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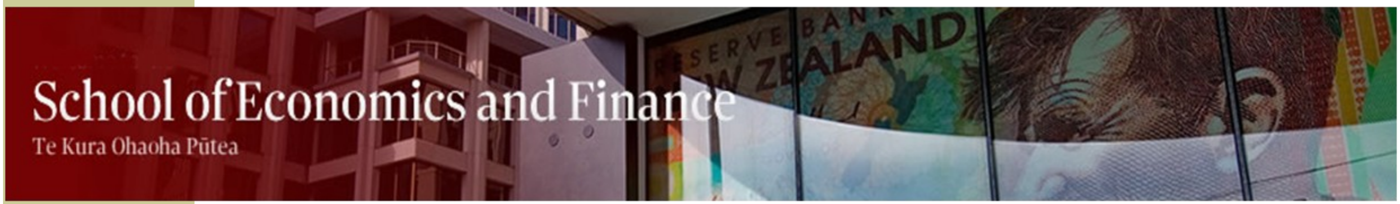

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OPTIMAL PRICING OF WATER: OPTIMAL DEPARTURES FROM THE INVERSE ELASTICITY RULE*

Yiğit Sağlam[†]
Victoria University of Wellington

Abstract

I consider the problem of water usage, developing a model to analyze the optimal pricing of water within a second-best economy. Consumers are assumed to have two main needs for water—drinking and non-drinking. Water is also used to produce food: The agricultural sector has a derived demand for water. As a water supplier, the local government may price discriminate across consumers and farmers. I introduce the second-best pricing scheme, derive conditions for the marginal-cost pricing and inverse-elasticity rules to apply, and analyze when it is optimal for the government to optimally deviate from these two pricing schemes.

Keywords: Ramsey Pricing, Inverse Elasticity Rule, Marginal Cost Pricing, Water Scarcity, Water Pricing

JEL Classification Numbers: D11, H23, L95, Q25.

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[†]School of Economics and Finance, Victoria University of Wellington, RH 312 Rutherford House, 23 Lambton Quay, Wellington, New Zealand. Telephone: +64-4-4639989, Fax: +64-4-463-5014. E-mail: Yigit.Saglam@vuw.ac.nz

1 Introduction

About seventy percent of water withdrawals in all OECD countries are by agriculture. As water becomes relatively scarce, government protection through subsidization of the agricultural sector has become increasingly questionable. Irrigation water does not only affect other agricultural input demands (such as capital, fertilizers, and labor), but it also has implications for the composition of agricultural output. Furthermore, inefficient use of water by the agricultural sector may cause overuse of water as well as water pollution, which is a problem in both developed and developing countries. In figure 1, I illustrate sectoral water prices in several OECD countries in late 1990s. The agricultural sector paid substantially less than industry and households; specifically, the ratio is around one percent; see the OECD [12, 13, 14]. For example, on average, farmers in the United States pay about \$0.05 per cubic meter, while households pay around \$1.25 per cubic meter. In France, these prices are \$0.08 and \$3.11 per cubic meter, respectively. Finally, in Spain, on average, farmers pay \$0.07 per cubic meter whereas households pay \$1.07 per cubic meter. In some countries (including Italy, Japan, and Turkey), marginal cost of using an additional unit of irrigation water equals zero, because of non-volumetric pricing schemes. Part of the difference between tap and irrigation prices can result from quality of water provided to these sectors. For example, households and industries may require pressurized water, whereas the agricultural sector does not require a high quality of water. However, one would not expect the effect of differences in quality to be this much.

