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# The Composition of Government Expenditure with Alternative Choice Mechanisms

John Creedy and Solmaz Moslehi\*

## Abstract

This paper investigates the choice of the composition of government expenditure using both positive and normative approaches. The former involves aggregation over selfish voters (simple majority voting and stochastic voting are examined), while the latter involves the choice by a single disinterested individual (considered to maximise a social welfare function). The approach allows direct comparisons of the choice mechanisms. The structures examined include a transfer payment combined with a pure public good, and a transfer payment with tax-financed education. Explicit solutions are obtained for the choice of expenditure components, and these are shown to depend on the proportional difference between the arithmetic mean and another measure of location of incomes, where the latter depends on the choice mechanism. In each case the expenditure composition depends on an inequality measure defined in terms of the proportional difference between a measure of location of the income distribution and the arithmetic mean, where the location measure depends on the decision mechanism.

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Keywords: Government expenditure; Majority voting; Stochastic voting; Public goods; Social welfare

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# 1 Introduction

This paper investigates the choice of the composition of government expenditure using both positive and normative approaches. The former involves aggregation over selfish voters: simple majority voting and stochastic voting are examined. The latter involves the choice by a single disinterested individual who is considered to maximise a social welfare function. Special attention is given to the relationship between the composition of expenditure and inequality. This allows consideration of the question of the extent to which higher inequality produces a choice in favour of a higher proportion of expenditure being devoted to equalising transfer payments. The approach provides a synthesis which allows direct comparisons of the choice mechanisms. In each model, the expenditure composition is found to depend on income inequality, defined in terms of the proportional difference between arithmetic mean income and another measure of location of the income distribution. The precise location measure depends on the particular choice mechanism investigated. It is useful to explore alternative modelling approaches in view of the substantial variations in the composition of government expenditure across countries.

The present analysis looks at the division between a transfer payment and a pure public good, and the division between a transfer payment and tax-financed education.<sup>1</sup> In addition, concentration is on the case where (as in most of the optimal tax literature) there is heterogeneity only with respect to abilities. Individuals are therefore assumed to have similar tastes. In view of the emphasis on comparing alternative choice mechanisms, for simplicity incomes are assumed to be fixed. The introduction of education clearly involves a different kind of trade-off. A higher transfer payment involves less education spending and reduces incomes, thereby affecting individuals' budget constraints (in addition to the change in the transfer). In the case where there is only a public good, lower public good expenditure feeds into

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<sup>1</sup>Expenditure on education is classified as a publicly provided private good in Bearse *et al.* (2001), Borck (2008) and Soares (2006). In these studies education is provided publicly but individuals can choose to use this or pay for private education at the market price. Publicly provided goods are financed by a proportional income tax.

individuals' utility functions directly. The synthesis is shown to carry over to the more complex choice problem. Comparisons between expenditure shares and inequality, and among choice mechanisms, are not significantly affected. However, when looking at the relationship between the transfer payment and the given tax rate, it is simply necessary to keep in mind that incentive effects are likely to produce a concave schedule.

Relatively few studies have concentrated on the composition of expenditure. The political economy literature has instead given more attention to the total government size or tax rate. Most studies focus on one type of government expenditure, either public goods expenditure or a redistributive transfer payments and consider voting on the tax rate. For example, Meltzer and Richard (1981) Krusell and Rios-Rull (1999), and Borck (2007) considered redistributive expenditure and Tridimas (2001) and Tridimas and Winer (2005) look at expenditure on public goods. However, Barse *et al.* (2001) examined majority voting over a transfer payment and public education, conditional on the tax rate, in a static framework. Creedy and Moslehi (2009) examined majority voting over government expenditure on transfer payments as well as public goods, with endogenous incomes, again within a static framework.

This paper follows the second line of approach and concentrates on the composition of government expenditure for a given tax. The income tax rate is assumed, as in Barse *et al.* (2001) and Creedy and Moslehi (2009), to be exogenously fixed and thus determined by a separate process. It may be thought of as determined by some conventional view regarding 'taxable capacity', or other constraints are imposed on governments regarding the rate. In practice, taxing and expenditure policies are indeed debated independently.<sup>2</sup>

Most earlier studies concentrated on majority voting outcomes, where only the median voter is decisive. However, more recent studies have examined stochastic voting involving the maximisation of a political support function, and where the mean, variance and skewness are relevant in de-

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<sup>2</sup>In some cases, a two-stage procedure may be envisaged in which voting over the tax rate is separate from that over the composition.

termining voting outcomes. Furthermore, multidimensional voting can be considered. Examples include Tridimas and Winer (2005) where individuals vote on tax rates and public goods. Tridimas (2001) considered different categories of expenditure, or public goods, and used probabilistic voting to study the allocation of public consumption. In addition, Hassler and *et al.* (2005) applied probabilistic voting in the context of overlapping generations with transfer payments and found that the voting outcome for redistribution is larger and more persistent than the social planner's choice with commitment. Dolmas (2009) applied probabilistic voting in a simple growth model with tax on consumption, labour and capital income, as well as a lump-sum transfer and exogenous government expenditure.

Section 2 considers the allocation of expenditure between transfer payments and a public good. A majority voting equilibrium is shown to exist and the ratio of expenditure levels is found to depend on the median voter's income as a ratio of arithmetic mean income. The stochastic voting case is then considered. These results are compared with results obtained using a general form of social welfare function. Section 3 combines a transfer payment with tax-financed public education expenditure instead of a public good. Brief conclusions are in Section 4.

## 2 Choice of Public Goods and Transfer Payment

This section examines choices regarding a transfer payment,  $b$ , and consumption of a public good,  $Q$ , under the alternative decision mechanisms. First, the specification of individuals' preferences, and the government budget constraint is described. This applies to all mechanisms. Consider person  $i$ 's preferences for private consumption,  $c_i$ , (where the consumer price index is normalised to unity) and consumption of a public good,  $Q$ , which is non-rival and non-excludable. Suppose  $i$ 's preferences are described by the general direct utility function:

$$U_i = U(c_i, Q) \tag{1}$$

All individuals pay a proportional income tax, at the rate  $\tau$ . As the pure public good is non-excludable, the individual's budget constraint is:

$$c_i = y_i(1 - \tau) + b \quad (2)$$

In this framework, where incomes are fixed, consumption is given simply by (2).

