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Optimal Tax Rules and Addictive Consumption*

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Abstract

This paper studies implementation of the social optimum in a model of habit formation. We consider taxes that address inefficiencies due to negative consumption externalities, imperfect competition, and self-control problems. Our contributions are to: i) account for producers' market power; and ii) require implementation to be robust and time consistent. Together, these features can imply significantly lower taxes. We provide a general characterization of the optimal tax rule and illustrate it with two examples.

Keywords: dynamic externalities, internalities, addiction, optimal taxation, time consistent implementation.

JEL: D11, D43, L13, H21.

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1 Introduction

Addictive goods often give rise to welfare distortions. Inefficiencies may arise from factors such as self-control problems, externalities, and imperfect competition. To implement the social optimum, the government may impose excise taxes. The purpose of our paper is to examine how such taxes might be set according to a state-contingent rule. We provide a general characterization of the efficiency inducing Markovian tax and show that each of the above distortions is represented by a separate component of the optimal tax rate. This decomposition is used to identify tax rules for specific examples. We argue that imperfect competition and time consistent implementation can have mutually reinforcing effects on optimal tax rates. Specifically, these two features together can imply lower tax rates for addictive goods.

The exposition focuses on the market for cigarettes, but our results might also be applied to government intervention in the gambling, fast food, and alcohol industries. Following the pioneering contribution of Becker and Murphy (1988), we model addiction by considering consumer preferences with intertemporal complementarities. This specification implies that current consumption affects future marginal utility. Individual consumers are viewed as price takers: they cannot influence aggregate variables. We also assume that all agents correctly forecast future prices and policies, both on and off the equilibrium path. Our setting departs from the classical habit formation framework by incorporating a number of realistic features that have important welfare and policy implications.

First, we explicitly consider the external costs of addiction (e.g. passive smoking, drunk driving, and crime). We account for them by introducing a negative stock externality: consumer utility is decreasing in the past consumption of other agents. There is abundant empirical evidence for the external costs of addictive goods. For example, Gruber and Köszegi (2001) report estimates of smoking externalities that are between 42 and 72 cents per pack for low birth weight babies, 19 to 70 cents per pack for second hand smoke, and 33 cents per pack for other externalities. Larger values are reported by Sloan, Ostermann,

Picone, Conover and Taylor (2004).

Second, our analysis recognizes that smokers have self-control problems. In the spirit of Phelps and Pollak (1968), Laibson (1997) and O’Donoghue and Rabin (1999), we use quasi-hyperbolic discounting to capture consumption externalities.¹ As a result of present bias, consumers will place too much emphasis on immediate gratification and too little on the subsequent harm from smoking.² Excessive consumption due to such self-control problems has been proposed as an important rationale for government intervention. For example, O’Donoghue and Rabin (2006) examine policies that correct externalities arising from the consumption of “sin goods” such as junk food. This idea is further developed by Gruber and Köszegi (2001), who study how the government can address self-control problems in a setting with addiction.

Third, we account for imperfect competition. The U.S. tobacco industry is heavily concentrated: in 2007, its Herfindahl index was 0.33.³ In principle, producers’ market power may lead to underprovision of the addictive good. Furthermore, the combination of rational expectations and intertemporal complementarities will cause imperfectly competitive firms to experience a different type of time consistency problem which arises even if they discount future profits exponentially. Driskill and McCafferty (2001) study the implications of habit formation for the laissez-faire equilibrium in an oligopolistic industry. They show that if firms are unable to precommit to future policies, their internal conflict will compound the effect of competition and reduce profits. Our analysis incorporates this effect of habit formation on market power and explores its consequences for government intervention.

Finally, we require tax policies to deliver time consistent implementation: no player would wish to deviate from the social optimum in any period, provided that her opponents also

¹The literature on these time preferences is surveyed in DellaVigna (2009).

²Another source of externality that we do not consider here, but is nonetheless plausible in the presence of addictive goods, is beliefs that exhibit projection bias. That is, consumers underestimate the extent to which their future state will differ from their current state. In particular, smokers may underestimate the degree of their future addiction, leading to overconsumption (Loewenstein, O’Donoghue and Rabin 2003).

³The index was computed using data from the economic fact sheet of the Center for Disease and Control (www.cdc.gov).

behave optimally. Specifically, we allow the policy maker to change the tax rate as consumers become more or less addicted. Thus we depart from Gruber and Köszegi (2001), who propose a constant tax rate to address self-control problems in a perfectly competitive setting with quadratic utility.⁴ Unless the industry is in an efficient steady state, such a policy would generally be neither first best nor time consistent. To attain the social optimum, the tax should be set equal to the difference between the private and the social valuations of a marginal change in addiction. This wedge will typically vary over time. Consequently, the social planner will be tempted to renege on past promises and change future tax rates.

Time consistent implementation can be attained with a policy rule that ties taxes to state variables. Such an instrument would allow the government to achieve efficiency robustly by adjusting its policy in response to both anticipated and unanticipated changes in the environment. One possibility is personalized tax rates that depend on the smokers' individual addiction stocks. However, this would be impractical. We follow an alternative approach suggested by Krusell et al (2005), conditioning the tax rate on the average (aggregate) stock. Even though no consumer could individually affect such a policy, oligopolistic firms do take into account the consequences of their decisions for future tax rates.

While our tax proposal attains time consistent implementation, it may fail to deliver the social optimum in some subgames. This tax rule will typically provide efficient incentives to all smokers only if their addiction stocks and preferences are identical. However, we show that our results apply to settings with heterogeneous consumers if payoffs are quadratic or homogeneous of degree one.

Our main contribution is to identify an interaction between time consistent implementation and producers' market power. We argue that these two features have significant and interdependent consequences for the level of corrective taxes. The combination of time consistency and oligopoly implies that the optimal tax rate at the efficient steady state is lower than what would be suggested by previous studies. Imperfect competition leads to higher

⁴A similar model is used by Gruber and Köszegi (2004) to calibrate the incidence of cigarette taxation when individuals belong to different income groups.

prices and hence reduces addictive consumption. Moreover, robust implementation of efficiency necessitates a state-contingent tax rule. One consequence of a Markovian structure for taxes is that rates should generally be higher in states with excessive addiction. The effect of such instruments is amplified by imperfect competition. Markovian taxes give oligopolistic firms an additional incentive to curtail production. An increase in output would not only result in more taxes paid in the current period, but also in higher future tax rates as the government responds to increased addiction stocks.

We obtain an expression for each of the welfare distortions noted above and derive a time consistent tax rule that addresses them all. Our results are illustrated with two examples. The first one assumes payoffs that are homogeneous of degree one. In this special case, the social optimum can be attained with a tax rate that is constant over time. In the second example, we adopt a quadratic specification which is then calibrated to match the U.S. market for cigarettes. Our model fits relatively well with actual data. We are able to separate the quantitative implications of imperfect competition and time consistent implementation for government policy. Finally, our results are compared to the existing literature.

The remainder of the paper is organized as follows. Section 2 specifies the instantaneous utility and time preferences of consumers, industry structure, government intervention and the equilibrium concept. In section 3, we characterize socially optimal consumption. Section 4 describes the decision-making process of consumers and firms; it also defines the laissez-faire equilibrium. In section 5, we derive a Markovian tax policy which implements the efficient feedback rule. We show that this tax can be decomposed into additively separable components that correspond to different welfare distortions. In section 6, we obtain a closed-form solution for a setting with linearly homogeneous utility. Section 7 analyzes a linear quadratic model which is calibrated in section 8 to match key facts about the U.S. tobacco industry. We also evaluate the quantitative effects of time consistent implementation and oligopolistic industry structure. Section 9 concludes the paper.

2 The Model

First we outline a model of an oligopolistic market for addictive goods.

2.1 Consumer Preferences

Consider a representative smoker of mass one who, at time t , purchases cigarettes, x_t , and a numéraire, m_t . The law of motion for her private addiction stock is

$$k_{t+1} = x_t + \theta k_t, \tag{1}$$

where the persistence of k_t is captured by the accumulation rate parameter $\theta \in (0, 1)$. Let K_t denote *aggregate* addiction stock. It evolves according to

$$K_{t+1} = X_t + \theta K_t, \tag{2}$$

where X_t is the aggregate consumption of the addictive good. We consider equilibria in which consumers behave identically: $K_t = k_t$ and $X_t = x_t$. This assumption is relaxed in appendix C. Smokers are price takers: an individual consumer's decisions have no effect on aggregate variables.

The representative consumer derives utility from consumption of both x_t and m_t . The marginal utility of the addictive good depends also on the private addiction stock, k_t . Moreover, passive smoking gives rise to an external cost $\varphi(K_t)$.⁵ Thus, the consumer's instantaneous payoff is defined as

$$u_t = m_t + v(x_t, k_t) - \varphi(K_t). \tag{3}$$

The function v satisfies $v_{x,t} > 0$, $v_{k,t} < 0$ and $v_{xx,t} < 0$, $\forall t$.⁶ Since habits generate com-

⁵In this paper, passive smoking takes the form of a stock externality. That is, instantaneous utility is affected by the aggregate addiction stock K_t . Alternatively, we could model passive smoking as a flow externality that depends on current aggregate consumption X_t . Our results would be qualitatively unchanged.