Putting cost differences aside, another factor is government protection of the agricultural sector. Subsidizing the agricultural sector can result in inefficiencies such as overuse of water consumption and water pollution, and without government protection, marginal-cost pricing implies equal prices across sectors which use water from the same reservoir. In the water literature, nonetheless, the main focus has mostly been on analyzing the market frictions and the resulting pricing schemes. Some authors have worked on the estimation of tap water demand, including Gaudin, Griffin, and Sickles [7], Kim [11], and irrigation water demand, including Iglesias, Garrido, and Gómez-Ramos [10], Appels, Douglas, and Dwyer [1], de Fraiture and Perry [4]. However, different sectors may often use water from the same reservoir, so analyzing the optimal pricing of water in one sector, while ignoring the changes in the demand by another sector, may have implications for the policy suggestions. For this reason, sectoral water prices are usually interconnected through a resource constraint. To account for multiple uses of water and to choose optimal prices simultaneously, some researchers (such as Diakité, Semenov, and Thomas [5], Garcia and Reynaud [6], Griffin [8]) have employed a static Ramsey pricing scheme. However, the intertemporal allocation of

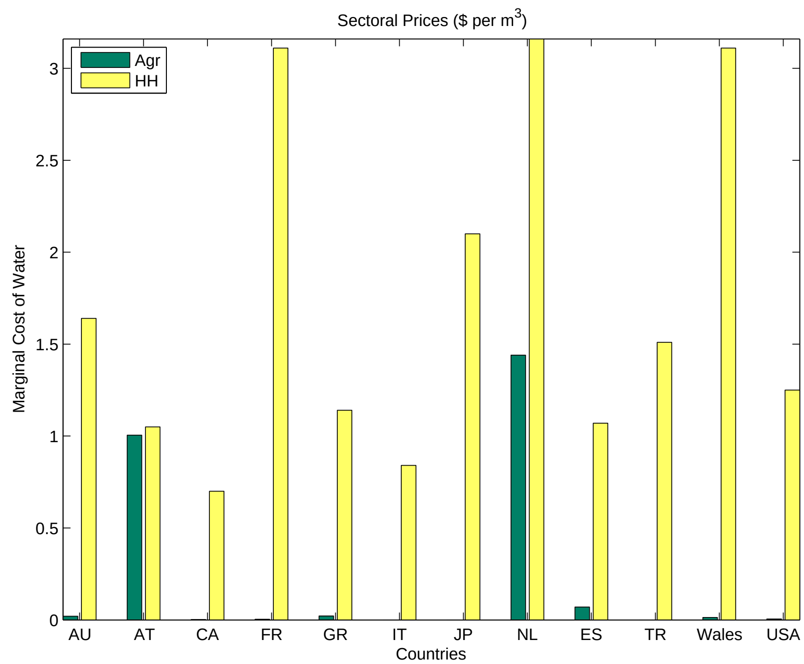

Figure 1: Water Prices for Different Sectors in OECD Countries

water may have important effects on the second-best water prices, as low volume of water can make the resource constraint tighter and cause increase in water price(s). Finally, the dynamic water reservoir management has been an important factor in water pricing; see Castelletti, Pianosi, and Soncini-Sessa [3], Howitt, Msangi, Reynaud, and Knapp [9], Schuck and Green [16]. However, dynamic water reservoir management mostly fails to consider multiple uses of water, as in Schuck and Green [16], or a government revenue constraint, as in Howitt, Msangi, Reynaud, and Knapp [9].

In this paper, I incorporate both dynamic resource and revenue constraints to attempt to explain why the observed pricing schemes may differ from marginal-cost pricing. I do not attempt to explain each of the many pricing schemes, but rather I concentrate on a commonly-observed pricing scheme: two-sector pricing. To investigate this price discrimination problem, I set up a simple model in which a water supplier, which I shall refer to as the (local) government, charges different customer groups different prices. I construct a partial-equilibrium model where both domestic households and agricultural producers demand water. The government seeks to maximize net social welfare of the economy, subject to both dynamic resource and revenue constraints. The government achieves its goal through price discrimination between domestic demand, which is the demand by households, and agricultural demand. Also, I ignore any government protection toward agriculture. Nonetheless, in an illustrative example, I showed that in the optimal allocation, the agriculture, which I assumed to have a relatively less inelastic demand, may be charged a lower price than what the households pay. This is because when water scarcity is severe enough, the dynamic resource constraint becomes dominant and the government's water allocation problem changes from a Ramsey pricing problem to a resource allocation problem with no revenue constraint. Also, since the households can shift part of their income to other commodities as a response to higher water price(s), the marginal-cost pricing is not necessarily optimal. Consequently, the optimal pricing rule can be quite different from marginal-cost pricing or Ramsey pricing schemes, implied by the inverse elasticity rule.