The public good is produced with a constant unit production cost of  $P$ . The total cost is recovered through the tax system, in view of its non-excludable nature, with a 'tax price' per person of  $PQ/N$ , where  $N$  is the population size. The latter thus may be thought to affect outcomes, as a higher population means that the cost is shared among more people. It seems useful to restrict attention to utility functions which avoid expenditure shares (rather than total amounts) depending on  $N$ .<sup>3</sup> This avoids outcomes involving a movement entirely to public good expenditure as  $N$  increases. It can be shown that the use of homothetic utility functions where, in addition, the marginal rate of substitution is a linear function of the ratio of quantities consumed, satisfies this requirement.<sup>4</sup> The following analysis thus uses the form:<sup>5</sup>

$$U_i = c_i Q^\gamma \quad (3)$$

which clearly has the desired properties mentioned above.

The government budget constraint requires the revenue from income taxation to be equal to the sum of expenditure on transfer payments and public good. Thus:

$$\tau \bar{y} = b + \frac{PQ}{N} \quad (4)$$

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<sup>3</sup>Hindriks and Myles (2006) examine a number of models involving the choice of a non-rival public good, in each of which population size,  $N$ , appears as a determinant. However, they do not discuss the implications of large  $N$ . Some authors have argued that population heterogeneity is increased as population size increases, which has quite different effects: see Shelton (2007, pp 2234–2235) for more discussion and a review of the literature.

<sup>4</sup>The need for the ratio of marginal utilities of private to public goods to depend linearly on  $Q_G/c_i$  restricts attention to special case of the more general 'linear preference system' of Allen and Bowley (1935).

<sup>5</sup>This is of course a simple monotonic transformation of the standard Cobb-Douglas form. For example if  $U = x^\alpha y^{1-\alpha}$ , taking the  $\alpha$ th root gives  $xy^{(1-\alpha)/\alpha}$ .

## 2.1 Voting Mechanisms

### 2.1.1 Majority Voting

In examining majority choice, the approach involves obtaining individuals' indirect utility functions in terms of one policy variable, using the government budget constraint to eliminate the other. Voting is therefore one-dimensional and a check can be made for single-peakedness, which guarantees a voting equilibrium. If this exists, the solution for the majority voting outcome is found by maximising the median voter's indirect utility with respect to the policy variable.

Substituting  $c_i$  from (2) and  $b$  from (4) into the direct utility function gives indirect utility,  $V_i$ , in terms of  $Q$ :

$$V_i = \left\{ y_i (1 - \tau) + \tau \bar{y} - \frac{PQ}{N} \right\} Q^\gamma \quad (5)$$

Since  $\partial^2 V_i / \partial Q^2 < 0$ , preferences regarding  $Q$ , for given  $\tau$ , are single-peaked. Hence majority voting is determined by the choice,  $Q_m$ , of the median voter, who has  $y_i = y_m$ . Both  $b_m$  and  $Q_m$  must be positive, so that  $0 < Q_m < \tau \bar{y}$ . Maximising  $V_m$  with respect to  $Q_m$  and some rearrangement gives:

$$\frac{PQ_m}{N} = \frac{\gamma}{(1 + \gamma)} \bar{y} \left\{ \tau + \frac{y_m}{\bar{y}} (1 - \tau) \right\} \quad (6)$$

Clearly a relatively low value of  $\gamma$ , the weight attached to consumption of the private good in utility functions, implies a relatively low choice of public goods.<sup>6</sup> As income distributions are positively skewed,  $y_m/\bar{y}$  is less than one. Also, with higher equality, or higher  $y_m/\bar{y}$ , majority voting results in higher expenditure on public goods. Since raising the tax rate increase the government per capita expenditure on public goods. The majority choice of expenditure on transfer payments, using the government budget constraint, becomes:

$$b_m = \frac{1}{(1 + \gamma)} \bar{y} \left\{ \tau - \gamma (1 - \tau) \frac{y_m}{\bar{y}} \right\} \quad (7)$$

Higher basic inequality leads to a more redistributive expenditure policy, for given  $\tau$ . Indeed the ratio  $b_m/\bar{y}$  falls linearly as  $y_m/\bar{y}$  increases, from

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<sup>6</sup>In the trivial case where  $\gamma = 0$ , and voters obtain no utility from the public good, this reduces to the simple form  $b_m = \tau \bar{y}$ .



$\tau/(1+\gamma)$  when  $y_m/\bar{y} = 0$  to  $(\tau - \gamma(1-\tau))/(1+\gamma)$  when  $y_m/\bar{y} = 1$ . Furthermore,  $b_m/\bar{y}$  increases linearly as  $\tau$  increases, from  $b_m/\bar{y} = 0$  when  $\tau = \{1 + (1/(\gamma y_m/\bar{y}))\}^{-1}$  to  $(1 - (\gamma y_m/\bar{y}))/ (1 + \gamma)$  when  $\tau = 1$ . The ratio,  $R_m = b_m/(PQ_m/N)$ , is given by:

$$R_m = \frac{\frac{1}{\gamma}\tau - \frac{y_m}{\bar{y}}(1-\tau)}{\tau + \frac{y_m}{\bar{y}}(1-\tau)} \quad (8)$$

The majority choice of the composition of expenditure, for given  $\tau$ , thus depends on the taste parameter  $\gamma$  and the median income relative to the arithmetic mean. The ratio is independent of the units of measurement of incomes and of the population size. Differentiation of  $R_m$  with respect to  $y_m/\bar{y}$  gives  $(\partial R_m/\partial (y_m/\bar{y})) < 0$ . Hence a reduction in  $y_m/\bar{y}$ , that is an increase in inequality, produces a higher proportion of expenditure devoted to the redistributive transfer payment.<sup>7</sup> Also,  $\partial^2 R_m/\partial (y_m/\bar{y})^2 > 0$  which implies that the ratio of transfer payment expenditure to public goods increases at an increasing rate as inequality rises. Furthermore, it can be shown that  $R_m$  increases with  $\tau$  at a decreasing rate but, because incentive effects are neglected here, it does not turn downwards.<sup>8</sup> Also, there is a negative relationship between the weight attached to public goods,  $\gamma$ , and the ratio of expenditure on transfer payment to public goods.

It is possible to define a set of income inequality measures,  $I_L = 1 - y_L/\bar{y}$ , where  $y_L$  is some measure of location. Hence the majority voting outcome is a function of inequality,  $I_m = 1 - y_m/\bar{y}$ , although the median voter, acting entirely selfishly, has no desire to reduce inequality except insofar as the median person gains from redistribution.