⁶Our convention for derivatives is as follows. The first subscript (before the comma) denotes the variable of differentiation. When the arguments of the derivative are suppressed, the second subscript (after the

plementarities between past and current consumption, the marginal utility of the addictive good is increasing in k : $v_{xk,t} > 0$. Also, the external cost is increasing in aggregate addiction stock, i.e. $\varphi_{K,t} > 0$.

In each period, the buyer receives a constant flow of income I . Let P_t denote the current consumer price of the addictive good. The price of the numéraire is normalized to one. The instantaneous budget constraint is thus given by

$$m_t + P_t x_t = I. \tag{4}$$

After substituting out the numéraire from (4) into (3) and suppressing I , utility becomes

$$u(x_t, k_t, K_t) = v(x_t, k_t) - \varphi(K_t) - P_t x_t.$$

In addition to the harm of passive smoking, the consumer also experiences a self-control problem. We model internalities by assuming (β, δ) time preferences. From the viewpoint of the period- t smoker, her lifetime utility is

$$U_t = u(x_t, k_t, K_t) + \beta \sum_{s=t+1}^{\infty} \delta^{s-t} u(x_s, k_s, K_s), \tag{5}$$

where $0 < \beta \leq 1$ and $0 < \delta < 1$. To understand her self-control problem, consider how she assesses the effect of period- $t + 1$ consumption on period- $t + 2$ utility. In period t , her discount factor for the trade-off between $t + 1$ and $t + 2$ is δ . However, in period $t + 1$, she will discount her period- $t + 2$ payoff by $\beta\delta$. Thus, if her preferences exhibit present bias (i.e. $\beta < 1$), in each period t she will anticipate excessive smoking in the future.⁷

comma) denotes the time at which they are evaluated.

⁷In the special case when $\beta = 1$ preferences are exponential and the self-control problem disappears.

2.2 Production

Cigarettes are manufactured in a symmetric n -firm oligopoly at a constant unit cost of c . Firms have market power: they account for the effect of their decisions on aggregate variables and prices. The symmetric industry structure allows us to focus on the problem of an arbitrarily chosen producer. Let q_t denote his period- t output level.

The producers' objective is maximization of lifetime profits. For simplicity, we assume that firms discount future profits exponentially. However, the analysis can be extended to include producers with (β, δ) -preferences.

Suppose that firms cannot commit up-front to future production. Thus, they must take into account the effect of their choices on future competition. Furthermore, consumer addiction generates a dynamic demand structure, which creates a strategic conflict between each producer and his future self. This internal conflict arises because period- $t + 1$ output decisions will not take into account their effect on the period- t price.

2.3 Decision Making

Our assumptions imply that both consumers and producers face time consistency problems: a future recalculation of their optimal schedules would drive them away from their previously preferred plans. The literature has considered several approaches to modeling such agents. For example, O'Donoghue and Rabin (1999) distinguish naive from sophisticated decision makers. Our analysis assumes that consumers and firms are sophisticated. That is, they anticipate subsequent temptations to deviate from the currently optimal plan. Also, all agents have rational expectations and correctly predict future market conditions for any K .

We model consumption and production choices as a dynamic game. Decision makers are viewed as sequences of players, each choosing her strategy in a single period. The subgame-perfect equilibrium of this game delivers a time consistent decision profile: given rational expectations, no player will want to deviate in any period (Strotz 1955).

2.4 Government Intervention

Suppose that a benevolent social planner wants to maximize welfare. In a symmetric setting where $k_t = K_t$ and $x_t = nq_t = X_t$, instantaneous welfare can be written as

$$\omega(nq, K) = v(nq, K) - \varphi(K) - cnq. \quad (6)$$

Specifying social time preferences is more controversial, because consumers do not discount their payoffs exponentially. The existing literature usually assumes that the social planner is concerned with the long run, i.e. her discount factor is δ . Thus, we define lifetime welfare as

$$\Omega_t = \sum_{s=t}^{\infty} \delta^{s-t} \omega(nq_s, K_s). \quad (7)$$

In this setting, the social optimum can be attained with a single policy instrument. Suppose that the government levies a per unit tax on the consumption of cigarettes. Tax revenues are given back to consumers as lump-sum transfers.⁸ Government intervention discourages smoking by raising effective consumer prices. Let the period- t tax rate and producer price be τ_t and p_t , respectively. The consumer price is thus $P_t = p_t + \tau_t$.

In each period, the timing is as follows: i) the government announces the tax rate; then ii) the firms and the buyer make their production and consumption decisions, respectively. We consider tax policies that have a Markovian structure. That is, the period- t tax rate is a differentiable function of the current aggregate addiction stock: $\tau_t = \tau(K_t)$. As explained in the introduction, such a tax rule allows for robust and time consistent implementation of efficiency. An individual price-taking consumer cannot affect K , and so has no influence over government policies. Firms, on the other hand, take into account the effects of their output decisions on future tax rates.

⁸This assumption is restrictive. Parry et al. (2006) note that the last two increases in U.S. federal alcohol taxes were part of a deficit reduction package.

3 The Socially Optimal Plan

First, let us characterize the output policy which maximizes lifetime welfare (7). In the remainder of the paper, we use hats on variables to denote efficiency. The socially optimal production of a representative firm solves:

$$\hat{\Omega}(K_t) = \max_{q_t \in \mathbb{R}_+} \{v(nq_t, K_t) - \varphi(K_t) - cnq_t + \delta\hat{\Omega}(nq_t + \theta K_t)\}, \quad (8)$$

where $\hat{\Omega}$ is the government's value function. Bellman equation (8) defines an efficient feedback rule $q_t = \hat{f}(K_t)$:

$$\hat{f}(K_t) = \arg \max_{q_t \in \mathbb{R}_+} \{v(nq_t, K_t) - \varphi(K_t) - cnq_t + \delta\hat{\Omega}(nq_t + \theta K_t)\}. \quad (9)$$

Standard dynamic programming arguments guarantee the existence and uniqueness of \hat{f} .

Definition 1 *Efficiency is characterized by i) a value function $\hat{\Omega}$ which solves (8); and ii) an output policy $q_t = \hat{f}(K_t)$ defined by (9).*

Bellman equation (8) yields the following efficiency Euler equation:

$$v_{x,t} - c = \delta\theta(v_{x,t+1} - c) - \delta(v_{K,t+1} - \varphi_{K,t+1}). \quad (10)$$

This condition has a standard interpretation: it compares the current net social benefits of a marginal increase in a firm's output with the corresponding future net social effects.

4 Individually Optimal Decisions

In this section, we analyze the market for the addictive good.

4.1 Consumption Decisions

To obtain the demand for cigarettes, we now examine the Markov-perfect equilibrium (MPE) of the consumer's intrapersonal game. We restrict the analysis to strategies that are differentiable functions of her current state $z_t = (k_t, K_t, P_t)$. Private and aggregate addiction stocks k_t and K_t evolve according to (1) and (2). In the spirit of Driskill and McCafferty (2001), we conjecture that the predicted future price depends only on the aggregate addiction stock at that time: $P_{t+1} = \Gamma(K_{t+1})$. This assumption is consistent with price-taking behavior: the consumer does not expect to influence K_{t+1} . The function $\Gamma(\cdot)$ will be pinned down by rational expectations.⁹

To characterize the Markov-perfect equilibrium, we use dynamic programming techniques similar to those in Harris and Laibson (2001). Let V and W be the consumer's current and continuation value functions. Suppose that her MPE consumption strategy is $x_t = h(z_t)$. Total consumption X_t of the addictive good is thus given by $g(K_t) = h(K_t, K_t, \Gamma(K_t))$. The strategy h solves the Bellman equation

$$V(z_t) = \max_{x_t \in \mathbb{R}_+} \{v(x_t, k_t) - \varphi(K_t) - P_t x_t + \beta \delta W(x_t + \theta k_t, g(K_t) + \theta K_t, \Gamma(g(K_t) + \theta K_t))\}. \quad (11)$$

Optimality requires that

$$h(z_t) = \arg \max_{x_t \in \mathbb{R}_+} \{v(x_t, k_t) - \varphi(K_t) - P_t x_t + \beta \delta W(x_t + \theta k_t, g(K_t) + \theta K_t, \Gamma(g(K_t) + \theta K_t))\}. \quad (12)$$

Moreover, the smoker's period- t self discounts future utility exponentially from period- $t + 1$ onward. Thus, the continuation value function W must also solve the recursive equation

$$W(z_t) = v(h(z_t), k_t) - \varphi(K_t) - P_t h(z_t) + \delta W(h(z_t) + \theta k_t, g(K_t) + \theta K_t, \Gamma(g(K_t) + \theta K_t)). \quad (13)$$

⁹Even though rational expectations imply $P_t = \Gamma(K_t)$, we need to treat P_t as a state variable in the consumer's game. This enables us to characterize consumption for any P_t and thus obtain demand.