This paper is in four more parts: In the next section, I provide a brief summary of the use of the Ramsey pricing on the water literature, while in section 3, I introduce the model. In section 4, I analyze the dynamic Ramsey pricing problem of the local government. Then, I focus on the static problem to analyze prices, derive conditions that make two-sector pricing efficient, and present a numerical example for an assumed objective function, a cost function, and constraints. Finally, I shall discuss the qualitative results of my paper in light of the inverse elasticity rule. I summarize and conclude the paper in section 5.

2 Model

In my model, water can be used for domestic and irrigation purposes. Domestic demand has two parts—drinking water and other uses of water, the second of which I shall refer to as *bathing water*. These are supplied from the same tap, so the two different uses cannot be priced differently. Drinking water is, however, more important than bathing water, but charging a sufficiently low price for drinking water may result in an "overuse" of bathing water. Agricultural output is as important to well-being as drinking water. Thus, charging low prices for agriculture and high prices for tap water is a solution which avoids the need to monitor bathing-water usage, while still providing for basic needs.

For simplicity, the following scenario may be useful: a representative agent has two taps that provide water. One tap provides water for drinking and bathing purposes, while the second tap can only be used for food production. As I shall discuss below, these three water uses are valued differently.

2.1 Households

Suppose households have a fixed income I in every period and do not save. Income is completely spent on three commodities—food f, tap water w_1, and a consumption good y which represents all other commodities except food and water. Tap water can be used for drinking and non-drinking purposes. Drinking use represents the necessary uses of water, while non-drinking use, which I shall refer to as bathing water, is associated with all other uses of water except drinking water[1].

With their income, households purchase food and consumption good as well as tap water. The prices of food and tap water are denoted by p_f and p_1, respectively. I assume that all the prices and income are in terms of the price of the composite good. The price of food may vary because water is used in agricultural production, and there it may have a different price. I shall discuss this in the agricultural-production section. Moreover, the reason why drinking and bathing water have the same price is that both are tap water. Even when drinking and bathing water are different commodities, they typically cannot be priced differently because they come out of the "same" tap. Thus, the price of drinking water and bathing water are assumed to be equal. The household's budget constraint then is

$$p_1 \ w_1 + p_f \ f + y = I. \tag{2.1}$$

At this point, I do not assume specific preferences for the per-period utility function, except to assume that preferences are locally non-satiated and strictly concave. Let w_{11} and w_{12} denote drinking and bathing water, respectively, and their sum equals the total tap water use w_1. The utility-maximization problem of the representative agent is a static maximization problem:

$$\max_{<f, w_{11}, w_{12}, y>} U(f, w_{11}, w_{12}, y)$$
$$\ni p_1 \ (w_{11} + w_{12}) + p_f \ f + y = I.$$

By solving the optimization problem, one can derive the indirect utility function $\Upsilon(\cdot)$, and the Marshallian demands as a function prices and income $(\mathbf{P}, I)$.

2.2 Producers

Producers are farmers who require water to produce food. Farmers demand irrigation water w_2 at its price p_2, which may be different from the price of tap water p_1, because the government may set different prices for different sectors. Note that, in this model, no quality differences exist between irrigation water and tap water, so I shall assume that the cost of

[1]In some countries, such as Italy, drinking and bathing water are priced differently. There, this problem does not arise.

supplying tap and irrigation water are the same[2].

As the price of irrigation water changes, producers determine how much inputs to employ and how much to produce. I assume perfect competition in agricultural markets, so maximized profits are zero in equilibrium. Land is taken as fixed, and consumers are assumed to supply labor inelastically. Agricultural production is assumed to follow constant returns-to-scale (CRS), having the following production technology:

$$f = f(w_2) = \kappa\, w_2,\ \ \kappa > 0. \tag{2.2}$$

A representative agricultural firm's profit maximization problem becomes:

$$\max_{<w_2>} \ \Pi(w_2; p_f, p_2) = \max_{<w_2>} \ p_f f(w_2) - p_2 w_2.$$

Three points are worth noting: first, since the market is competitive, the price of food equals the marginal cost of producing food, so p_f equals (p_2/κ). Second, the equilibrium profits will be driven to zero. Finally, since equilibrium profits are zero, the volume of water used as input is found from the Marshallian demand for food. This means that w_2 equals (f/κ). In other words, the agricultural sector has a derived demand for water.