A negative relationship between inequality and government size, or the tax rate, is a core result in the theoretical literature inspired by Romer (1975)

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<sup>7</sup>This result is consistent with Creedy and Moslehi (2009) which concentrates on transfer payment and public goods for a given tax with endogenous labour supply. In addition, see Meltzer and Richard (1981) and Romer (1975), who examined majority voting over the tax rate, with an unconditional transfer payment. There is considerable literature associated with these studies. However, empirical evidence concerning this relationship, based on cross-sectional data for a range of countries, has been found to be mixed: see Lind (2005).

<sup>8</sup>Thus  $\partial^2 R_m/\partial \tau^2 < 0$  and when  $\tau = 1$ ,  $\partial R_m/\partial \tau = y_m/\gamma\bar{y}$ .

and Meltzer and Richard (1981). However, empirical studies are mixed and Lind (2005) discussed some different reasons for these mixed result.<sup>9</sup> Alesina and Glaeser (2004) suggested that differences in the composition of government expenditure cannot be explained by economic theories in which higher inequality leads to higher redistribution. Instead, they suggested (2004, p.220) that there are different attitudes toward redistribution, based partly on views regarding income mobility. In the present context such differences in expenditure patterns can in fact be consistent with inequality differences, if there are different preferences for public and private goods across countries. Furthermore, increasing  $y_m/\bar{y}$  reduces  $R_m$  at a decreasing rate for all tax rates and preference parameters. Numerical results (not reported here) show that, over the most relevant range of  $y_m/\bar{y}$ , the response of the expenditure ratio to a change in inequality is expected to be relatively small. This may be another reason why empirical studies obtain mixed results.

### 2.1.2 Stochastic Voting

This section considers stochastic voting instead of the simple majority voting mechanism discussed above. Suppose the population consists of  $K$  groups of individuals, where group  $k$  has population proportion  $n_k$ , for  $k = 1, \dots, K$ , so that  $\sum_{k=1}^K n_k = 1$ . Within each group individuals have the same income, so that if there are two political parties, A and B, with associated policies, individuals within each group have the same indirect utility except for a stochastic element. Voter  $i$  in group  $k$  prefers party A if:

$$V_{k,A} > V_{k,B} + s_{i,k} + g \quad (9)$$

The term  $s_{i,k}$  represents member  $i$  of group  $k$ 's additive bias towards party B, and for the  $k$ th group  $s_{i,k}$  lies between  $-s_k^*$  and  $+s_k^*$ .<sup>10</sup> This bias is

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<sup>9</sup>These include the existence of multiple social contracts, prospect of upward income mobility, multi-dimensional policies, race and redistribution versus social insurance.

<sup>10</sup>An alternative specification of bias in which the additive form in (9) is replaced by a multiplicative form, whereby what matters are the relative sizes of  $V_{k,A}$  and  $V_{k,B}$  rather than the absolute sizes, leads to a formulation which is additive in the logarithms. The objective function is thus expressed as a weighted geometric mean of indirect utilities, rather than a weighted arithmetic mean, such that  $S = \sum_i \eta_i \log V_i$ . The voting outcomes

considered to arise from factors which are not related to the policies of the parties. In addition  $g$  represents a population-wide additive bias towards party B. The  $s_{i,k}$  are random variables, along with  $g$ , with expected values of zero. The introduction of random components involves a substantial change to the voting framework, compared with the deterministic model in which the probability of an individual voting for party A switches from 0 to 1, as  $V_{k,A}$  switches from being less than, to greater than,  $V_{k,B}$ .

Following Persson and Tabellini (2000), the situation facing party B is symmetric with that of A and, where each party is assumed to be trying to maximise its chances of winning the election, the policies of the two parties converge. They can be regarded as having the objective function:

$$S = \sum_{k=1}^K \eta_k V_k \quad (10)$$

where of course  $V_k$  is regarded as a function of policy variables. and  $\eta_k$  is  $n_k/s_k^*$ .

This framework does not place any restriction on the number of policy instruments under consideration. Hence there are no problems of the existence of a voting equilibrium, such as those which can arise with deterministic voting. However, two conditions need to hold in the stochastic voting model. First, voters' utilities should be concave functions of political platforms. Second, the density of ideologies should not be a sharply increasing function: in the present model the distribution of  $s_{i,k}$  is uniform, which satisfies this condition.<sup>11</sup>

This approach can thus be applied to the present context of the division of expenditure between transfers and a public good, as follows. Each party maximises the support function, the expected vote in (10), subject to the government budget constraint. Indirect utility for an individual in group  $k$

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is considerably sensitive to the form specification of bias. Dolmas (2009) looks at equilibrium of taxes in a probabilistic voting model. He finds that the assumption about the form, additive or multiplicative, of random non-policy element affects considerably the equilibrium of taxes. Here, only the additive form of bias is examined.

<sup>11</sup>See Acemoglu and Robinson (2006, p. 365) for more details about the conditions required for probabilistic voting.

takes the form  $\{y_k(1 - \tau) + b\} Q^\gamma$ . The first-order conditions with respect to transfer payments and public goods are as follows:

$$\sum_{k=1}^K \eta_k Q_S^\gamma = \lambda \quad (11)$$

$$\sum_{k=1}^K \eta_k \gamma \{y_k(1 - \tau) + b_S\} Q_S^\gamma = \lambda \frac{pQ_S}{N} \quad (12)$$

where  $\lambda$  is the relevant Lagrange multiplier. Also,  $pQ_S/N$  and  $b_S$  are the per capita expenditure on public goods and transfer payments under stochastic voting. The above first-order conditions with the government budget constraint give:

$$\frac{pQ_S}{N} = \frac{\gamma \bar{y}}{(1 + \gamma)} \left\{ \tau + (1 - \tau) \frac{\tilde{y}}{\bar{y}} \right\} \quad (13)$$

$$b_S = \frac{\bar{y}}{(1 + \gamma)} \left\{ \tau - \gamma(1 - \tau) \frac{\tilde{y}}{\bar{y}} \right\} \quad (14)$$

Where  $\tilde{y} = \sum_{k=1}^K y_k \left( \eta_k / \sum_{k=1}^K \eta_k \right)$  and is a weighted average income. The weights,  $\eta_k / \sum_{k=1}^K \eta_k$ , are a function of political influence,  $\eta_k$ . Recall that  $\eta_k = n_k / s_k^*$  which shows that the group size,  $n_k$ , as well as the group density,  $s_k^*$ , affect the weighted average income under stochastic voting. The latter shows the sensitivity of the voters in each group to the economic policies.