Definition 2 *The MPE of the consumer's intrapersonal game is characterized by i) a current value function V and a continuation value function W which solve equations (11) and (13); and ii) a consumption strategy function $x_t = h(z_t)$ which is a fixed point of the mapping defined by (12).*

In a similar setting, Judd (2003) proves existence and uniqueness of MPE for β sufficiently close to one. However, lower values of this parameter may lead to multiplicity or non-existence of equilibria.

Dynamic programming enables us to obtain a generalized Euler equation that describes the intertemporal choice of the consumer:

$$v_{x,t} - P_t = \delta[\theta + (1 - \beta)h_{k,t}](v_{x,t+1} - \Gamma(K_{t+1})) - \beta\delta v_{k,t+1}. \quad (14)$$

Equation (14) captures the direct and the intrapersonal strategic effects of a marginal increase in current consumption. It defines a relationship between the current market price and individually optimal consumption. Since P_t is a state variable, this relationship will hold also for prices off the equilibrium path. Thus, we can use (14) to derive industry demand.

The assumption of a representative smoker implies that $x_t = X_t$ and $k_t = K_t$. Given a tax rule $\tau(K)$ and a price prediction function $\Gamma(K)$, an adjusted accumulation rate for a sophisticated consumer can be written as

$$\sigma_t = \sigma(K_t) = \theta + (1 - \beta)h_k(K_t, K_t, \Gamma(K_t)).$$

Substituting x_t and k_t into (14) yields the following equilibrium condition:

$$v_x(X_t, K_t) - \Gamma(K_t) = \delta\sigma(K_{t+1})[v_x(X_{t+1}, K_{t+1}) - \Gamma(K_{t+1})] - \beta\delta v_k(X_{t+1}, K_{t+1}), \quad (15)$$

where

$$X_t = g(K_t), \quad K_{t+1} = g(K_t) + \theta K_t, \quad X_{t+1} = g(g(K_t) + \theta K_t).$$

The left-hand side of (15) represents the net benefit of consuming an extra cigarette today. The first term on the right-hand side comprises the discounted value of future net benefits induced by a marginal increase in current consumption, corrected for the intraper-sonal strategic effect with the adjusted accumulation rate, $\sigma(K_{t+1})$. The second term on the right-hand side reflects the direct impact of higher addiction stock tomorrow, again modified to account for present bias. Note that $\varphi(K)$ is absent from (15); without a corrective tax, each consumer will ignore the external cost of passive smoking that she imposes on others.

4.2 Production Decisions

Next, we study oligopolistic provision of the addictive good. Suppose that the government specifies a tax rule $\tau(K)$. Let Q_t denote industry output. Aggregation yields $x_t = X_t = Q_t$ and $k_t = K_t$. Rearranging the smoker's generalized Euler equation (14) delivers the period- t inverse industry demand for the addictive good:

$$P_t = v_x(Q_t, K_t) + \beta\delta v_k(Q_{t+1}, K_{t+1}) - \delta\sigma(K_{t+1})[v_x(Q_{t+1}, K_{t+1}) - \Gamma(K_{t+1})]. \quad (16)$$

As (16) demonstrates, P_t depends also on anticipated future prices and policies. In a setting with rational expectations, this will give rise to a time consistency problem for oligopolistic firms. Since Q_{t+1} is determined in period $t + 1$, future producers will not internalize the consequences of their decisions for period- t payoffs. Driskill and McCafferty (2001) show that the resulting internal conflict will reduce firms' market power.

Suppose that, for any K_t , agents correctly infer future consumption and addiction stocks. Then the producers' price p_t can be determined from a rational expectations inverse demand function:

$$p_t = P(Q_t, K_t) - \tau(K_t),$$

where $P(Q_t, K_t)$ is derived from the right-hand side of (16) by setting $K_{t+1} = \theta K_t + X_t$ and $X_{t+1} = g(\theta K_t + X_t)$.

We now focus on the Markov-perfect equilibrium of the producers' game: their period- t output strategies are assumed to be differentiable functions of the aggregate addiction stock K_t . Consider the problem of an arbitrarily chosen producer. His instantaneous profit is

$$\pi_t = [P(Q_t, K_t) - \tau(K_t) - c]q_t.$$

Let his MPE output strategy be $q_t = f(K_t)$. Since firms are identical, this strategy solves the Bellman equation

$$\Pi(K_t) = \max_{q_t \in \mathbb{R}_+} \{[P(q_t + (n-1)f(K_t), K_t) - \tau(K_t) - c]q_t + \delta\Pi(q_t + (n-1)f(K_t) + \theta K_t)\}, \quad (17)$$

where

$$f(K_t) = \arg \max_{q_t \in \mathbb{R}_+} \{[P(q_t + (n-1)f(K_t), K_t) - \tau(K_t) - c]q_t + \delta\Pi(q_t + (n-1)f(K_t) + \theta K_t)\}. \quad (18)$$

Finally, we require that consumers correctly forecast future prices off as well as on the equilibrium path. Thus:

$$\Gamma(K) = P(nf(K), K), \quad \forall K. \quad (19)$$

Definition 3 *The MPE of the producers' game is characterized by i) a value function Π which solves equation (17); ii) an output strategy function f which is a fixed point of the mapping defined by (18); and iii) a price prediction function Γ which satisfies (19).*

From Bellman equation (17) we can derive a generalized Euler equation that describes the intertemporal trade-off of a representative oligopolist:

$$P_{Q,t}q_t + P_t - \tau_t - c = \delta(P_{Q,t+1}q_{t+1} + P_{t+1} - \tau_{t+1} - c)\rho_{t+1} - \delta[(\rho_{t+1} - \theta)P_{Q,t+1} + P_{K,t+1} - \tau_{K,t+1}]q_{t+1}, \quad (20)$$

where

$$\rho_t = \rho(K_t) = (n-1)f_K(K_t) + \theta$$

is the effect of a marginal change in K_t on the future aggregate addiction stock K_{t+1} . The left-hand side of (20) is the increment in current profit from an extra unit of output. The right-hand side captures the present discounted value of future marginal profits corrected for the external and the internal strategic effects. The derivatives of the rational expectations inverse demand function $P(Q, K)$ can be obtained by differentiating the right-hand side of (16) after imposing $K_{t+1} = \theta K_t + X_t$ and $X_{t+1} = g(\theta K_t + X_t)$:

$$P_{Q,t} = v_{xx,t} + \beta \delta (g_{K,t+1} v_{xk,t+1} + v_{kk,t+1}) + \\ \delta (g_{K,t+1} v_{xx,t+1} + v_{xk,t+1} - \Gamma_{K,t+1}) \sigma_{t+1} + \delta (v_{x,t+1} - \Gamma_{t+1}) \sigma_{K,t+1}$$

and

$$P_{K,t} = v_{xk,t} + \theta \{ \beta \delta (g_{K,t+1} v_{xk,t+1} + v_{kk,t+1}) + \\ \delta (h_{K,t+1} v_{xx,t+1} + v_{xk,t+1} - \Gamma_{K,t+1}) \sigma_{t+1} + \delta (v_{x,t+1} - \Gamma_{t+1}) \sigma_{K,t+1} \}.$$

4.3 Laissez-Faire Equilibrium

As a benchmark, consider the equilibrium in an industry that is free of government intervention: $\tau(K) \equiv 0$.

Definition 4 *The laissez-faire industry equilibrium is characterized by i) a strategy function $\tilde{q}_t = \tilde{g}(K_t)/n = \tilde{f}(K_t)$ that solves the producer's generalized Euler equation (20) and; ii) a price prediction function $\tilde{\Gamma}(K_t)$ that satisfies the rational expectations condition (19), where $\tau(K) \equiv 0$.*

The laissez-faire equilibrium gives rise to three sources of inefficiency. First, the negative externality of passive smoking causes the buyer to consume too much relative to the social optimum. Second, the buyer's self-control problem implies that her subsequent selves will smoke excessively as assessed with her current preferences over future behavior. Third,

producers' market power will cause a distortion in the opposite direction, potentially leading to underprovision of cigarettes.

5 Implementation of Efficiency

In this section, we derive the optimal tax rule $\tau(K)$ which implements the efficient output policy as a solution to the producer's generalized Euler equation: $f(K_t) \equiv \hat{f}(K_t)$. This instrument allows the government to attain efficiency robustly in a non-cooperative time consistent equilibrium. However, it may also generate other equilibria that are socially suboptimal (Akao 2008).

Consider MPE in which the efficient policy rule, \hat{f} , solves the producer's equilibrium condition (20). In order to keep the notation simple, let $\bar{\rho}_r = 1$ if $r = t$ and $\bar{\rho}_r = \hat{\rho}_r = (n-1)\hat{f}_K + \theta$ if $r > t$. Similarly, let $\bar{\sigma}_r = 1$ if $r = t$ and $\bar{\sigma}_r = \hat{\sigma}_r = (1-\beta)\hat{h}_K + \theta$ if $r > t$. Suppose that $v_x(n\hat{f}(K), K)$, $v_k(n\hat{f}(K), K)$ and the right-hand side of (20) are bounded, and that $0 < \hat{\sigma}_t < 1$, $0 < \hat{\rho}_t < 1$.

