goods like heating and lighting) may be ‘public goods’ within the household; thus those goods can be consumed simultaneously by

several people. In addition, there may be economies from purchasing larger quantities of some goods.

Instead of dividing total household income by the number of people in the household (irrespective of their ages or gender), a measure of household size can be obtained by giving each person a separate weight, using a set of 'adult equivalence scales'. Such scales are typically imposed by the analyst and are based, loosely speaking, on perceived relative needs of different types of individual in the household and economies of scale within the household. However, value judgements cannot be avoided in the choice of scales. In practice they are often taken from other studies, often relating to other countries, without consideration of their rationale. Equivalence scales are also implicit in the tax and transfer system, but of course it could not be implied that they are consciously used (and the system is typically made up of a range of overlapping benefits).[7]

A simple but very flexible adult equivalence scale is the following, where, n_a and n_c denote respectively the number of adults and children in the household, and *m* is the adult equivalent size of the household:[8]

$$m = \left(n_a + \theta n_c\right)^{\alpha} \qquad (1)$$

Here θ and $\alpha \leq 1$ are parameters reflecting the relative 'cost' of a child and economies of scale respectively. Using (1), with $\theta = 0.5 \text{ and } \alpha = 0.8$, the equivalent sizes of the four hypothetical households are respectively: 1; 1.74; 2.08; and 2.41.

Having obtained the adult equivalent size of each household, it is then a simple matter to obtain the total income per adult equivalent person. The resulting gross and disposable income per adult equivalent for each household is shown in Table 2. It can be seen that the movement from market income per adult equivalent to

---

7 For an example of the calculation of implicit scales, see van de Ven and Creedy (2005).

8 This form was suggested by Cutler and Katz (1992) and Jenkins and Cowell (1994). For an analysis of a wide range of scales using this formula, see Creedy and Sleeman (2005).

disposable income per adult equivalent involves a 'reranking' of households 2 and 3, if the households are ranked in ascending order.

Table 2 Household Income per Adult Equivalent Person

Household	m	Gross income/m	Disposable income/m
1	1	100	80
2	1.74	57.5	46.0
3	2.08	48.1	48.1
4	2.41	41.5	37.34

The further assumption is made that each member of the household receives the income per adult equivalent person; this is the new welfare metric. It is clearly an artificial contruct. Comparisons then depend on the choice of unit of analysis in combination with this welfare metric. It turns out that this choice is not as straightforward as has often been assumed. In fact, three further pairs of distributions may be considered. First, comparisons can be made using the household as the basic unit of analysis (as with the first two distributions considered in the previous subsection): this approach compares

[100, 57.5, 48.1, 41.5]

with

[80, 46.0, 48.10, 37.34].

However, the rationale for this choice of unit is not entirely clear.

Second, perhaps the simplest and most natural choice is to make comparisons using the individual as the basic unit of analysis. This compares the distribution

[100, 57.5, 57.5, 48.1, 48.1, 48.1, 41.5, 41.5, 41.5, 41.5]

with

[80, 46.0, 46.0, 48.1, 48.1, 48.1, 37.34, 37.34, 37.34, 37.34].

Here the elements are ordered by taking each household and individual in turn from Table 1. Again it can be seen that some reranking of individuals is involved when comparing the two distributions: this is discussed further in Section 3 below.

When using the individual as the unit of analysis, each person 'counts for one' irrespective of the household to which they belong. Inequality remains unchanged when one person is replaced by another person with the same metric (income per adult equivalent) but belonging to a different type of household. It thereby satisfies an 'anonymity principle'. However, it does not necessarily satisfy the 'principle of transfers'.[9] This principle is the inequality-disliking value judgement which takes the view (in the context of homogeneous individuals) that an income transfer from a richer to a poorer individual (which leaves the relative rank of the two people unchanged) is judged to reduce inequality and represent an improvement.[10] But if rich large households are highly efficient at generating welfare (in terms of the choice of this metric), given large economies of scale, it is possible, when using the individual as unit, for evaluations to be inequality-preferring.