2.3 Government

I assume that the government manages water supply and provides water to consumers and producers. This assumption is reasonable as it is estimated that less than ten percent of the world's population is provided drinking water through private sector services; see the OECD [15]. In each period, the government is responsible for supplying water and taxing consumers as well as the agricultural sector. It does so by choosing the price of tap water, p_1, and of irrigation water, p_2. The two water prices may be equal, when the government prefers to charge both sectors the same price, or they may be different, when the government price discriminates across the two sectors.

The government seeks to maximize the net social welfare of households and producers. Since profits in the agricultural sector are zero in equilibrium, the only component of the government's objective function is the indirect utility function of the households. The indirect utility function is taken as a criterion function for the maximization problem. The government has to satisfy two constraints. The first constraint is that it must collect enough tax revenues to fund supplying water; i.e., it must generate enough revenue to cover fixed capital investment, operating and maintenance costs of the water supply. In doing this, the

[2]Even though this assumption is not critical, it is useful to compare the two water prices.

government may choose to invest in purchasing bonds b_{t+1}, which has a fixed return r. The second constraint on the government involves intertemporal resource allocation of water. Specifically, it must decide how much water to save for the future.

The optimization problem faced by the government is to maximize the net social welfare in the economy subject to the resource constraint as well as raising revenues. In this respect, I shall refer to this problem as the Ramsey problem. That the government must collect revenues to cover its costs introduces a distortion in the economy. However, even without this distortion, it is not necessarily the case that the solution to this problem is no price discrimination, in which both domestic households and agricultural producers are charged the same price. Let $\mathbf{P_t}$ represents the vector of water prices (p_{1t}, p_{2t}). The water prices depend on the current water stock and the amount of bonds at time t. Within this framework, the stochastic version of the Ramsey problem is as follows[3]:

$$\max \mathcal{E}\left[\sum_{t=0}^{\infty} \beta^t \Upsilon(\mathbf{P_t}; I)\right] \tag{2.3a}$$

$$\ni p_{1t}\ w_{1t} + p_{2t}\ w_{2t} - \Psi\left(w_{1t} + w_{2t}\right) + b_{t+1} \geq \tau + r\ b_t, \tag{2.3b}$$

$$w_{t+1} \leq S_t\left(w_t, E_t\right) - \left(w_{1t} + w_{2t}\right), \tag{2.3c}$$

$$p_{ft} = p_{2t}/\kappa; \tag{2.3d}$$

$$w_{11t}, w_{12t}, w_{2t}, w_{t+1} \geq 0, \tag{2.3e}$$

$$w_{1t} = w_{11t} + w_{12t}, \tag{2.3f}$$

$$w_0 \text{ is given} \tag{2.3g}$$

where the water quantities $\{w_{11t}, w_{12t}, w_{2t}\}$ depend on $(\mathbf{P_t}; I)$, due to the consumer and producer optimization problems. The parameter τ denotes the fixed cost of production. The available water stock at time t, denoted by $S_t\left(w_t, E_t\right)$, depends on both the water saved from previous period and a random shock E_t which may follow a specific, stochastic Markovian process. The available stock may also be limited by capacity constraint. Under the assumption that there are no quality differences among different water uses, the cost of water, denoted by $\Psi\left(.\right)$, is a function of only the total withdrawals[4].

[3]Note that the water supplier in this problem may tax more than τ in order to discourage water use, thus ensuring a larger stock in the future. What is done with this surplus in tax revenue is currently not modeled.

[4]Since this is a surface water management problem, there are no abstraction costs that depend on the water stock or the elevation.