Proposition 1 *The optimal time-consistent tax rule satisfies*

$$\tau_t + \sum_{s=1}^{\infty} \delta^s \left[\tau_{K,t+s} \frac{\hat{X}_{t+s}}{n} \prod_{r=0}^{s-1} \bar{\rho}_{t+r} \right] = d_t^1 + d_t^2 + d_t^3, \quad (21)$$

where d_t^1, d_t^2, d_t^3 are defined as

$$d_t^1 = \delta \sum_{s=1}^{\infty} (\delta\theta)^{s-1} \varphi_{K,t+s}, \quad (22)$$

$$d_t^2 = \beta \sum_{s=1}^{\infty} \delta^s \left[v_{k,t+s} \prod_{r=0}^{s-1} \bar{\sigma}_{t+r} \right] - \delta \sum_{s=t}^{\infty} (\delta\theta)^{s-1} v_{k,t+s}, \quad (23)$$

$$d_t^3 = \hat{P}_{Q,t} \frac{\hat{X}_t}{n} + \sum_{s=1}^{\infty} \delta^s \left[[(\hat{\rho}_{t+s} - \theta) \hat{P}_{Q,t+s} + \hat{P}_{K,t+s}] \frac{\hat{X}_{t+s}}{n} \prod_{r=0}^{s-1} \bar{\rho}_{t+r} \right]. \quad (24)$$

Proof. See appendix A. ■

The left-hand side of (21) represents the current and future tax obligations due to a marginal increase in the current output of a given firm. The right-hand side comprises three components, representing the distortions due to externalities, present-biased preferences and imperfect competition, respectively.

The first component, d_t^1 , reflects externalities from smoking. It is equal to the social valuation of the external cost due to a marginal increase in aggregate addiction stock. This term is analogous to expressions obtained in the environmental economics literature.

The second component, d_t^2 , accounts for consumption internalities. It is the difference between the private and the social valuations of a marginal increase in private addiction. While the period- t consumer discounts the period- $t + s$ harm from addiction by $\beta\delta^s$, the social discount factor for that harm is δ^s . Moreover, the consumer's rate of accumulation is adjusted to account for intrapersonal strategic effects. This component is zero when consumer preferences are exponential ($\beta = 1$).

The third component, d_t^3 , captures distortions caused by market power. It also accounts for the welfare consequences of the producers' time consistency problem. Since $\hat{P}_Q < 0$, this term is non-positive. If the number of firms is sufficiently low, d_t^3 may offset the other distortions, perhaps even implying a subsidy rather than a tax.

In sections 6 and 7, we present two examples which yield consumption policies that are linear in addiction stock. This linearity suggests that the adjusted accumulation rates for consumers and producers are constant over time: $\hat{\sigma}_t = \hat{\sigma}$ and $\hat{\rho}_t = \hat{\rho}$. Therefore, the components of the efficient tax rule will take the following forms:

$$d_t^1 = \delta \sum_{s=1}^{\infty} (\delta\theta)^{s-1} \varphi_{K,t+s}, \quad (25)$$

$$d_t^2 = \beta\delta \sum_{s=1}^{\infty} (\delta\hat{\sigma})^{s-1} v_{k,t+s} - \delta \sum_{s=1}^{\infty} (\delta\theta)^{s-1} v_{k,t+s}, \quad (26)$$

$$d_t^3 = \hat{P}_{Q,t} \frac{\hat{X}_t}{n} + \delta \sum_{s=1}^{\infty} (\delta\hat{\rho})^{s-1} [(\hat{\rho} - \theta)\hat{P}_{Q,t+s} + \hat{P}_{K,t+s}] \frac{\hat{X}_{t+s}}{n}. \quad (27)$$

In the limit case of a perfectly competitive industry (i.e. $n \rightarrow \infty$), we would have

$$d_t^3 = 0, \quad \sum_{s=1}^{\infty} \delta^s \left[\tau_{K,t+s} \frac{\hat{X}_{t+s}}{n} \prod_{r=0}^{s-1} \bar{\rho}_{t+r} \right] = 0.$$

With perfect competition, individual firms cannot influence prices with output, either directly or by contributing to the addiction stock. Thus, the tax rule becomes $\tau(K_t) = d_t^1 + d_t^2$. Alternatively, condition (40) suggests that this tax policy can also be written as $\tau(K) = \hat{\Gamma}(K) - c$. That is, the corrective tax should be set equal to the difference between the efficient consumer price and marginal cost.

6 Homogeneous Payoffs

In this section, we derive closed-form results for utility functions that are homogeneous of degree one. This setting allows time consistent implementation with the policy instrument considered by Gruber and Köszegi (2001), i.e. a tax that is constant over time. We show that optimal prices and taxes are independent of K . Therefore, our results will hold even if consumers differ in their addiction stocks.

Suppose that $v(x, k)$ exhibits the following property:¹⁰

$$v(\alpha x, \alpha k) = \alpha v(x, k), \forall \alpha > 0.$$

We also assume that the external harm from passive smoking is proportional to the aggregate addiction stock: $\varphi(K) = \xi K$. This class of utility functions has derivatives, $v_x(x, k)$ and $v_k(x, k)$, that are homogeneous of degree zero, and implies an efficient consumption policy that is linear in K :

$$X_t = \hat{g}_K K_t.$$

¹⁰An example of a functional form that satisfies this requirement is $v(x, k) = (a_1 x^\phi + a_2 k^\phi)^{1/\phi} - \zeta k$, where ζ is sufficiently large to ensure that $v_k < 0$ for the relevant values of x and k .

Thus, along the optimal path, marginal utilities will be constant:

$$v_x(\hat{g}_K K, K) = v_x(\hat{g}_K, 1), \quad v_k(\hat{g}_K K, K) = v_k(\hat{g}_K, 1). \quad (28)$$

Appendix B1 provides a condition that determines the policy parameter \hat{g}_K . It also derives an expression for the rational expectations demand and shows that $\hat{P}_{Q,t}$, $\hat{P}_{K,t}$ are given by

$$\hat{P}_{Q,t} = -\frac{v_{xk}(\hat{g}_K, 1)}{\hat{g}_K K_t}, \quad \hat{P}_{K,t} = \frac{v_{xk}(\hat{g}_K, 1)}{K_t}. \quad (29)$$

Since marginal utilities and marginal profits do not change over time, efficient consumer and producer prices are constant. Thus, the social optimum can be attained with tax rates that are independent of the addiction stock: $\tau(K_t) \equiv \tau$. The decomposition derived in section 5 implies that the optimal tax rate τ can be written as $\tau = d^1 + d^2 + d^3$.

- Component d^1 internalizes the external cost of passive smoking. In this example, condition (25) takes the following form:

$$d^1 = \frac{\delta \xi}{1 - \delta \theta}.$$

- Component d^2 reflects distortions due to present bias of consumer preferences. Our payoff specification implies that $\hat{\sigma} = \theta + \hat{g}_K(1 - \beta)$. Thus, condition (26) delivers

$$d^2 = v_k(\hat{g}_K, 1) \left(\frac{\beta \delta}{1 - \delta[\theta + \hat{g}_K(1 - \beta)]} - \frac{\delta}{1 - \delta \theta} \right).$$

- Component d^3 corrects inefficiencies caused by imperfect competition. On the optimal path, we have $\hat{\rho} = (n - 1)\hat{g}_K/n$ and $\hat{X}_t = \hat{g}_K K_t/n$. Substituting the expressions for the derivatives of the inverse demand function (29) into (27) yields

$$d^3 = - \left(\frac{1 - \delta(\hat{g}_K + \theta)}{n[1 - \delta(\hat{g}_K + \theta)] + \delta \hat{g}_K} \right) v_{xk}(\hat{g}_K, 1).$$

The resulting tax rule is summarized in proposition 2.

Proposition 2 *When payoffs are linearly homogeneous as described in section 6, the optimal time-consistent tax rule is given by*

$$\tau = \frac{\delta\xi}{1 - \delta\theta} + \left(\frac{\beta\delta v_k(\hat{g}_K, 1)}{1 - \delta[\theta + \hat{g}_K(1 - \beta)]} - \frac{\delta v_k(\hat{g}_K, 1)}{1 - \delta\theta} \right) - \left(\frac{[1 - \delta(\hat{g}_K + \theta)]v_{xk}(\hat{g}_K, 1)}{n[1 - \delta(\hat{g}_K + \theta)] + \delta\hat{g}_K} \right).$$

Proof. See appendix B1 and the above analysis. ■

7 Quadratic Payoffs

When marginal utilities vary over time, the social planner will want to revise the tax rate as K changes. We now adopt a linear-quadratic payoff structure which exhibits this property. To attain robust and time consistent implementation, the government could follow a policy rule that is contingent on the aggregate addiction stock: $\tau_t = \tau(K_t)$.