A third possibility uses the equivalent adult as the income unit. This artificial income unit is thus combined with its corresponding artificial income measure, income per adult equivalent. In this case there are not necessarily integer numbers of equivalent adults (except for the single-adult, who is household number 1 in Table 1) and the distributions cannot be written simply as vectors. Thus the equivalent adult size must be treated as a household weight in obtaining inequality or other measures. To illustrate this case, the arithmetic mean gross adult equivalent income per adult equivalent person, denoted $\overline{y}$, is:

$$\overline{y} = \frac{\left[100 + (1.74)(57.5) + (2.08)(48.1) + (2.41)(41.5)\right]}{(1 + 1.74 + 2.08 + 2.41)} \qquad (2)$$

The use of the artificial equivalent adult as the unit of analysis means that the income unit and the income concept are treated consistently. Each individual's contribution to

9 This was first pointed out by Glewwe (1991).

10 Although the implications of adopting this principle have been widely investigated, it is important always to keep in mind that it is a value judgement.

inequality depends on the demographic structure of the household to which that individual belongs.[11] Thus an adult in a one-person household 'counts for one'. But an adult counts for 'less than one' (has a weight less than 1) when placed in a multi-person household.

Importantly, the use of this income unit is consistent with the principle of transfers, described above. This can be useful because there are general results linking this value judgement to Lorenz curves, which are widely used to depict distributions. For an individual income distribution, first arrange individuals in ascending order, that is from lowest to highest income. The Lorenz curve plots the cumulative proportion of people (on the horizontal axis) against the cumulative proportion of total income (on the vertical axis of the diagram).

Consider the Lorenz curves of two distributions which have the same arithmetic mean income. Suppose the Lorenz curve of one distribution, say A, lies everywhere inside the other distribution, say B: that is, A's curve is closer to the upward sloping diagonal line of equality which arises if all incomes are equal. One way that distributions can be evaluated is as follows. For a distribution $(x_1, x_2, ..., x_n)$, suppose the evaluation function – representing the value judgements of an independent judge - takes the form, $W = \sum_{i=1}^{n} U(x_i)$, where $U(x_i)$ is a concave function representing the contribution of individual i's income to *W*. The concavity of *U* reflects adherence to the principle of transfers and the degree of concavity reflects the extent of aversion to inequality.[12] It has been established that all functions of this general kind would judge the first distribution to be better than the second in that it gives a higher value of *W*.[13] This result is true irrespective of the precise extent of aversion to inequality.

If the arithmetic means of the two distributions differ, the same result applies instead to the concept of the Generalised Lorenz curve: this plots the product of the

11 Its use was first suggested by Ebert (1997). For detailed analysis of the choice of income unit, see Shorrocks (2004).

12 An evaluation function of this type is commonly referred to as an additive, individualistic and Paretian 'social welfare function'. Despite the name, it does not represent society's views, but those of a single person who is not part of the distribution but who is making the evaluation.

13 This result was established by Atkinson (1970).

proportion of total income and the arithmetic mean income against the corresponding proportion of people. Thus the vertical axis of the Lorenz curve is 'stretched' by an amount depending on the arithmetic mean.

Importantly, it cannot be assumed that comparisons are insensitive to the choice of income unit. Indeed, it is quite possible for a tax reform to be judged differently, changing inequality and welfare comparisons in opposite directions, when using the individual and the equivalent adult as income units.[14]

The discussion has so far been in terms of distributions of market and disposable incomes. Some household surveys contain detailed information about household expenditures, and this can be used to compute an additional metric, that of disposable income after the deduction of indirect taxes. If the indirect tax system has considerable selectivity, this task is complicated by the need for detailed expenditure data for each category. But if there is a broad-based goods and services tax (such as a value-added tax), perhaps combined with limited excises (for example, on tobacco, alcohol and petrol), the allocation is less complex.[15] Hence additional distributions can be produced in terms of a welfare metric described as 'income after direct taxes and transfers and after indirect taxes'. However, no new basic issues arise in terms of choice of income unit or equivalence scale. For this reason, this metric is not considered separately here.

2.2 The Use of Allocation Rules

In the previous subsection the welfare metric was based on an assumption of equal sharing within the household to produce the measure of income per adult equivalent person. Further distinctions were then made depending on the choice of income unit. Yet another approach is to use an explicit sharing rule to allocate disposable income to individual members of each household. The particular sharing rule used may be based on special surveys which provide information about income sharing, or it may

---

14 Examples are given by Decoster and Ooge (2002) and Creedy and Scutella (2004).

15 For example, if x denotes expenditure by an individual and v is the tax-exclusive indirect tax rate, then the corresponding tax-inclusive rate is $v/(1+v)$ and expenditure after the deduction of tax is $x/(1+v)$.

be rather more ad hoc.[16] Suppose that the allocation rule is based on an additive household size, *s*, defined as:

$$s = 1 + 0.5(n_a - 1) + 0.3n_c \qquad (3)$$

Hence the first adult is given a weight of 1, while all other adults are given a weight of 0.5 and all children are given a weight of 0.3. These values are chosen arbitrarily, merely for illustrative purposes.[17] For example, household 2 has a total disposable income of 45+35=80. The size of this household for sharing purposes is 1.5, so the amount allocated to the first adult is 80/1.5=53, and the amount allocated to the second adult is (0.5)(80)/1.5=27. Using this approach, the distribution of individual disposable incomes is shown in the left-hand panel of Table 3.