The ratio of expenditure on transfer payments to public goods with stochastic voting,  $R_S$ , therefore becomes:

$$R_S = \frac{\frac{1}{\gamma} \tau - (1 - \tau) \frac{\tilde{y}}{\bar{y}}}{\tau + (1 - \tau) \frac{\tilde{y}}{\bar{y}}} \quad (15)$$

Comparing the stochastic voting results with the median voter's choice of public goods and transfer payment indicates that the two expressions are identical except for the fact that the majority choice depends on  $y_m/\bar{y}$  whereas stochastic voting depends on  $\tilde{y}/\bar{y}$ . In the median voter framework only the median individual is the decisive voter and the relative position of the median to the arithmetic mean plays the crucial role. With probabilistic voting, the

group size plays an important role, but also the responsiveness of voters in each group affects the outcome.

Comparative statics results with respect to  $\tau$  and  $\gamma$  are similar to the majority voting case. In addition,  $b_S$  and  $pQ_S/N$  are linearly decreasing and increasing in  $\tilde{y}/\bar{y}$ , respectively. Moreover, there is a negative relationship between  $\tilde{y}/\bar{y}$  and the ratio of expenditure on transfer payments to public goods. The weight of different groups in the political support function,  $\eta_k$ , is one of the main determinates of the composition of government expenditure and the derivatives of per capita expenditure on transfer payments and public goods with respect to  $\eta_k$  are given by:

$$\frac{\partial (pQ_S/N)}{\partial \eta_k} = \frac{\gamma(1-\tau)(y_k - \tilde{y})}{(1+\gamma) \sum_{k=1}^K \eta_k} \quad (16)$$

$$\frac{\partial b}{\partial \eta_k} = -\frac{\gamma(1-\tau)(y_k - \tilde{y})}{(1+\gamma) \sum_{k=1}^K \eta_k} \quad (17)$$

Suppose income in group  $k$  is lower than the weighted average income,  $\tilde{y}$ . Consequently, voters in this group support less expenditure on public goods and more expenditure on transfer payments. For instance, suppose there are three groups of individuals: poor, middle-income and rich. If the political influence of the group of poor falls, the expenditure on transfer payments decreases relative to public goods. The political influence of the poor can fall by reducing the share of poor in the whole population or increasing the responsiveness of voters in the group of poor to changes in the economic policy.

## 2.2 A Social Welfare Function

This subsection examines the policy decision regarding the composition of expenditure where choices are made by an independent judge, whose value judgements are summarised by a social welfare function,  $W$ , defined as a function of indirect utilities. Hence:

$$W = W(V_1, \dots, V_N) \quad (18)$$

which is assumed to be concave with respect to  $b$  and  $Q$ . This implies that the associated social indifference curves, giving combinations of  $Q$  and  $b$  which

leave  $W$  unchanged, are convex. An optimal allocation of expenditure is obtained as a point of tangency of the highest social indifference curve which can be reached subject to the government's budget constraint relating  $Q$  and  $b$ . The optimal policy is thus given from the condition:

$$\left. \frac{db}{dQ} \right|_{\tau} = \left. \frac{db}{dQ} \right|_W \quad (19)$$

Differentiating  $W$  totally with respect to public good expenditure and the transfer payment gives:

$$dW = \left( \sum_{i=1}^N \frac{\partial W}{\partial V_i} \frac{\partial V_i}{\partial b} \right) db + \left( \sum_{i=1}^N \frac{\partial W}{\partial V_i} \frac{\partial V_i}{\partial Q} \right) dQ \quad (20)$$

Consider social indifference curves relating combinations of  $Q$  and  $b$  for which  $W$  is constant. Setting  $dW = 0$  gives the slope of indifference curves,  $\left. \frac{db}{dQ} \right|_W$ , as:

$$\left. \frac{db}{dQ} \right|_W = - \frac{\sum_{i=1}^N \frac{\partial W}{\partial V_i} \frac{\partial V_i}{\partial Q}}{\sum_{i=1}^N \frac{\partial W}{\partial V_i} \frac{\partial V_i}{\partial b}} \quad (21)$$

This slope can be expressed more conveniently by defining  $v_i$  as the welfare weight attached to an increase in  $i$ 's income; that is:

$$v_i = \frac{\partial W}{\partial V_i} \frac{\partial V_i}{\partial b} \quad (22)$$

and:

$$\sum_{i=1}^N \frac{\partial W}{\partial V_i} \frac{\partial V_i}{\partial Q} = \sum_{i=1}^N v_i \left( \frac{\partial V_i / \partial Q}{\partial V_i / \partial b} \right) \quad (23)$$

Substituting (22) and (23) into (21) gives:

$$\left. \frac{db}{dQ} \right|_W = - \sum_{i=1}^N v'_i \left( \frac{\partial V_i / \partial Q}{\partial V_i / \partial b} \right) \quad (24)$$

with  $v'_i = v_i / \sum_{i=1}^N v_i$ . The slope of social indifference curves is therefore a weighted sum of the ratio of  $\partial V_i / \partial Q$  to  $\partial V_i / \partial b$ .

Hence differentiating  $V_i = \{y_i(1 - \tau) + b\} Q^\gamma$  gives the result that  $\partial V_i / \partial Q = \gamma V_i / Q$  and  $\partial V_i / \partial b = Q^\gamma$ . Appropriate substitution, and writing  $\tilde{y} = \sum_i^N v'_i y_i$ , gives:

$$\left. \frac{db}{dQ} \right|_W = -\frac{\gamma}{Q} \{ \tilde{y}(1 - \tau) + b \} \quad (25)$$

The government budget constraint in this model takes the simple linear form in (4). Hence:

$$\left. \frac{db}{dQ} \right|_\tau = -\frac{P}{N} \quad (26)$$

Equating slopes, and using  $PQ/N = \tau \bar{y} - b$ , gives the optimal transfer,  $b_W$ , as the solution to:

$$b_W = \frac{\bar{y}}{(1 + \gamma)} \left\{ \tau - \gamma(1 - \tau) \frac{\tilde{y}}{\bar{y}} \right\} \quad (27)$$

This takes the same basic form as equation (7), except that  $b_m$  is replaced by  $b_W$  and the median income,  $y_m$ , is replaced by the welfare-weighted average,  $\tilde{y}$ : the corresponding inequality measure is thus  $I_W = 1 - \tilde{y}/\bar{y}$ . Hence the optimal transfer compares with the solution given in (6) above and the expression for the expenditure ratio takes the same basic form as in equation (8).