3 Analysis

The Ramsey problem stated in the previous section is a two-stage maximization problem. In the first stage, the government chooses w_{t+1} and b_{t+1} optimally; the rest of the water supply will be released for sectors in the current period. In the second stage, the government chooses the two water prices optimally for efficient use of water by the sectors. Since the focus of this paper is on the differential prices, analyzing the prices in the static setup is sufficient. Hence, I assume that the government chooses w_{t+1} and b_{t+1} optimally, and I will focus on the static version of the problem where the government chooses the water prices given the water supply and the fixed cost[5]. Suppose that the government collects the revenue from selling water and does not use the money for any purposes. In this case, households have no other income source than their per-period income. Thus, their budget constraint is given in equation (2.1).

3.1 Analysis with Resource Constraint

First, I shall focus on the resource constraint and thus, I shall ignore the revenue constraint in this first part of the analysis. The static version of the dynamic Ramsey problem becomes:

$$\max_{<p_1(w),p_2(w)>} \Upsilon(\mathbf{P}; I) \tag{3.1a}$$

$$\ni w_1(\mathbf{P}; I) + w_2(\mathbf{P}; I) = WD(\mathbf{P}; I) \leq \bar{w}, \tag{3.1b}$$

$$w_1(\mathbf{P}; I), w_2(\mathbf{P}; I) \geq 0, \tag{3.1c}$$

$$p_f = p_2/\kappa; \tag{3.1d}$$

$$f(\mathbf{P}; I) = \kappa w_2(\mathbf{P}; I) \tag{3.1e}$$

where WD denotes the total withdrawals from the water stock, and $\bar{w}$ is the available water supply. For notational simplicity, I shall continue with the following notation:

$$w_1(\mathbf{P}; I) = w_{11}(\mathbf{P}; I) + w_{12}(\mathbf{P}; I).$$

Let λ and δ be the Lagrange multipliers on the resource constraint and the budget constraint of households, respectively. Note that the budget constraint is not given in the maximization problem since using Marshallian demands and the indirect utility function will

[5] I abuse the notation here and denote the fixed cost in the static problem by τ. The fixed cost in the static version in fact equals the fixed cost plus $(r\ b_t - b_{t+1})$.

make the budget constraint hold. The economic interpretations of λ and δ are the marginal value of water and marginal utility of income, respectively. The Lagrange multiplier δ can be ignored through normalization, so I shall interpret the ratio (λ/δ) as the marginal value of water. The first order conditions (henceforth, FOCs) of the static Ramsey problem are

$$\begin{aligned}
\frac{\partial \Upsilon(\cdot, I)}{\partial p_1} &= MU_1 \frac{\partial w_1(\cdot, I)}{\partial p_1} + MU_2 \frac{\partial w_2(\cdot, I)}{\partial p_1} + MU_y \frac{\partial y(\cdot, I)}{\partial p_1} \\
&= \lambda \left[\frac{\partial w_1(\cdot, I)}{\partial p_1} + \frac{\partial w_2(\cdot, I)}{\partial p_1} \right] \\
\frac{\partial \Upsilon(\cdot, I)}{\partial p_2} &= MU_1 \frac{\partial w_1(\cdot, I)}{\partial p_2} + MU_2 \frac{\partial w_2(\cdot, I)}{\partial p_2} + MU_y \frac{\partial y(\cdot, I)}{\partial p_2} \\
&= \lambda \left[\frac{\partial w_1(\cdot, I)}{\partial p_2} + \frac{\partial w_2(\cdot, I)}{\partial p_2} \right]
\end{aligned}$$

where MU_i is the marginal utility with respect to commodity i. Note that the first components of the partial derivative of the indirect utility function with respect to the two water prices equal:

$$MU_1 \frac{\partial w_1(\cdot, I)}{\partial p_i} = MU_{11} \frac{\partial w_{11}(\cdot, I)}{\partial p_i} + MU_{12} \frac{\partial w_{12}(\cdot, I)}{\partial p_i}; \ \forall\, i = 1, 2.$$

The solution to household's utility-maximization problem requires that the marginal utility of a water use equals its price. Using this property, one can rewrite the FOCs in the following way:

$$\begin{aligned}
\frac{\lambda}{\delta} =& p_1 + (p_2 - p_1) \frac{\partial w_2(\cdot, I)/\partial p_1}{\partial WD(\cdot, I)/\partial p_1} + \frac{\partial y(\cdot, I)/\partial p_1}{\partial WD(\cdot, I)/\partial p_1} && \text{(3.2a)} \\
=& p_2 + (p_1 - p_2) \frac{\partial w_1(\cdot, I)/\partial p_2}{\partial WD(\cdot, I)/\partial p_2} + \frac{\partial y(\cdot, I)/\partial p_2}{\partial WD(\cdot, I)/\partial p_2}. && \text{(3.2b)}
\end{aligned}$$