Table 3 Individual Post-Sharing Disposable and Final Incomes

	Disposable income				Final income			
	A1	A2	C1	C2	A1	A2	C1	C2
1	80	-	-	-	78	-	-	-
2	53	27	-	-	58	34	-	-
3	55	28	17	-	60	35	25	-
4	43	21	13	13	49	30	22	22

This type of explicit income-sharing rule is naturally associated with the use of the individual as the income unit. The distribution of individual disposable income, after application of the sharing rule, is thus:

[80, 53, 27, 55, 28, 17, 43, 21, 13, 13].

In examining the redistributive effect of taxes and transfers, care is then needed in selecting the comparison distribution of gross or pre-tax incomes. For example, comparing the above individual distribution with market income per adult equivalent

16 Early studies of income sharing include Lazear and Michael (1988), Jenkins (1991) and Borooah and McKee (1994). See also Bonke and Browning (2003).

17 These values coincidentally correspond to the 'after housing costs' adult equivalence scales used by the OECD.

person, on an individual basis, would retain the same number of observations but would be spurious.[18] This is because each household's adult equivalent income is assumed to be obtained equally by each individual in the household: as shown above, that distribution is:

[100, 57.5, 57.5, 48.1, 48.1, 48.1, 41.5, 41.5, 41.5, 41.5].

The closest comparison is obtained by constructing an individual distribution of market income, on the similar (though again artificial) assumption that market income is shared, though of course the tax and transfer system is not applied to the shared market incomes. This distribution is found to be:

[100, 67, 33, 55, 28, 17, 48, 24, 14, 14].

As mentioned earlier, it is sometimes possible to allow for indirect taxes and thus to construct an additional distribution of (individual) disposable income after the deduction of existing indirect taxes. This is somewhat more complex than when adult equivalent scales are used, because certain goods (such as alcohol and tobacco) need to be attributed to adults, while other goods (such as children's clothing) are necessarily consumed by children. This distribution is again not treated separately here, only because it raises no special issues from the point of view of comparisons.

Fiscal incidence studies go further and attempt to allocate some components of government expenditure to individuals. In particular, health expenditure can be allocated based on age, gender and summary information about individuals' use of publicly financed health services. Similarly primary, secondary and tertiary education expenditure can be allocated to individuals based on age.[19]

The right-hand block of Table 3 shows a hypothetical distribution of 'final' income resulting from the allocation of some government expenditure and the deduction of indirect taxes. These numbers again are not based on specific assumed policies but reflect for example the tendency for children to benefit most from education and

---

18 This is true even if the sharing or allocation structure in (3) were also used as the adult equivalence scales.

19 There would of course be little point in obtaining a measure of final income per adult equivalent (by aggregating the individual final incomes in the household and then dividing by the number of adult equivalents) since, for example, education benefits cannot be thought of as being shared equally within the household. However, in a wider context the absence of tax-financed government expenditure of this kind would require an intra-household transfer.

health policies (and, by assumption, the adults are of working age). The distribution of final income, with the individual as unit of analysis, is thus:

[78, 58, 34, 60, 35, 25, 49, 30, 22, 22].

3 Inequality and Tax Progressivity

This section summarises the various distributions discussed in the previous section and presents the resulting inequality and tax progressivity measures.

3.1 Thirteen Distributions

The various distributions discussed in the previous section can be summarised in Table 4. This shows thirteen distributions although, as discussed above, these could easily be extended by treating indirect taxes separately, both using adult equivalent scales and the explicit sharing or allocation rule.[20]

The number of households is *H*; the number of individuals in household *i* is n_i; the adult equivalent size of household *i* is m_i; and the number of individuals with positive market income is N_W. The number of individuals is $N = \sum_{i=1}^{H} n_i$ and the number of adult equivalents is $N_E = \sum_{i=1}^{H} m_i$.

The numerical example was constructed so that the four hypothetical households have the same total market income of 100. After the application of a progressive tax and transfer system, the four household disposable incomes are:

[80, 80, 100, 90].