However, (27) is actually far from straightforward in view of the complexity of the welfare weights,  $v'_i$ , even for simple forms of the social welfare function. Equation (27) does not in fact provide a closed-form solution for  $b_W$ . As in standard optimal tax analyses, the solution to the nonlinear equation (27) can be obtained numerically using a simulated distribution of incomes and searching for the value of  $b$  which gives the highest  $W$ , while making use of the government budget constraint.

It would clearly be convenient if  $\tilde{y}$  could be replaced by a term that did not itself depend on the optimal value of  $b$ , thereby providing an approximation to the optimal value which could easily be expressed (and, if desired, calculated) using (27). This would facilitate analysis of the comparative-static properties of the model. This problem is examined further in the appendix, which considers the widely used form of social welfare function:

$$W = \sum_{i=1}^N \frac{V_i^{1-\varepsilon}}{1-\varepsilon} \quad (28)$$

for  $\varepsilon > 0$  and  $\varepsilon \neq 1$ .<sup>12</sup> It is shown that a good approximation can be obtained in which the weighted income  $\tilde{y}$  is replaced by the equally-distributed equivalent income,  $y_{ede}$ : this is the income which, if equally distributed, gives the same social welfare as the actual distribution, for a welfare function defined in terms of incomes and taking the same iso-elastic form as above (but with  $y_i$  replacing  $V_i$ ). Thus  $\partial W/\partial y_i = y_i^{-\varepsilon}$  and  $\varepsilon$  measures the absolute value of the elasticity of marginal valuation. In this context  $\varepsilon$  reflects relative inequality aversion, and  $I_W$  is approximated by the Atkinson (1970) inequality measure. Hence optimal values can be examined without the need for simulation analyses of the kind described above, by replacing  $\tilde{y}$  in the above expression by its approximation,  $\tilde{y}_A$ , where  $\tilde{y}_A = y_{ede}$ .

Furthermore, if the distribution of income is lognormal it is shown in the appendix that the following relationship holds:

$$\frac{y_{ede}}{\bar{y}} = \left( \frac{y_m}{\bar{y}} \right)^\varepsilon \quad (29)$$

The optimal expenditure levels and their ratio can thus be expressed in terms of  $y_m/\bar{y}$ , just as in the majority voting framework, except that there is an additional degree of nonlinearity in the expressions, involving the term  $\varepsilon$ . The majority voting outcome and the social welfare maximising outcome are approximately the same in the special case where  $\varepsilon = 1$ . These results suggest that it may be difficult empirically to discriminate between majority voting and optimal allocation frameworks.

One property of the model is that the chosen value of the transfer payment increases with the given tax rate. As a result, the ratio,  $R$ , increases continually with  $\tau$ . However, this arises because labour supply incentive effects are not modelled. This is the major result that is likely to be modified when allowance is made for incentive effects of taxation. An increase in  $\tau$  has, in addition to a ‘tax rate effect’ which causes revenue to increase, a ‘tax base effect’ whereby total income falls. At some point the second effect dominates and total revenue falls as  $\tau$  increases at higher levels. The profiles of  $b$  and of  $R$  against  $\tau$  are thus likely eventually to turn downwards at some

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<sup>12</sup>For  $\varepsilon = 1$ , the term  $V_i^{1-\varepsilon}/(1-\varepsilon)$  is replaced by  $\log V_i$ .



stage.<sup>13</sup>

Returning to the objective, in (10), which applies in the case of stochastic voting, this has in some respects the appearance of a social welfare function expressed as a weighted sum (or weighted arithmetic mean) of indirect utilities. This is a form of utilitarian welfare function, except that the weights do not depend on value judgements of the decision-maker or judge whose views are represented by the social welfare function. The weights are determined by the bias characteristics of the different income groups. A special case of (10) arises if all  $s_k^* = s^*$  for all  $k$ , in which case the objective function to be maximised is exactly the same as the ‘classical utilitarian’ social welfare function.<sup>14</sup> Hence stochastic voting results take similar forms to the utilitarian choices, except that  $\tilde{y}$  is replaced by  $\tilde{\tilde{y}}$ . Basically,  $\tilde{y}$  and  $\tilde{\tilde{y}}$  are just different weighted average incomes. The weight in the stochastic voting depends on the group size and the group densities whereas the weight in the social welfare case depends on the value judgment of the decision-maker.<sup>15</sup>

### 3 Choice of Education and Transfer Payment

Instead of the choice of allocation of tax revenue between a transfer payment and a tax-financed non-rival public good, suppose it is required to choose the combination of a redistributive transfer payment,  $b$ , and a quantity of tax-financed public education per person,  $E$ . The same amount of education is received by each individual, considered to be a private rather than a public good. This involves total spending on education of  $NpE$ , where  $p$  is

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<sup>13</sup>This is confirmed by Creedy and Moslehi (2009) in a cross-sectional study of 24 democratic countries.

<sup>14</sup>The utilitarian form has also been rationalised in terms of a social contract among individuals making a type of ‘constitutional choice’ behind a veil of ignorance, in which all outcomes are treated as being equally likely. This argument was first stated by Edgeworth. A social contract arising from extreme risk aversion leads to the Rawlsian maxi-min evaluation function.

<sup>15</sup>In the case of no bias, where  $\eta_k = 1$ , comparison shows that the voting equilibrium coincides with  $\varepsilon = 0$ ; that is, it is the same as the optimal allocation resulting from a ‘classical utilitarian’ welfare function where only the sum of utilities matters. More generally, covariances are required, which means that further structure needs to be added to the model regarding the joint distribution of  $\eta$  and  $y$ .

the cost per unit of education. The role of education is to increase human capital, reflected in a higher pre-tax income of each individual. The following subsections consider alternative choice mechanisms in turn.

The present approach contrasts with Bearnse *et al.* (2001) who examine transfers and public education, both of which are financed by a proportional income tax, and voting is on the composition of government expenditure for a given tax rate. However, individuals decide whether to use the publicly provided education or buy private education, which leads to double-peaked preferences where the single crossing condition can be violated.<sup>16</sup>

For simplicity, suppose there is no externality of the kind where the education of one person enhances the productivity of other individuals (for example, where teamwork is involved). Nevertheless there is an important effect arising via the tax structure because education raises the tax base. The assumption used in previous sections regarding incentives is retained, namely that labour supplies and hence pre-tax incomes are not influenced by the tax and transfer system. Nevertheless  $y_i$  is endogenous as it is influenced by the choice of  $E$ . It is determined via the following Cobb-Douglas human capital production function:

$$y_i = \delta y_{0,i}^\theta E^{1-\theta} \quad (30)$$

where  $y_{0,i}$  is  $i$ 's 'base' value of income. Rearrangement of (30) gives:

$$\frac{y_i}{y_{0,i}} - 1 = \delta \left( \frac{E}{y_{0,i}} \right)^{1-\theta} - 1 \quad (31)$$

which implies that equal education spending per person produces a proportional increase in income above the base value that is higher for lower-income individuals. In this simple model, education is therefore equalising, though in a different manner from that of the transfer payment. Furthermore, the transfer payment has no effect on the tax base, unlike education expenditure.