Suppose that $v(x, k)$ takes the following form:

$$v(x, k) = b_x x - \frac{b_{xx}}{2}(x)^2 - b_k k - \frac{b_{kk}}{2}(k)^2 + b_{xk} x k, \quad (30)$$

where

$$b_x, b_{xx}, b_k, b_{kk}, b_{xk} > 0, \quad b_{xx}b_{kk} - (b_{xk})^2 > 0.$$

In addition, each consumer bears an external cost $\varphi(K) = \xi K$ from passive smoking. This specification yields a linear inverse industry demand and quadratic profits. Therefore, the social optimum will involve a linear price prediction function and consumption strategies:

$$\hat{\Gamma}(K) = \hat{\Gamma}_0 + \hat{\Gamma}_K K, \quad h(k, K, P) = h_0 + h_k k + h_K K + h_P P, \quad g(K) = n f(K) = \hat{g}_0 + \hat{g}_K K.$$

On the efficient path, the marginal utilities $v_{x,t}$ and $v_{k,t}$ are linear in the aggregate ad-

diction stock. They can be written as $v_{x,t} = \hat{\lambda}_0 + \hat{\lambda}_K K_t$ and $v_{k,t} = \hat{\mu}_0 + \hat{\mu}_K K_t$, where

$$\hat{\lambda}_0 = b_x - b_{xx}g_0, \quad \hat{\lambda}_K = -b_{xx}\hat{g}_K + b_{xk} \quad (31)$$

$$\hat{\mu}_0 = -b_k + b_{xk}\hat{g}_0, \quad \hat{\mu}_K = -b_{kk} + b_{xk}\hat{g}_K. \quad (32)$$

The addiction stock evolves according to $K_{t+1} = \hat{g}_0 + \hat{\gamma}_K K_t$, where $\hat{\gamma}_K = \theta + \hat{g}_K$. We use forward iteration to obtain an expression for future addiction stocks K_{t+s} in terms of K_t :

$$K_{t+s} = \hat{g}_0 \frac{1 - (\hat{\gamma}_K)^s}{1 - \hat{\gamma}_K} + (\hat{\gamma}_K)^s K_t. \quad (33)$$

Appendix B2 provides equations for the optimal policy parameters \hat{g}_0, \hat{g}_K . It also shows that the derivatives of the rational expectations demand are given by

$$\hat{P}_Q = \frac{\beta\delta\hat{\mu}_K}{1 - \delta\hat{\sigma}\hat{\gamma}_K} - b_{xx}, \quad \hat{P}_K = b_{xk} + \theta b_{xx} + \theta \left[\frac{\beta\delta\hat{\mu}_K}{1 - \delta\hat{\sigma}\hat{\gamma}_K} - b_{xx} \right]. \quad (34)$$

By definition, the good is addictive if past consumption reinforces current consumption while holding prices fixed. Thus, we also require that the parameters satisfy $\hat{P}_K > 0$.

Next, we show that efficiency can be implemented with a linear tax rule $\tau(K) = \tau_0 + \tau_K K$.

To determine τ_0 and τ_K , we use the decomposition described in section 5.

- Since the marginal harm of the externality is constant, the term that represents this welfare cost is the same as with the homogeneous utility example.

$$d^1 = \frac{\delta\xi}{1 - \delta\theta}.$$

- The component that accounts for present bias, d_t^2 , is equal to the discrepancy between the private and the social valuations of a marginal increase in private addiction. These valuations take the form of discounted sums of the marginal disutility of addiction along the optimal path, $v_{k,t+s} = \hat{\mu}_0 + \hat{\mu}_K K_{t+s}$. Substituting out K_{t+s} with (33) allows

us to express these valuations in terms of K_t . Their difference is:

$$d_t^2 = \left(\hat{\mu}_0 + \frac{\hat{\mu}_K \hat{g}_0}{1 - \hat{\gamma}_K} \right) \left(\frac{\delta \beta}{1 - \delta \hat{\sigma}} - \frac{\delta}{1 - \delta \theta} \right) + \left(K_t - \frac{\hat{g}_0}{1 - \hat{\gamma}_K} \right) \left(\frac{\delta \beta \hat{\mu}_K \hat{\gamma}_K}{1 - \delta \hat{\sigma} \hat{\gamma}_K} - \frac{\delta \hat{\mu}_K \hat{\gamma}_K}{1 - \delta \theta \hat{\gamma}_K} \right).$$

- The third component also depends on the aggregate addiction stock because optimal output changes over time. Imposing the efficient consumption policy, $\hat{X}_{t+s} = \hat{g}_0 + \hat{g}_K K_{t+s}$, on (23) and substituting out K_{t+s} with (33), we obtain:

$$d_t^3 = \frac{\hat{g}_0 + \hat{g}_K K_t}{n} \hat{P}_Q + \frac{(\hat{\rho} - \theta) \hat{P}_Q + \hat{P}_K}{n} \left[\frac{\hat{g}_0}{1 - \delta \hat{\rho}} \left(1 + \frac{\hat{g}_K}{1 - \delta \hat{\rho} \hat{\gamma}_K} \right) + \frac{\hat{g}_K \hat{\gamma}_K K_t}{(1 - \delta \hat{\rho} \hat{\gamma}_K)} \right].$$

Let the total distortion be $D_t = d^1 + d_t^2 + d_t^3$. Since all of its components are linear in K_t , it can also be written as $D_t = D_0 + D_K K_t$. Therefore, equation (21) becomes:

$$\tau_0 + \tau_K K + \frac{\delta}{n} \left[\frac{\hat{g}_0}{1 - \delta \hat{\rho}} \left(1 + \frac{\hat{g}_K}{1 - \delta \hat{\rho} \hat{\gamma}_K} \right) + \frac{\hat{g}_K \hat{\gamma}_K K}{(1 - \delta \hat{\rho} \hat{\gamma}_K)} \right] \tau_k = D_0 + D_K K. \quad (35)$$

Applying the method of undetermined coefficients to (35) delivers the next result.

Proposition 3 *When payoffs are quadratic as described in section 7, the optimal time-consistent tax rule is given by $\tau(K) = \tau_0 + \tau_K K$, where*

$$\tau_K = \frac{n(1 - \delta \hat{\rho} \hat{\gamma}_K) D_K}{n(1 - \delta \hat{\rho} \hat{\gamma}_K) + \delta \hat{g}_K \hat{\gamma}_K}, \quad \tau_0 = D_0 - \frac{\delta}{n} \frac{\hat{g}_0}{1 - \delta \hat{\rho}} \left(1 + \frac{\hat{g}_K}{1 - \delta \hat{\rho} \hat{\gamma}_K} \right) \tau_K.$$

Proof. See appendix B2 and the above analysis. ■

In appendix C, we also show that this tax rule can be applied to a quadratic setting with heterogeneous addiction stocks.

8 Numerical Exercise

In our model, four factors contribute to the optimal tax rule: time consistent implementation, externalities, internalities and imperfect competition. In proposition 1 we were able to disentangle three of those elements as independent components of the optimal tax rate. In this section, we use a numerical example to quantitatively assess the contribution of each of these four factors and to gauge their importance.

8.1 Baseline Calibration

We calibrate our quadratic model to match some stylized facts about the U.S. tobacco industry. The parameters used in our numerical example are presented in Table 1. The economic fact sheet from the Center for Disease and Control reports that, in 2009, three companies accounted for 85 percent of all sales in the U.S. To reflect this high degree of concentration, we choose $n = 3$. Since the operating costs of tobacco producers are usually quite low, the unit cost c is set to 0. Following the existing literature, we assume a discount factor $\delta = 0.96$, while the present bias parameter β is set to 0.65.¹¹ Furthermore, we adopt the view of Gruber and Köszegi that the external cost of smoking is relatively small; we assign ξ a value of 1.5. The coefficients b_x , b_k , b_{xx} , b_{kk} and b_{xk} of the instantaneous utility function are chosen to match empirical observations of cigarette consumption, prices and taxes, as well as estimations of the short-run and the long-run elasticities of demand.

Rather than drawing on recent data, we use a summary of time series of state cross sections by Becker, Grossman and Murphy (1994) which covers the period from 1955 to 1985. The long time horizon of this study is appropriate, given that we use the steady state as a reference point. Becker et al. (1994) report a per capita mean consumption of 126 packs per year. In 1967 cents, the mean retail price and tax per pack are 29.8 and 6.68, respectively. To reproduce econometric estimations of the price elasticities of demand, we

¹¹Laibson, Repetto, Tobacman, Center and Room (2007) analyze a structural model of lifecycle consumption and find that $\delta = 0.96$ and $\beta = 0.7$. Paserman (forthcoming) uses data on unemployment spells and accepted wages from the NLSY to estimate a job search model and finds that $\delta = 0.99$ and $\beta \in (0.4, 0.89)$.

refer to a meta-analysis of the empirical literature provided by Gallet and List (2003). It reports a short-run elasticity in the region of -0.4 and a long-run elasticity of around -0.6.