Hence many people would argue that a comparison of the inequality of distributions 1 and 2 from Table 4 would serve little value; tax and transfer systems are designed to allow for differences in the demographic structure of households. Hence a simple

[20] Hence there could be three distributions (for three income units) of income per adult equivalent after direct taxes and transfers and the deduction of indirect taxes, plus a distribution of individual post tax and transfers income after sharing and the deduction of indirect taxes.

comparison of this kind, which ignores differences in household composition, would be meaningless (in this example it would suggest that the tax structure is inequality increasing). However, as discussed in Section 2, not everyone would agree with this judgement.

Table 4 Alternative Distributions

No.	Welfare metric	Unit	Sharing	No of Units
1	Total market HH income	Household	N/A	*H*
2	Total disposable HH income	Household	N/A	*H*
3	Total market HH inc per adult equiv	Household	N/A	*H*
4	Total disp HH inc per adult equiv	Household	N/A	*H*
5	Total market HH inc per adult equiv	Individual	Equal	*N*
6	Total disp HH inc per adult equiv	Individual	Equal	*N*
7	Total market HH inc per adult equiv	Equiv indiv	Equal	N_E
8	Total disp HH inc per adult equiv	Equiv indiv	Equal	N_E
9	Individual market income	Individual	No	N_W
10	Indiv disposable income	Individual	No	N_W
11	Individual market income	Individual	Yes	*N*
12	Individual disposable income	Individual	Yes	*N*
13	Individual final income	Individual	Yes	*N*

3.2 Redistributive Effects

Table 4 shows that a range of possible comparisons exists. Another comparison (other than of distributions 1 and 2) involving the household as unit is between distributions 3 and 4 (that is, pre- and post-tax-and-transfer incomes), each of which adjusts household incomes using adult equivalence scales. Various inequality

measures could then be computed. In some cases they are linked with a particular 'social welfare function', reflecting the value judgements of an independent observer, and a degree of inequality aversion can be specified.[21] In practice it is valuable to examine results using a range of measures, thereby considering the effects of adopting different value judgements.

However, for present purposes, where the emphasis is on the various distributions rather than the precise measurement of inequality, it is sufficient to use a single inequality measure. Results are reported for the standard well-known Gini measure. This can be calculated in a variety of ways, but a convenient form is the following, where x_i denotes unit i's income, $\overline{x}$ is arithmetic mean, and individuals are arranged in ascending order:

$$G_x = 1 + \frac{1}{n} - \frac{2}{n^2}\sum_{i=1}^{n}(n+1-i)\left(\frac{x_i}{\overline{x}}\right) \qquad (3)$$

The Gini inequality measures of distributions 3 and 4 are shown in the first row of Table 5. Although the adjusted household income measures are to some extent more comparable than with distributions 1 and 2, this comparison ignores the number of individuals involved.

Comparisons between two distributions of market and disposable income per adult equivalent, which allow for the differing compositions of the households, are between numbers 5 and 6 (using the individual as unit) and between 7 and 8 (using the equivalent adult as unit) in Table 4. The Gini comparisons are reported in Table 5.

The simplest comparison is of the effect of the tax and transfer system on the market and disposable incomes of those individuals who have some market income; this compares distributions 9 and 10.[22] However, this comparison would suggest some

---

21 For a short introduction see, for example, Creedy (1999), and for extensive analysis, see Lambert (1992).

22 There may be a temptation to include all individuals, and hence zero incomes for those not working. It is not clear why one would wish to add zero values and of course the resulting inequality measures are much higher. In the present example, the addition of 4 zeros to each of the distributions 9 and 10 produces Gini values of 0.5425 and 0.5314 respectively.

horizontal inequity in that the two individuals with market incomes of 100 (in households 1 and 4) are treated differently. This judgement clearly ignores the fact that the tax and particularly the transfer structure does not regard the two individuals as being similar, a fact which is ignored when only those with positive market incomes are considered.