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<sup>16</sup>Borck (2008) uses a similar framework to study voters' preferences for centralised versus local public education. Numerical simulations show the group of rich and poor prefer centralisation and the middle class prefers local education. Soares (2006) extends the political economy model of public funded education by using the general equilibrium overlapping generation model with altruism.

The government's budget constraint is:

$$b + pE = \tau \bar{y} \quad (32)$$

where  $\bar{y}$  is arithmetic mean income, given by:

$$\bar{y} = \frac{1}{N} \sum y_i = \delta E^{1-\theta} \left( \frac{1}{N} \sum y_{0,i}^\theta \right) \quad (33)$$

It is convenient to write  $\bar{y}_{0,\theta} = \left( \frac{1}{N} \sum y_{0,i}^\theta \right)^{1/\theta}$ . Although this notation involves the use of a bar above the variable,  $y$ , the  $\theta$  subscript in  $\bar{y}_{0,\theta}$  indicates that it is a generalised mean rather than an arithmetic mean.<sup>17</sup> Hence:

$$\bar{y} = \delta E^{1-\theta} \bar{y}_{0,\theta} \quad (34)$$

Substituting into the budget constraint (32) gives:

$$b = \tau \delta E^{1-\theta} \bar{y}_{0,\theta} - pE \quad (35)$$

Assume that individuals obtain no direct consumption benefits from education. Hence, it is appropriate simply to consider their net income, or consumption, as the maximand. Hence individuals are concerned with:

$$c_i = y_i(1 - \tau) + b \quad (36)$$

## 3.1 Voting Mechanisms

### 3.1.1 Majority Voting

Consider first the choice of expenditure policy by majority voting. Substituting for  $b$  from (35) into (36), consumption can be expressed in terms of  $E$ , whereby:

$$c_i = \delta E^{1-\theta} \{ y_{0,i}^\theta (1 - \tau) + \tau \bar{y}_{0,\theta}^\theta \} - pE \quad (37)$$

It can be seen that  $\partial^2 c_i / \partial E^2 < 0$ , confirming that preferences regarding  $E$  are single peaked. Therefore the per capita majority choice of expenditure

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<sup>17</sup>The generalised mean also contrasts with the 'moment of order  $\theta$  about the origin' which is  $\frac{1}{N} \sum y_{0,i}^\theta$ . Hence the units of the generalised mean correspond to those of the original variable,  $y_{0,i}$ .

on education,  $pE_m$ , is determined by the preferences of the median voter, with  $y_{0,m}$ . Setting  $\partial c_m/\partial E_m = 0$  and solving for  $E_m$  gives:<sup>18</sup>

$$pE_m = (P_E)^{1-\frac{1}{\theta}} \bar{y}_{0,\theta} \{\delta(1-\theta)\}^{\frac{1}{\theta}} \left\{ \tau + (1-\tau) \left( \frac{y_{0,m}}{\bar{y}_{0,\theta}} \right)^\theta \right\}^{\frac{1}{\theta}} \quad (38)$$

Thus, using government budget constraint:

$$b_m = (\tau \delta \bar{y}_{0,\theta}^\theta E_m^{-\theta} - p) E_m \quad (39)$$

The majority choice of the ratio of the transfer payment to education expenditure,  $R_m$ , is:

$$R_m = \frac{1}{1-\theta} \left\{ 1 + \left( \frac{1}{\tau} - 1 \right) \left( \frac{y_{0,m}}{\bar{y}_{0,\theta}} \right)^\theta \right\}^{-1} - 1 \quad (40)$$

and is independent of  $\delta$  and  $p$ . It can be shown that  $\partial R_m/\partial \tau > 0$ , implying that expenditure on transfers increases relative to education, though this result is likely to be different when incentive effects are modelled.

Differentiation of  $R_m$  with respect to  $\theta$  gives  $\frac{\partial R_m}{\partial \theta} > 0$ . Hence a rise in  $\theta$  produces a higher proportion of expenditure devoted to the redistributive transfer payment. Higher  $\theta$  implies that education is less effective in increasing the average level of productivity, so voters vote on lower education and higher transfer payment.

### 3.1.2 Stochastic Voting

As before, suppose the population is divided to  $K$  groups, with the proportion in each group of  $n_k$ , so that  $\sum_{k=1}^K n_k = 1$ . Each party maximises the number of votes and the policies of the two parties converge. The support function is:

$$S = \sum_{k=1}^K \eta_k V_k \quad (41)$$

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<sup>18</sup>If  $U_i = c_i^\alpha$  is considered instead of simply maximising  $c_i$ , indirect utility,  $V_i$ , can be written in terms of  $Q_E$  using (36) and the first-order condition for  $i$ 's most preferred value of  $Q_E$  is  $\frac{\partial V_i}{\partial Q_E} = \frac{\alpha V_i}{c_i} \frac{\partial c_i}{\partial Q_E} = 0$ . The trivial solution where  $V_i = 0$  can be ignored, and the condition for a maximum is, as above,  $\frac{\partial c_i}{\partial Q_E} = 0$ .

where  $\eta_k$  is  $n_k/s_k^*$  and  $V_k$  is the indirect utility of group  $k$ , regarded as a function of policy variables, and is:

$$V_k = \delta y_{0,k}^\theta E^{1-\theta} (1 - \tau) + b \quad (42)$$

The probabilistic voting outcomes are obtained by maximizing the support function subject to the government budget constraint. Therefore:

$$pE_S = (p)^{1-\frac{1}{\theta}} \bar{y}_{0,\theta} \{\delta(1-\theta)\}^{\frac{1}{\theta}} \left\{ \tau + (1-\tau) \frac{\sum_{k=1}^K \eta_k y_{0,k}^\theta}{\bar{y}_{0,\theta}^\theta \sum_{k=1}^K \eta_k} \right\}^{\frac{1}{\theta}} \quad (43)$$

$$b_S = (\tau \delta \bar{y}_{0,\theta}^\theta E_S^{-\theta} - p) E_S \quad (44)$$

Where  $pE_S$  and  $b_S$  are per capita expenditure on public goods and transfer payments at stochastic voting. Also,  $\eta_k$  is equal to  $n_k/s_k^*$  and shows the political influence of the group  $k$ . In this case  $y_{0,m}^\theta$  is replaced by  $\sum_{k=1}^K \eta_k y_{0,k}^\theta / \sum_{k=1}^K \eta_k$ , defined as the weighted generalized mean, where each weight is a function of the political influence. As before, the general properties of the alternative mechanisms are similar, but further progress requires assumptions about the joint distribution of  $\eta_k$  and  $y_{0,k}$ , about which *a priori* information is difficult to specify.