We use the parameters from Table 1 to obtain the steady-state equilibrium levels of output, addiction stock and prices in the setting with quadratic payoffs. The results for our baseline scenario are reported in Table 2. They are consistent with the basic empirical observations outlined in Becker et al. (1994). Table 3 shows how this equilibrium would be affected if we changed one parameter at a time.

Our model yields a steady state with a consumption of 102.41 cigarette packs per year and a price of $\hat{P} = 25.25$. The steady-state addiction stock is $\hat{K} = 204.82$. The corresponding laissez-faire benchmarks are $\tilde{X} = 117.62$, $\tilde{K} = 235.24$ and $\tilde{P} = 18.80$. Industry demand is:

$$X_t = 32.71 - 1.69P_t + 0.88P_{t+1} + 0.48X_{t+1} + 0.09K_t. \quad (36)$$

It implies a short-run elasticity of -0.42 and a long-run elasticity of -0.58. These values are close to the estimates of Gallet and List (2003). Imposing rational expectations on (36) yields $P(X, K) = 66.99 - 0.55X + 0.07K$. The efficient consumption policy is $g(K) = 99.2 + 0.016K$.

When $n = 3$ efficient consumption can be implemented with the following tax rule:

$$\hat{\tau}(K; 3) = -5.891 + 0.062K. \quad (37)$$

It corresponds to a steady-state tax of 6.71 cents per pack in 1967 prices (43 cents in 2009 prices). This number is similar to the statistic in Becker et al. (1994), but lower than current federal tax on cigarettes.¹²

¹²We do not believe that the tobacco industry is currently in a steady state. CDC data shows that per capita cigarette consumption has been declining steadily since 1980 (<http://www.cdc.gov>).

8.2 Discussion

In this section we investigate how taxes are affected by the requirement for time consistent implementation and by oligopolistic industry structure. We also evaluate the externality and internality components of the optimal tax rate.

8.2.1 Time Consistent Implementation

First we explore the impact of the requirement for time consistent implementation. We show how it can make a non-negligible difference to the implied tax rate. To compare our approach to that of Gruber and Köszegi (2001), we begin by assuming a perfectly competitive industry, i.e. $p_t = c, \forall t$. In such a setting, the state-contingent tax rule is:

$$\hat{\tau}(K; \infty) = 12.055 + 0.0644K. \quad (38)$$

The addiction stock in the efficient steady state would be $\hat{K} = 204.82$.

To evaluate the significance of the requirement for time consistent implementation, consider a fixed tax rate set at the steady-state value of the state-contingent tax schedule: $\hat{\tau}_t = \hat{\tau}(\hat{K}; \infty) = 25.25, \forall t$. This would be the social planner's choice if she committed to a constant tax rate when the industry was in an efficient steady state.¹³ Figure 1 plots the two policy instruments for a range of addiction stocks. It demonstrates that their values can differ significantly if the industry is far from the efficient steady state. For example, the laissez-faire equilibrium yields a steady-state addiction stock $\tilde{K} = 323.9$. The state-contingent policy rule would then imply a tax rate of 32.9 cents, which is 30.3 percent higher than the proposed fixed tax of 25.25 cents. Even if the government could commit to maintain a fixed tax rate, welfare could be improved further by introducing a higher tax rate initially and then by moderating it as the addiction stock declines.

Gruber and Köszegi (2001) also assume that the government can precommit to a fixed

¹³If this fixed tax rate was in place, the efficient steady state would (eventually) be reached, but the transition path would be suboptimal.

tax rate. However, they consider a policy which is tied to the initial value of state variable rather than to \hat{K} . If their policy was introduced at the laissez-faire steady state, it would imply a tax rate of 25.8. This number is close to the fixed tax rate $\hat{\tau}(\hat{K}; \infty)$ the government would choose at the efficient steady state, but quite different from the value that the optimal tax rule would take at the laissez-faire steady state, $\hat{\tau}(\tilde{K}; \infty)$.

8.2.2 Imperfect Competition

In order to examine the implications of market power, we compare the optimal time consistent tax rate under perfect competition with the corresponding tax rate for $n = 3$. Figure 2 illustrates the schedules $\hat{\tau}(K; 3)$ and $\hat{\tau}(K; \infty)$ as specified by (37) and (38), respectively.¹⁴ These schedules show that ignoring the oligopolistic structure of the tobacco industry leads to a substantial upward bias in taxation. This result is driven by two features of the model.

First, consider the fixed tax rate that can maintain efficiency when aggregate addiction stock is at the socially optimal steady-state level \hat{K} :

$$\hat{\tau} = d^1 + d^2(\hat{K}) + d^3(\hat{K}; n), \forall t. \quad (39)$$

The parameter specification of Table 1 yields $\hat{\tau} = 10.67$, which is well above $\hat{\tau}(\hat{K}; 3) = 6.71$. A comparison between (39) and (21) shows that the expression for the optimal tax schedule has an extra term $\delta \sum_{s=1}^{\infty} (\delta \hat{\rho})^{s-1} \tau_{K,t+s} (\hat{X}_{t+s}/n)$ which accounts for future tax obligations. Since $\tau_K > 0$, the Markovian policy rule would imply a steady-state tax below (39). Firms have a disincentive to overproduce because extra production leads to higher future addiction stocks and hence to higher future taxes. This extra term decreases with n . The point is that there is an interaction between time consistent implementation and producers' market power. In a perfectly competitive setting, each producer would have a negligible influence on aggregate addiction stock, and hence on current and future time consistent taxes.

¹⁴If K was close to zero, (37) would imply a negative tax rate. However, this subsidy will not induce excessive output, as firms realize that this would reduce future subsidies or turn them into taxes.

Second, equation (39) implies that the optimal policy would depend on n even if the government committed to a fixed tax rate. The reason is that an oligopolistic market structure creates a welfare distortion: firms will limit production to increase prices and boost profits. Since addiction gives rise to dynamic demand, firms will experience a time consistency problem. This will reduce their market power, but the distortion will not be eliminated completely. Thus, optimal tax policies will have to account for potential underprovision of the addictive good. Using the parameters from Table 1, we obtain $d^3(\hat{K}; 3) = -14.58$. The absolute value of this term is decreasing in the number of firms. When the industry becomes perfectly competitive, $d^3(K, n)$ approaches zero.

8.2.3 Externalities Versus Internalities

Finally, it is instructive to compare the externality and the internality components of the tax rate. They are represented by the terms d^1 and $d^2(\hat{K})$. Our numerical example yields $d^1 = 2.77$ and $d^2(\hat{K}) = 22.48$. This is consistent with the position taken by Gruber and Köszegi (2001) that consumer self-control problems are the primary reason for government intervention.

9 Conclusion

This paper studies government intervention in an oligopolistic industry producing an addictive good. All agents have rational expectations and perfect foresight. We construct a tax policy which corrects inefficiencies arising from: i) a negative stock externality; ii) consumption internalities; and iii) producers' market power. Habit formation has important implications for tax policy design. In particular, the government may be tempted to renege on previously determined tax plans and revise its policy. The reason is that the wedge between private and social values of marginal changes in addiction typically varies over time.

To allow for robust and time consistent implementation, we consider tax rules that are

contingent on the aggregate addiction stock. Our paper provides a general characterization of the optimal time consistent tax rule. We illustrate our analysis with two examples. When payoffs are homogeneous of degree one, equilibrium marginal utilities and marginal profits do not change over time. Thus, efficiency can be implemented with a constant tax rate. If, however, payoffs are quadratic, the optimal tax rate will depend on the aggregate addiction stock. We calibrate the linear quadratic utility model to match the U.S. market for cigarettes and show how time consistency affects the implied tax rate.

Our analysis has some caveats. First, we do not deal with the lifecycle aspects of addiction, whereby many people begin smoking as teenagers and some manage to quit. Incorporation of these aspects would render the model intractable. Second, we assume that consumers are identical. In some settings we are able to cope with heterogeneities in addiction stocks. However, there may be other differences across smokers, e.g. in their utilities or discount factors. Third, we do not explore all normative frameworks or explanatory models. Alternative value judgements or models of addiction may lead to different implications. Finally, the paper focuses exclusively on sophisticated smokers. The appropriate modelling approach for consumer behavior is still debated in the literature. Naive smokers will also benefit from such taxes, but they will not deem time consistent tax rules credible.