Table 5 Gini Inequality Measures

Distributions Before and after	Gini before	Gini after	Reduction (per cent)	
3 and 4	0.1871	0.1538	0.0333	(18 %)
5 and 6	0.1405	0.1171	0.0234	(17 %)
7 and 8	0.1371	0.1125	0.0246	(18 %)
9 and 10	0.2375	0.2190	0.0185	(8 %)
11 and 12	0.3530	0.3269	0.0261	(7 %)
11 and 13	0.3530	0.2404	0.1126	(32 %)
12 and 13	0.3269	0.2404	0.0864	(26 %)

Consider comparisons which allow for the allocation of indirect taxes and some items of government expenditure. Since the use of an income sharing or income allocation rule (which differs from the use of an adult-equivalence scale) is crucial, the comparisons are necessarily based on the individual as the unit of analysis. Comparisons are thus between distributions 11 and 12, between 12 and 13 and between 11 and 13. These are also reported in Table 5, along with the percentage reductions. Clearly any judgement about the redistributive effects of taxes and transfers, and of some components of government expenditure, depends crucially on which comparisons are selected.

The comparisons moving from a measure of market income to disposable income are obtained by comparing distributions 3 to 4; 5 to 6; 7 to 8; 9 to 10; and 11 to 12.

The last two comparisons reveal much smaller percentage reductions in the Gini inequality measure than the first three. Of those using adult equivalence scales (the first three) the absolute Gini values differ slightly although in this case there are small differences in percentage reductions. However, in practice it is possible for the use of different income units to come to different, even opposite, conclusions about the effects of a tax change.

The allocation of some government expenditure to individuals (comparisons involving distribution 13) produces the largest reductions in inequality. This is perhaps not surprising in view of the fact that the Gini measure depends on relative incomes (as well as on the ranks of individuals), and government expenditure in this example involves relatively larger amounts going to the households with children.

3.3 Progressivity Measures

Comparisons between distributions may also involve the use of progressivity measures. In view of the use of the standard Gini inequality measure, it is convenient to use the Kakwani (1986) progressivity measure. This reflects the disproportionality of taxation. It involves the concentration measure of tax payments, which is precisely like the Gini inequality measure, except that in ranking the units in ascending order, the rank used for tax payments is the same as that used for pre-tax incomes. For example, in comparing distributions 3 and 4, the market incomes per adult equivalent are, when ranked in ascending order, given by

[41.5, 48.1, 57.5, 100],

and the net tax paid (the effect of direct taxes and transfers) by those households is [4.16, 0, 11.5, 20].

The concentration measure of taxation is simply obtained by using the expression for the Gini measure in equation (3) above, but keeping the order as in the previous sentence. The actual Gini measure of tax payments would of course be obtained by arranging households in the order

[0, 4.16, 11.5, 20].

Another phenomenon, discussed briefly above, arises when the ranking of households or individuals changes when moving from pre-tax to post-tax incomes. Reranking can be measured by the difference between the Gini measure of post-tax

incomes and the (smaller) concentration measure of post-tax incomes. These are obviously equal when there is no reranking.

In the present context, where transfer payments and some other government expenditure items are allocated, it is not possible to consider progressivity measures for all the distributions examined in Table 5. This is because in some cases there are negative effective tax payments and the basic Gini and concentration measures cannot apply for negative values. Where comparisons are possible, the results are shown in Table 6. The tax ratio shown in the final column of the table is defined as total (effective) tax payments divided by total pre-tax income.[23]

Table 6 Progressivity Measure

Before and after	Progressivity	Reranking	Tax ratio
3 and 4	0.2267	0.0050	0.1443
5 and 6	0.2252	0.0054	0.1135
7 and 8	0.2142	0.0060	0.1251
11 and 12	0.1950	0.0017	0.1250

4 Comparisons over Time

The previous sections of this paper have discussed alternative income distribution comparisons for a single time period. The fact that the redistributive effect of any tax system cannot be evaluated independently of the population (the pre-tax income distribution) raises the question of how comparisons can be made over time, where typically both the population and the tax structure are different. In fiscal incidence studies the question is thus: has the income tax and transfer system become more or less redistributive? The difficulty is therefore to isolate the marginal effect of the tax policy change from that of the population change.

[23] Kakwani (1986) showed that the redistributive effect (the reduction in the Gini inequality measure when moving from pre- to post-tax income) is equal to the progressivity measure multiplied by $g/(1-g)$, where g denotes the tax ratio, less the reranking measure. This can be confirmed using the results in Tables 5 and 6.

Suppose that two cross-sectional household surveys are available. Let T_i denote the tax structure for $i = 0,1$ (an initial period and subsequent period respectively). Similarly let P_i denote the population in period *i*. There are therefore four possible Gini inequality measures of both gross market income and disposable income; denote these by $G_m\left(P_i,T_j\right)$ and $G_d\left(P_i,T_j\right)$ for $i, j = 0,1$. Indeed, these four Gini measures could be obtained using each of the combinations of income concept and unit of analysis discussed above. It is assumed here that each survey contains enough information about the characteristics of households so that the disposable incomes of each population can be computed for each of the tax structures.