### 3.2 A Social Welfare Function

Consider the maximisation of a social welfare function expressed in terms of the net income of each individual, so that:<sup>19</sup>

$$W = W(c_1, \dots, c_N) \quad (45)$$

Following the same procedure as discussed in the previous section, but with  $E$  replacing  $Q$ , the optimal value corresponds to a tangency solution where the highest social indifference curve touches the government budget constraint. The slope of social indifference curves is given by:

$$\left. \frac{db}{dE} \right|_W = - \sum_{i=1}^N v_i' \left( \frac{\partial c_i / \partial E}{\partial c_i / \partial b} \right) \quad (46)$$

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<sup>19</sup>Hence  $c_i$  replaces  $V_i$  in the previous expression for  $W$ . The use of, say,  $V_i = c_i^\alpha$  makes no difference to the result.

with  $v'_i = v_i / \sum_{i=1}^N v_i$  and  $v_i = \frac{\partial W}{\partial c_i} \frac{\partial c_i}{\partial b}$ . The slope of the budget constraint is:

$$\left. \frac{db}{dE} \right|_{\tau} = (1 - \theta)\tau \delta E^{-\theta} \bar{y}_{0,\theta}^{\theta} - p \quad (47)$$

It can be shown that substitution for  $\partial c_i / \partial E$  and  $\partial c_i / \partial b$ , and solving for  $E_W$  gives the same result as in equation (38), but with  $y_{0,m}$  replaced by  $\tilde{y}_{0,\theta}$ , defined as the weighted generalised mean:

$$\tilde{y}_{0,\theta} = \left( \sum_{i=1}^N v'_i y_{0,i}^{\theta} \right)^{1/\theta} \quad (48)$$

Despite the different type of choice problem, the synthesis obtained earlier carries over. A general property of these types of model is that the expression giving the solution for optimal values takes the same basic form as that for majority voting outcomes, with the difference that the median income is replaced by a welfare-weighted mean income.

Again, the complexities associated with the welfare-weighted term,  $\tilde{y}_{0,\theta}$ , may be overcome by approximating it using the equally distributed equivalent value of basic income,  $y_{0,i}$ , and assuming lognormality of the distribution of  $y_{0,i}$ : this follows the same approach as set out for the previous model, as discussed in the appendix.<sup>20</sup> The result corresponding to (29) is:<sup>21</sup>

$$\frac{\tilde{y}_{0,\theta}}{\bar{y}_{0,\theta}} = \left( \frac{y_{0,m}}{\bar{y}_{0,\theta}} \right)^{\varepsilon} \quad (49)$$

where  $\varepsilon$  is the degree of relative inequality aversion. Comparison with (40) shows that  $\theta$  and  $\varepsilon$  have a similar effect on the variation in  $R$  with  $y_{0,m} / \bar{y}_{0,\theta}$ , as the exponent on the latter is the product,  $\theta\varepsilon$ .

## 4 Conclusions

This paper has investigated the choice of the composition of government expenditure in simple static models, in terms of the outcome of voting and the

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<sup>20</sup>In this case, lognormality also implies that in general the equally distributed equivalent, raised to the power, say  $\eta$ , of a variable  $x$  is the same as the equally distributed equivalent of  $x^\eta$ . And, as noted in the appendix, the equally distributed equivalent for aversion of  $\varepsilon$  is just the generalised mean for a power of  $1 - \varepsilon$ .

<sup>21</sup>Furthermore, if the mean and variance of logarithms of the distribution of  $y_0$  are  $\mu_0$  and  $\sigma_0^2$ , then  $\bar{y}_{0,\theta} = \exp\left(\mu_0 + \frac{\theta\sigma_0^2}{2}\right)$ .

maximisation of a social welfare function which makes the value judgements of a disinterested judge explicit. Choices were considered to be conditional on an exogenously fixed tax rate. The structures examined include a transfer payment combined with a pure public good, and a transfer payment with tax-financed education.

It was shown that, in each case, explicit solutions can be obtained for the majority choice of expenditure components, and these were shown to depend on a measure of inequality defined in terms of the ratio of the median to the arithmetic mean pre-tax income. A higher degree of skewness was found to be associated with a more redistributive expenditure structure (that is, relatively more revenue being devoted to a universal transfer payment or basic income). This corresponds to the property of those models which focus on the determination of the tax rate, rather than the composition of expenditure, in models having only a redistributive transfer payment. With stochastic voting, maximisation of a support function was found to have technical similarities with maximisation of a welfare function, though the interpretation is very different indeed. The stochastic voting framework has the advantage, unlike simple majority voting, that it does not rely on single-peaked preferences.

In the case of maximisation of a social welfare function, the median income of simple majority voting is replaced by a welfare-weighted mean income, with weights depending on the degree of inequality aversion of the judge. Although this does not provide a closed-form solution, it was shown that explicit solutions for the expenditure share can be approximated using an equally distributed income measure in place of the welfare-weighted mean income. The majority choice was consequently found to be approximately the same as the optimal choice in the case where the social welfare function displays constant relative inequality aversion of unity.

When the allocation is between a transfer payment and expenditure on education, similar results were found to apply when comparing alternative choice mechanisms, except that instead of the distribution of income, the relevant distribution was of initial ability.

Despite the simplicity of the models, for example in terms of the spec-

ification of the degree of population heterogeneity and tax structure, it is suggested that they can provide useful insights into the nature of the basic relationships involved, particularly regarding the relationship between expenditure shares and inequality, and the comparisons between alternative choice mechanisms which are seen to hinge on different measures of inequality, all based on the relative difference between arithmetic mean income and a measure of location.