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Appendix A: Optimal Time-Consistent Tax

In this appendix we prove proposition 1. Forward iteration of (20) yields:

$$\hat{P}_t - c - \tau_t + \hat{P}_{Q,t} \frac{\hat{X}_t}{n} + \sum_{s=1}^{\infty} \delta^s \left[[(\hat{\rho}_{t+s} - \theta) \hat{P}_{Q,t+s} + \hat{P}_{K,t+s} - \tau_{K,t+s}] \frac{\hat{X}_{t+s}}{n} \prod_{r=0}^{s-1} \bar{\rho}_{t+r} \right] = 0. \quad (40)$$

Iterating the consumer's equilibrium condition, (15), gives us an expression for the efficient consumer price,

$$\hat{P}_t = v_{x,t} + \beta \sum_{s=1}^{\infty} \delta^s \left[v_{k,t+s} \prod_{r=0}^{s-1} \bar{\sigma}_{t+r} \right]. \quad (41)$$

Substituting (41) into (40) yields:

$$\tau_t = v_{x,t} - c + \hat{P}_{Q,t} \frac{\hat{X}_t}{n} + \sum_{s=1}^{\infty} \delta^s \left\{ \beta v_{k,t+s} \prod_{r=0}^{s-1} \bar{\sigma}_{t+r} + [(\hat{\rho}_{t+1} - \theta) \hat{P}_{Q,t+s} + \hat{P}_{K,t+s} - \tau_{K,t+s}] \frac{\hat{X}_{t+s}}{n} \prod_{r=0}^{s-1} \bar{\rho}_{t+r} \right\}. \quad (42)$$

Finally, the iterated version of the efficiency Euler equation (10) is

$$v_{x,t} - c + \delta \sum_{s=1}^{\infty} (\delta\theta)^{s-1} (v_{k,t+s} - \varphi_{K,t+s}) = 0. \quad (43)$$

The socially optimal tax rule reconciles (42) with (43). This reconciliation occurs when $\tau(K)$ satisfies (21).

Appendix B: Examples

B1: Homogeneous Payoffs

First we pin down the optimal policy parameter \hat{g}_K . Substituting the expressions (28) for the marginal utilities in the efficiency Euler equation (10) delivers a condition for \hat{g}_K :

$$[v_x(\hat{g}_K, 1) - c](1 - \delta\theta) + \delta[v_k(\hat{g}_K, 1) - \xi] = 0. \quad (44)$$

Next we derive expressions for $\hat{P}_{Q,t}$, $\hat{P}_{K,t}$. The efficient equilibrium price is constant over time: $\hat{\Gamma}_{t+1} = \hat{\Gamma}$. Substituting the derivatives of the utility function into (16) gives us

$$\hat{P}_t = v_x(X_t, K_t) + \beta\delta v_k(\hat{g}_K, 1) - \delta[\theta + \hat{g}_K(1 - \beta)][v_x(\hat{g}_K, 1) - \hat{\Gamma}]. \quad (45)$$

Consumers correctly predict equilibrium prices. Imposing $\hat{P}_t = \hat{\Gamma}$ on (45) delivers:

$$\hat{\Gamma} = v_x(\hat{g}_K, 1) + \beta\delta v_k(\hat{g}_K, 1) - \delta[\theta + \hat{g}_K(1 - \beta)][v_x(\hat{g}_K, 1) - \hat{\Gamma}]. \quad (46)$$

Solving (46) for the anticipated price yields

$$\hat{\Gamma} = v_x(\hat{g}_K, 1) + \frac{\beta\delta v_k(\hat{g}_K, 1)}{1 - \delta[\theta + \hat{g}_K(1 - \beta)]}. \quad (47)$$

Substituting $\hat{\Gamma}$ in (45) delivers the following rational expectations inverse demand:

$$\hat{P}(Q, K) = v_x(Q, K) + \frac{\beta\delta v_k(\hat{g}_K, 1)}{1 - \delta[\theta + \hat{g}_K(1 - \beta)]}. \quad (48)$$

To obtain $\hat{P}_{Q,t}$, $\hat{P}_{K,t}$, we differentiate (48). Note that $v_{xx}(x, k)$ and $v_{xk}(x, k)$ are homogeneous of degree -1: $v_{xx}(\hat{g}_K K, K) = v_{xx}(\hat{g}_K, 1)/K$, $v_{xk}(\hat{g}_K K, K) = v_{xk}(\hat{g}_K, 1)/K$. Also, Euler's theorem gives us $v_{xx}(g_K, 1) = -v_{xk}(g_K, 1)/g_K$. Therefore, $\hat{P}_{Q,t}$, $\hat{P}_{K,t}$ can be rewritten as (29).

B2: Quadratic Payoffs

First we obtain conditions for the optimal policy parameters \hat{g}_0 , \hat{g}_K . Applying the method of undetermined coefficients to the efficiency Euler equation (10) yields

$$(\hat{\lambda}_0 - c)(1 - \delta\theta) + \delta[\hat{\mu}_0 + (\hat{\mu}_K - \theta\hat{\lambda}_K)g_0 - \xi] = 0, \quad \hat{\lambda}_K + \delta\hat{\gamma}_K(\hat{\mu}_K - \theta\hat{\lambda}_K) = 0,$$

where $\hat{\lambda}_0, \hat{\lambda}_K$ are defined by (31) and $\hat{\mu}_0, \hat{\mu}_K$ are defined by (32). The above equations can be used to compute \hat{g}_0, \hat{g}_K .

Next we characterize rational expectations demand. From (14) we can obtain h_k :

$$h_k = \frac{b_{xk} + \theta\delta [(\beta h_k - \theta - h_k)(-b_{xx}h_k + b_{xk}) + \beta(-b_{kk} + b_{xk}h_k)]}{b_{xx} - \delta [(\beta h_k - \theta - h_k)(-b_{xx}h_k + b_{xk}) + \beta(-b_{kk} + b_{xk}h_k)]}. \quad (49)$$

This allows us to compute the consumer's adjusted rate of accumulation: $\hat{\sigma} = \theta + (1 - \beta)h_k$.

Inverse industry demand is given by

$$\hat{P}(Q_t, K_t) = b_x + \beta\delta\hat{\mu}_0 - \delta\hat{\sigma}(\hat{\lambda}_0 - \hat{\Gamma}_0) + \hat{P}_Q Q_t + \hat{P}_K K_t, \quad (50)$$

where \hat{P}_Q and \hat{P}_K are

$$\hat{P}_Q = \beta\delta\hat{\mu}_K - \delta\hat{\sigma}(\hat{\lambda}_K - \hat{\Gamma}_K) - b_{xx}, \quad \hat{P}_K = b_{xk} + \theta[\beta\delta\hat{\mu}_K - \delta\hat{\sigma}(\hat{\lambda}_K - \hat{\Gamma}_K)]. \quad (51)$$

Finally, rational expectations imply that

$$\hat{\Gamma}_0 + \hat{\Gamma}_K K = b_x + \beta\delta\hat{\mu}_0 - \delta\hat{\sigma}(\hat{\lambda}_0 - \hat{\Gamma}_0) + \hat{P}_Q(\hat{g}_0 + \hat{g}_K K) + \hat{P}_K K.$$

Therefore, $\hat{\Gamma}_0$ and $\hat{\Gamma}_K$ must satisfy

$$\hat{\Gamma}_0 = b_x + \beta\delta\hat{\mu}_0 - \delta\hat{\sigma}(\hat{\lambda}_0 - \hat{\Gamma}_0) + \hat{P}_Q\hat{g}_0, \quad \hat{\Gamma}_K = \hat{P}_Q\hat{g}_K + \hat{P}_K.$$

Substituting $\hat{\Gamma}_K$ in the derivatives of the inverse demand function (51) yields (34).

Appendix C: Heterogeneous Consumers

Consider the linear-quadratic specification of section 7, but suppose that there are m consumers with addiction stocks k_t^1, \dots, k_t^m . The optimal tax τ_t must solve:

$$\hat{\Omega}(k_t^1, \dots, k_t^m) = \max_{\tau_t} \left\{ \sum_{i=1}^m [v(x^i(\tau_t), k_t^i) - \xi k_t^i - c x^i(\tau_t)] + \delta \hat{\Omega}(x^1(\tau_t) + \theta k_t^1, \dots, x_t^m(\tau_t) + \theta k_t^m) \right\}.$$

The first-order condition with respect to τ_t is

$$\sum_i [v_x(x_t^i, k_t^i) - c] \frac{\partial x^i}{\partial \tau_t} + \delta \sum_i \hat{\Omega}_{k^i, t+1} \frac{\partial x^i}{\partial \tau_t} = 0. \quad (52)$$

Summing across all envelope conditions yields:

$$\sum_i \hat{\Omega}_{k^i, t} = \sum_i [v_k(x_t^i, k_t^i) - \xi] + \delta \theta \sum_i \hat{\Omega}_{k^i, t+1}. \quad (53)$$

Our specification implies that $v_x(x^i, k^i) = b_x - b_{xx}x^i + b_{xk}k^i$ and $v_k(x^i, k^i) = -b_k - b_{kk}k^i + b_{xk}x^i$. Also, $\partial x^i / \partial \tau_t$ is the same across consumers. Let $X_t = \sum_{i=1}^m x_t^i / m$ and $K_t = \sum_{i=1}^m k_t^i / m$. Substitute $\sum_i \hat{\Omega}_{k^i, t+1}$ from (52) into (53), sum up over i and divide by m to get

$$b_x - b_{xx}X_t + b_{xk}K_t - c + \delta[-b_k - b_{kk}K_{t+1} + b_{xk}X_{t+1} - \xi] + \delta\theta[b_x - b_{xx}X_{t+1} + b_{xk}K_{t+1} - c] = 0.$$

This optimality condition is identical to the efficiency Euler equation with homogeneous consumers. Furthermore, utility maximization generates linear personal demand schedules whose slopes $\partial p_t / \partial x_t$ and $\partial p_t / \partial k_t$ are identical across all smokers. Aggregation will yield industry demand which also depends on the average addiction stock. Therefore, our results of section 7 will carry through in a setting with heterogeneous addiction stocks. Moreover, a mean preserving spread of these addiction stocks will not affect the tax rate. Finally, our argument also applies when there are heterogeneities in the utility parameters b_x and b_k .