To simplify the discussion using the hypothetical households introduced above (and minimise the data to be presented), suppose the only difference between the two populations is that in the second period there is a fifth household consisting of two children and two adults with market incomes of 80 and 20 (so again the households all have the same total income). Suppose the second-period disposable incomes are as show in Table 7: these may be compared, for the first four households, with those in Table 1. The total household disposable incomes are thus

[78, 78, 100, 87, 102]

and the disposable incomes per adult equivalent person are

[78, 45, 48.1, 36.1, 42.3].

Table 7 Individual Gross and Disposable Incomes for Second Tax Structure and Second Population

	Gross market income				Disposable Income			
HH	A1	A2	C1	C2	A1	A2	C1	C2
1	100	-	-	-	78	-	-	-
2	60	40	-	-	43	35	-	-
3	75	25	0	-	68	32	0	-
4	100	0	0	0	87	0	0	0
5	80	20	0	0	72	30	0	0

Consider comparisons using distributions 5 and 6 in Table 4, that is, the distributions of income per adult equivalent person, using the individual as unit of analysis. It may be tempting to compare the Gini measures of disposable income in each period, giving $G_d\left(P_0,T_0\right)=0.1171$ and $G_d\left(P_1,T_1\right)=0.1016$. This comparison would conclude that the policy reform has reduced inequality. But this would be a spurious comparison. Alternatively, it may be tempting to compare, for the two periods, the percentage reduction in the Gini when moving from market to disposable income: in this case they are both 17%, suggesting no change in the redistributive effect of taxes as a result of the policy change.[24] However, the separate effects of tax and population changes can be obtained as follows.

In order to identify the appropriate marginal effects of tax policy and population changes, it is first useful to consider the following decomposition:[25]

$$G_d\left(P_1,T_1\right)-G_d\left(P_0,T_0\right)=\left[G_d\left(P_1,T_1\right)-G_d\left(P_0,T_1\right)\right]+\left[G_d\left(P_0,T_1\right)-G_d\left(P_0,T_0\right)\right] \quad (4)$$

The first term in square brackets on the right hand side of (4) is the population effect given tax structure 1, and the second term in square brackets is the tax policy effect given initial population 0. Appropriate computation for the hypothetical data gives:

$$0.1016-0.1171=\left[0.1016-0.1220\right]+\left[0.1220-0.1171\right] \quad (5)$$

The reduction in inequality of disposable income per adult equivalent person is the term on the left hand side of (5), that is 0.1016 - 0.1171 = -0.0155. The population effect is negative, since 0.1016 – 0.1220 = -0.0204). The policy effect is actually positive, since 0.1220 – 0.1171 = 0.0049. Thus the effect of the tax policy change, measured using the population of the initial year, is to increase inequality.

---

24 The percentages are rounded to the nearest integer here as in Table 5, but in this case they are equal when given to two decimal places, both being 16.65%.

25 This kind of decomposition can be extended to allow for labour supply responses to the tax policy change, but rapidly becomes more complex as the number of decompositions increases. See Bargain (2010) and, for a range of extensions including the use of a money metric welfare measure, see Creedy and Hérault (2011).

However, there is another possible decomposition of the change in inequality, since:

$$G_d(P_1,T_1) - G_d(P_0,T_0) = \left[G_d(P_1,T_0) - G_d(P_0,T_0)\right] + \left[G_d(P_1,T_1) - G_d(P_1,T_0)\right] \quad (6)$$

The first term in square brackets on the right hand side of (6) is the population effect given tax structure 0, while the second term is the tax policy effect given population structure 1. Computation gives:

$$0.1016 - 0.1171 = \left[0.0984 - 0.1171\right] + \left[0.1016 - 0.0984\right] \quad (7)$$

In this case the population effect is again negative (-0.0187) and the policy effect is again positive (0.0032). Both effects are smaller in absolute terms but of course give the same overall reduction in the Gini measure.[26] Faced with two values for each of the marginal effects, one approach is to obtain the unweighted arithmetic mean, giving a tax policy effect of 0.00405 and a marginal population effect of -0.01955.[27] The overall reduction in inequality of disposable income per adult equivalent person in the present example (comparing the second cross-sectional dataset with the first) arises because the inequality-reducing marginal effect of the population change outweighs the inequality-increasing marginal effect of the tax policy change.