## Appendix: Approximating Welfare-Weighted Income

In considering the optimal composition of government expenditure, welfare-weighted average income,  $\tilde{y}$ , plays a crucial role. The expressions given for optimal values do not strictly provide closed-form solutions, so that numerical solution procedures must be used. This appendix shows that in a wide range of situations it is possible to use an approximation for  $\tilde{y}$  in the case where the social welfare function takes the form:

$$W = \frac{1}{1-\varepsilon} \sum_{i=1}^N V_i^{1-\varepsilon} \quad \varepsilon \neq 1, \varepsilon > 0 \quad (\text{A.1})$$

The aim is to obtain an approximation which allows the earlier results to be treated as closed-form solutions and thus to consider more easily their comparative static properties. The individual utility functions used in Section 2.2 take the form:

$$V_i = \{y_i(1-\tau) + b\} Q^\gamma \quad (\text{A.2})$$

Hence  $\partial V_i / \partial b = Q^\gamma$  and  $\partial W / \partial V_i = V_i^{-\varepsilon}$ , and:

$$v_i = \frac{\partial W}{\partial V_i} \frac{\partial V_i}{\partial b} = \{y_i(1-\tau) + b\}^{-\varepsilon} Q^{\gamma(1-\varepsilon)} \quad (\text{A.3})$$

Suppose  $b$  is small relative to  $y_i$ . In this case an approximation for  $\tilde{y} = \sum y_i (v_i / \sum v_i)$ , denoted  $\tilde{y}_A$ , is obtained as:

$$\tilde{y}_A = \frac{\frac{1}{N} \sum y_i^{1-\varepsilon}}{\frac{1}{N} \sum y_i^{-\varepsilon}} \quad (\text{A.4})$$

This can be further simplified using the assumption that income is lognormally distributed as  $\Lambda(y | \mu, \sigma^2)$ , with mean and variance of logarithms of  $\mu$  and  $\sigma^2$  respectively. Using the properties of the lognormal:

$$\tilde{y}_A = \exp \left[ \mu + \frac{\sigma^2}{2} (1 - 2\varepsilon) \right] = \exp \left[ \mu + \frac{\sigma^2}{2} (1 - \varepsilon) \right] \exp \left[ \frac{-\varepsilon \sigma^2}{2} \right] \quad (\text{A.5})$$

However, the use of the assumption that  $b$  can be neglected in (A.3) attaches too much weight to the lower incomes, and thus imparts a downward bias

to the approximation. One approach is thus to ‘correct’ for this bias by excluding the last term in (A.5). This gives:

$$\tilde{y}_A = \exp \left[ \mu + \frac{\sigma^2}{2} (1 - \varepsilon) \right] \quad (\text{A.6})$$

A feature of this result in (A.6) is that  $\tilde{y}_A$  is closely related to Atkinson’s (1970) measure of inequality. Let  $y_{ede}$  denote the ‘equally distributed equivalent’ income, representing the equal income which gives the same welfare as the actual distribution, using a welfare function of the form  $W = \frac{1}{1-\varepsilon} \sum_{i=1}^N y_i^{1-\varepsilon}$ . This is the same as the above but with  $V$  replaced by  $y$ . Thus:

$$y_{ede} = \left( \frac{1}{N} \sum_{i=1}^N y_i^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \quad (\text{A.7})$$

Again using the lognormal properties, the median and mean income are  $y_m = e^\mu$  and  $\bar{y} = e^{\mu + \sigma^2/2}$ . The term  $y^{1-\varepsilon}$  has mean and variance of logarithms of  $(1 - \varepsilon)\mu$  and  $(1 - \varepsilon)^2 \sigma^2$ , so that:

$$y_{ede} = \exp \left( (1 - \varepsilon) \mu + (1 - \varepsilon)^2 \frac{\sigma^2}{2} \right)^{\frac{1}{1-\varepsilon}} = \exp \left( \mu + (1 - \varepsilon) \frac{\sigma^2}{2} \right) \quad (\text{A.8})$$

Hence:

$$\tilde{y}_A = y_{ede} \quad (\text{A.9})$$

Furthermore:

$$\frac{y_{ede}}{\bar{y}} = \frac{\exp \left( \mu + (1 - \varepsilon) \frac{\sigma^2}{2} \right)}{\exp \left( \mu + \frac{\sigma^2}{2} \right)} = \left( \exp \left( -\frac{\sigma^2}{2} \right) \right)^\varepsilon \quad (\text{A.10})$$

and using the fact that  $y_m = \exp \mu$ :

$$\frac{y_{ede}}{\bar{y}} = \left( \frac{y_m}{\bar{y}} \right)^\varepsilon \quad (\text{A.11})$$

If  $\tilde{y}_A$  is approximated by  $y_{ede}$ , (A.11) gives the relationship between income ratios reported in Section 2.2.

To test the value of this approximation, values of the expenditure components using the approximation  $\tilde{y}_A = y_{ede}$  were compared with those obtained

using a simulated population of size 15000 drawn at random from a lognormal distribution with  $\mu = 9.0$  and  $\sigma^2 = 0.5$ , which imply that  $\bar{y} = 10405$  and  $y_m/\bar{y} = 0.78$ . Using the simulated distribution, a range of values of  $b$  were investigated (the value of  $b$  was increased by 10 each time). For each  $b$  the government budget constraint was used to obtain  $Q$  and the resulting values were used to calculate each individual's level of utility. These were then used to obtain social welfare, using the iso-elastic function with a specified inequality aversion parameter,  $\varepsilon$ . Finally, given a large number of  $W$  measures, the maximum was determined, giving the optimal composition. In order to compare welfare values, the composition obtained from the approximation was used with the simulated population. Table 1 gives the results for a range of parameter values for  $\gamma$  and inequality aversion,  $\varepsilon$ , with  $\tau = 0.35$ .

Table 1: Optimal Composition of Expenditure: Alternative Solutions

$\varepsilon$	Approximation			Simulation			%ΔW
	$b$	$PQ/N$	$R$	$b$	$PQ/N$	$R$	
$\gamma = 0.2$							
0.8	2111.82	1529.77	1.38	2180.1	1461.5	1.49	-0.0049
0.5	2039.95	1601.65	1.27	2090.1	1551.5	1.35	-0.0067
0.2	1962.47	1679.12	1.17	1990.1	1651.5	1.20	-0.0024
$\gamma = 0.5$							
0.8	582.05	3059.53	0.19	810.1	2831.49	0.29	-0.0546
0.5	438.3	3203.29	0.14	620.1	3021.49	0.21	-0.0710
0.2	283.35	3358.24	0.09	380.1	3261.49	0.12	-0.0243

These results show that the approximation gives values of expenditure levels and ratios which are close to those obtained using a large simulated population.<sup>22</sup> The percentage differences, shown in the final column of the table, are in each case found to be small.

<sup>22</sup>Furthermore, comparisons showed that the use of the equally distributed equivalent income was superior to that of (A.5).

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