Parameter	β	δ	c	n	b_x	b_{xx}	b_k	b_{kk}	b_{xk}	ξ	θ
Value	0.65	0.96	0	3	70	0.3	1	0.3	0.2	1.5	0.5

Table 1: parameters of the baseline scenario.

Variables	\hat{X}	\hat{P}	\hat{K}	τ	$\xi\hat{K}/\hat{X}$	ε_{LR}	ε_{SR}
Value	102.41	25.25	204.82	6.71	3.00	-0.58	-0.42

Table 2: steady-state equilibrium of the baseline scenario.

Changed Variable	\hat{X}	\hat{P}	\hat{K}	τ	$\xi\hat{K}/\hat{X}$	ε_{LR}	ε_{SR}
$n = 1$	102.41	25.25	204.82	-28.21	3.00	-0.58	-0.42
$n = 2$	102.41	25.25	204.82	-2.28	3.00	-0.58	-0.42
$n = 4$	102.41	25.25	204.82	11.28	3.00	-0.58	-0.42
$n = 5$	102.41	25.25	204.82	14.04	3.00	-0.58	-0.42
$\beta = 0.5$	102.41	33.29	204.82	16.49	3.00	-0.94	-0.58
$\beta = 0.6$	102.41	28.00	204.82	10.04	3.00	-0.69	-0.47
$\beta = 0.7$	102.41	22.42	204.82	3.30	3.00	-0.49	-0.36
$\beta = 0.8$	102.41	16.43	204.82	-3.85	3.00	-0.32	-0.25
$\delta = 0.90$	118.85	25.20	237.70	5.03	3.00	-0.58	-0.37
$\delta = 0.92$	113.06	25.20	226.11	5.60	3.00	-0.58	-0.38
$\delta = 0.94$	107.58	25.21	215.17	6.16	3.00	-0.58	-0.40
$\delta = 0.98$	97.50	25.31	195.01	7.27	3.00	-0.58	-0.43
$\xi = 0.0$	106.75	23.41	213.49	4.09	0.00	-0.52	-0.37
$\xi = 0.5$	105.30	24.02	210.60	4.97	1.00	-0.54	-0.39
$\xi = 2.5$	99.52	26.48	199.04	8.47	5.00	-0.63	-0.45
$\xi = 3.0$	98.07	27.09	196.14	9.34	6.00	-0.65	-0.47
$\theta = 0.45$	126.43	23.40	229.88	2.34	2.73	-0.52	-0.32
$\theta = 0.55$	80.67	27.20	179.27	11.27	3.33	-0.66	-0.56
$\theta = 0.60$	61.55	29.29	153.88	15.96	3.75	-0.75	-0.78
$\theta = 0.65$	45.23	31.57	129.24	20.80	4.29	-0.86	-1.14
$b_{xk} = 0.10$	63.91	25.02	127.82	12.05	3.00	-0.57	-0.73
$b_{xk} = 0.15$	78.70	25.00	157.40	9.71	3.00	-0.57	-0.57
$b_{xk} = 0.25$	146.55	26.08	293.10	2.41	3.00	-0.61	-0.28
$b_{xk} = 0.30$	257.58	28.44	515.15	-5.27	3.00	-0.71	-0.17

Table 3: steady-state equilibrium when changing one parameter at a time.

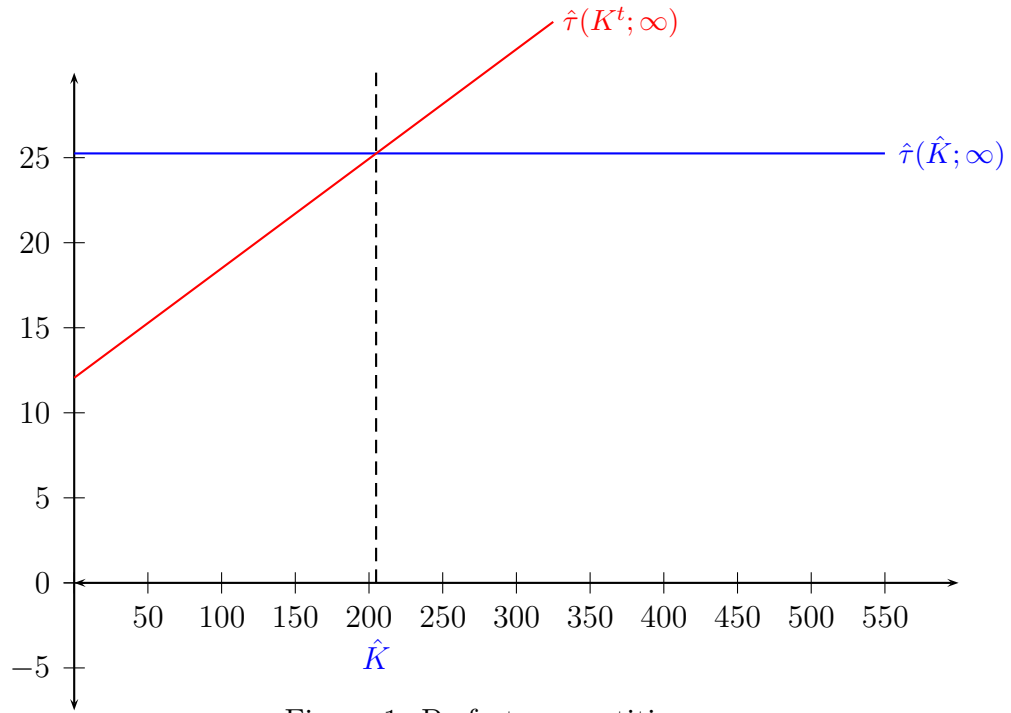


Figure 1: Perfect competition

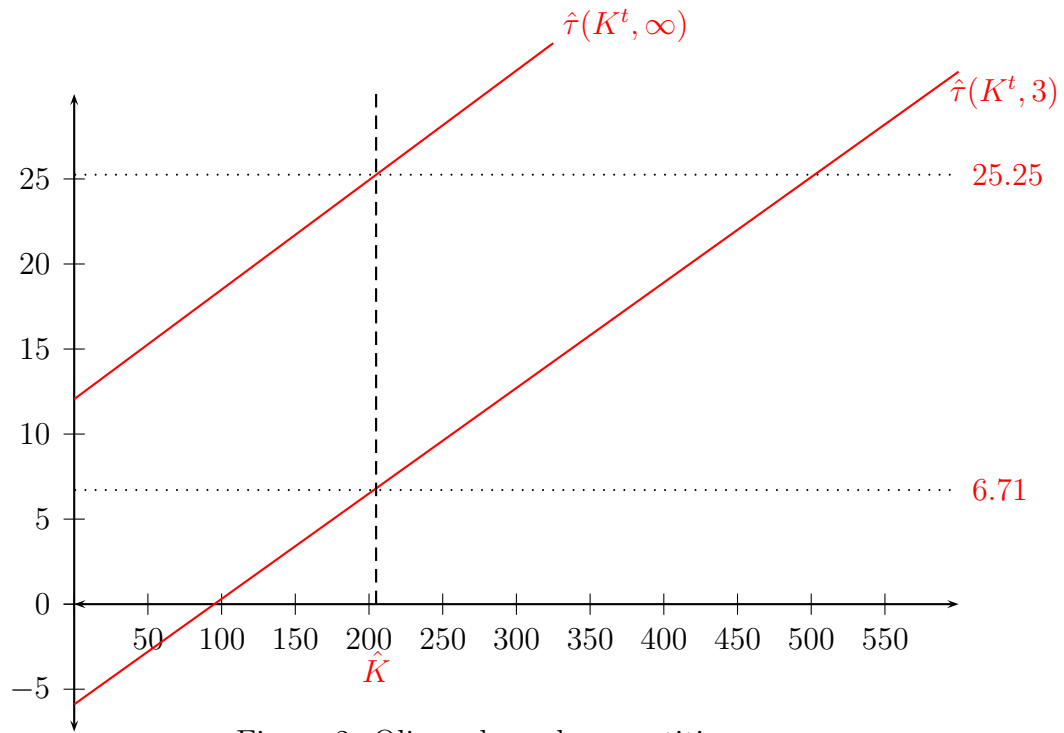


Figure 2: Oligopoly and competition