5 Conclusions

The aim of this paper has been to emphasise the need to avoid spurious comparisons of inequality and poverty by paying particular attention to the choice of welfare metric (or income concept) and unit of analysis. A wide range of possible distributions can be constructed so that great care is needed when making comparisons. It is especially important to make explicit the various value judgements that are inevitably involved.

---

26 Computations based on the use of the equivalent person as the unit of analysis (distributions 7 and 8 in Table 4), give a percentage reduction in the Gini when moving from market to disposable income in the second period of 20% (compared with 18%, as shown in Table 5). This could not be used to suggest that the policy change has produced a more redistribute tax structure. Use of this alternative unit of analysis produces positive marginal policy effects on the change in inequality of disposable income, in agreement with the use of the individual as unit (though absolute values are smaller).

27 This average is recommended by Shorrocks (2011), who links it to the Shapley Value, familiar from game theory.

The paper has concentrated on clarifying the nature of alternative distributions, rather than examining the more technical issues relating to formal measures of inequality, on which there is a vast literature.

In the literature on inequality measurement, many analytical results regarding inequality and tax progressivity have been developed in the context of populations consisting of individuals receiving different (exogenous) incomes but otherwise being identical. It was found possible to make important connections between clearly stated value judgements, such as the principle of transfers, and comparisons of Lorenz and Generalised Lorenz curves. Widely used inequality measures could be linked explicitly to value judgements and associated social welfare (or evaluation) functions. However, in practice the need to deal with heterogeneous individuals and households, combined with the sharing of resources within households, has meant that applied economists typically deal with somewhat artificial income concepts as well as income units.

When the artificial income concept of household income per adult equivalent is used, a similarly artificial unit, the equivalent adult, needs to be used as the unit of analysis if it is desired to apply all the established welfare results regarding Lorenz curves and the principle of transfers. But if an anonymity principle – the value judgement that all individuals should 'count as one' irrespective of the household to which they belong – is instead held, the appropriate unit is the individual. However, in this case the principle of transfers can no longer be relied on: indeed it is possible for such an approach to be inequality preferring if rich large households are judged to benefit from large economies of scale.

Instead of using equivalence scales where equal sharing is implicit, an explicit sharing rule could be used to allocate total household income among household members. This can be further extended to allow for the assignment of some government expenditure to individuals: thus the expenditure is assumed to involve publicly funded private goods. It must be recognised that results can depend on the sharing rule assumed, and of course the application of a common sharing rule to all households is a strong assumption. Furthermore, a comparison of final income with

market income involves the artificial income measure obtained by applying the same sharing rule to market incomes.

It has also been shown that income distribution comparisons over time, and particularly evaluations of tax policy changes, need to be made with care. This is because both the population structure (and thus the pre-tax income distribution) and the tax policy are subject to changes. However, the marginal effects of each change can be computed based on a range of decomposition analyses.

Despite the many problems, there will undoubtedly continue to be much interest in inequality and the redistributive role of government. There will therefore continue to be a strong demand for the empirical evaluation of policies in terms of their effects on inequality. Policies which are designed for quite different purposes may be thought to have implications for inequality that need to be explored. Hence applied economists cannot afford to be nihilistic. A substantial amount of pragmatism, using measures and approaches that have known limitations, is needed in the face of considerable complexity and fundamental difficulties, including data and modelling limitations.

It is therefore extremely important to be as clear as possible about the approach used, to provide a wide range of results to allow readers to use their own judgement, and to exercise caution in interpreting results.

A final additional word of caution is perhaps also warranted. Even if concern were purely with market incomes which unambiguously involve the individual as the unit of analysis, assessing the impact of taxation is not straightforward, in view of endogeneity effects involved. In particular, the tax and transfer structure is designed to allow for differences in the demographic structure of households, and individual incomes are affected by joint labour supply decisions of partners. Household formation and fertility are jointly determined, along with labour supply. Thus pre-tax incomes themselves depend on the tax structure, so the term ‘redistributive effect of taxes’ must be used with caution. Importantly, a tax change designed to be more redistributive may in fact lead to lower inequality of post-tax incomes, but also to higher inequality of pre-tax incomes.

References

Aaberge, R. and Colombino, U. (2008) Designing optimal taxes with a microeconometric model of household labour supply. Centre for Household Income, Labour and Demographic Economics Working Paper 06/2008

Atkinson, A.B. (1970) On the measurement of inequality. Journal of Economic Theory, 2, pp. 244-263.

Bargain, O. (2010) Back to the future: decomposition analysis of distributive policies using behavioural simulations. Institute for the Study of Labour (IZA) Discussion Paper 5226.

Bonke, J. and Browning, M. (2003) The distribution of well-being and income within the household. CAM Working Paper 2003-01, University of Copenhagen. Department of Economics. Centre for Applied Microeconometrics.

Borooah, V.K. and McKee, P.M. (1994) Intra-household income transfers and implications for poverty and inequality in the UK. In Poverty, Taxation and Income Distribution (ed. by J. Creedy). Cheltenham: Edward Elgar.

Creedy, J. (1996) Measuring income inequality. Australian Economic Review, 2'96, pp. 236-246.

Creedy, J. (1997a) Inequality, mobility and income distribution comparisons. Fiscal Studies, 18, no.3, pp. 293-302.

Creedy, J. (1997b) Lifetime inequality and tax progressivity with alternative income concepts. Review of Income and Wealth, 43, no.3, pp. 283-295.

Creedy, J. (1999) Taxation, redistribution and progressivity: an introduction. Australian Economic Review, 32, no. 4, pp. 410-422.

Creedy, J. and Scutella, R. (2004) The role of the unit of analysis in tax policy reform evaluations of inequality and social welfare. Australian Journal of Labour Economics, 7, pp. 89-108.

Creedy, J. and Sleeman, C. (2005) Adult equivalence scales, inequality and poverty. New Zealand Economic Papers, 39, pp. 51-83.

Creedy, J. and Hérault, N. (2011) Decomposing inequality and social welfare changes: the use of alternative welfare metrics. University of Melbourne Department of Economics Working Paper.

Creedy, J. and Hérault, N. (2012) Welfare-improving income tax reforms: a microsimulation analysis. Oxford Economic Papers, 64, pp. 128-150.

Cutler, D. M. and Katz, L. (1992) Rising inequality? Changes in the distribution of income and consumption in the 1980s. American Economic Review, 82, pp. 546-551.

Decoster, A. and Ooghe, E. (2002) Weighting with individuals, equivalent individuals or not weighting at all: does it matter empirically? Catholic University of Leuven Centre for Economic Studies Discussion Paper, no. DPS 02.15.

Decoster, A. and Haan, P. (2010) Empirical welfare analysis in random utility models of labour supply. Institute for the Study of Labour (IZA) Discussion Paper 5301.

Donaldson, D. (1992) On the aggregation of money measures of well-being in applied welfare economics. Journal of Agricultural and Resource Economics, 17, pp. 88-102.

Ebert, U. (1997) Social welfare when needs differ: an axiomatic approach. Economica, 64, pp. 233-244.

Ericson, P. and Flood, L. (2009) A microsimulation approach to an optimal Swedish income tax. IZA Discussion Paper no. 4379.

Kakwani, N. (1986) Analysing Redistribution Policies: A Study using Australian Data. Cambridge: Cambridge University Press.

Glewwe, P. (1991) Household equivalence scales and the measurement of inequality: transfers from the poor to the rich could decrease inequality. Journal of Public Economics, 44, pp. 214-216.

Jenkins, S.P. (1991) Poverty measurement and the within-household distribution: agenda for action. Journal of Social Policy, 20, pp. 457-483.

Jenkins, S. P. and Cowell, F. A. (1994) Parametric equivalence scales and scale relativities. Economic Journal, 104, pp. 891-900.

Lambert, P. (1992) The Distribution and Redistribution of Income. Oxford: Blackwell.

Lazear, E.P. and Michael, R.T. (1988) Allocation of Income within the Household. Chicago: University of Chicago Press.

Shorrocks, A.F. (2004) Inequality and welfare evaluation of heterogeneous income distributions. Journal of Economic Inequality, 2, pp. 193-218.

Shorrocks, A.F. (2011) Decomposition procedures for distributional analysis: a unified framework based on the Shapley value. Journal of Economic Inequality (forthcoming).

van de Ven, J. and Creedy, J. (2005) Taxation, reranking and equivalence scales. Bulletin of Economic Research, 57, pp. 13-36.

About the Author

John Creedy is Visiting Professorial Fellow at Victoria Business School, Victoria University of Wellington, New Zealand, and a Principal Advisor at the New Zealand Treasury. He is on leave from the University of Melbourne where he is The Truby Williams Professor of Economics.
Email: john.creedy@vuw.ac.nz

VICTORIA UNIVERSITY OF WELLINGTON
TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

Chair in Public Finance
Victoria Business School

Working Papers in Public Finance