To see the relationship between the two water prices, I derive a condition that follows from the two prices being equal. Assuming that equal prices solve these FOCs, the FOCs can be simplified to

$$p_1 = p_2 = \frac{\lambda}{\delta} - \frac{\partial y(\cdot, I)/\partial p_1}{\partial WD(\cdot, I)/\partial p_1} = \frac{\lambda}{\delta} - \frac{\partial y(\cdot, I)/\partial p_2}{\partial WD(\cdot, I)/\partial p_2}$$

where (λ/δ) is the marginal value of water. Notice that, without a composite good, equal sectoral water prices would solve the system of equations, so the optimal pricing scheme would be the marginal-cost pricing rule. Thus, including a non-water-related commodity is crucial here to ensure that income spent by the consumers on water and food changes with the water prices. Nonetheless, it is still possible to have the two water prices equal, if the following condition is met:

$$\frac{\partial y/\partial p_1}{\partial y/\partial p_2} = \frac{\partial WD/\partial p_1}{\partial WD/\partial p_2} \Leftrightarrow \frac{\epsilon_{y,1}}{\epsilon_{y,2}} = \frac{\epsilon_{WD,1}}{\epsilon_{WD,2}}$$

where $\epsilon_{WD,i}$ and $\epsilon_{y,i}$ denote the elasticity of total withdrawals and the composite good with respect to the water price p_i, for $i = 1, 2$, respectively. The condition above implies that the two-sector pricing policy and the marginal-cost pricing rule coincide, and it is interpreted in the following way: the local government sets p_1 equal to p_2, as long as the ratio of the price elasticity of y with respect to p_1 and p_2 equals the ratio of elasticity of total withdrawals with respect to p_1 and p_2, at the optimum. The last terms on the right-hand side of both FOCs in equations (3.2); i.e., the change in the consumption good with respect to changes in the water prices, leads to this condition. If the consumption good is unaffected by the water prices, then changing water prices will have no effect on the consumption good, and water prices could be set equal to the marginal value of water. Since I assume that the consumption good is a substitute (or a complement) of water-related commodities, the consumption good changes with at least one of the water prices.

It is important to note that the inverse-elasticity rule, described in Baumol and Bradford [2], does not necessarily apply in this case. Specifically, although households are assumed to have relatively more elastic demand for water, they may be charged a higher price than the agricultural producers. To see the intuition behind this, consider the situation: First, suppose that there is no consumption good. Further assume that the cross-price elasticities of different water uses are zero and that households have a relatively more elastic demand for water than the agricultural producers do. If the government aims to maximize utility subject to only the revenue constraint, households would pay a lower price, because of the inverse-elasticity rule; i.e., enough revenue can be generated by charging a higher price for the relatively less elastic demand. With only the resource constraint, the government would adopt marginal-cost pricing rule; i.e., both sectors are charged the same price which is the marginal value of water. As a result, without any consumption good, the revenue constraint would imply inverse-elasticity rule, while the resource constraint would lead to marginal-cost pricing rule.

Suppose now that households can also purchase a non-water-related commodity; i.e., a composite good non-taxable by the local government. Further assume that the consumption good is affected only by the price of tap water, and unaffected by the price of irrigation water. In this case, the water supplier prefers to charge a higher price to households. This is because, in the absence of the consumption good, enough revenue may be generated by setting p_1 higher than p_2, but at a larger cost, which means less utility than optimal. However, utility may now be maximized by setting p_1 higher than p_2 since increasing p_1 also increases the demand for the consumption good. In other words, more income can be allocated to the consumption good. As an optimal solution, the water supplier prefers to allocate as much income as possible for the consumption good, which is equivalent to collecting as less revenue as possible. Thus, it charges more to the households who have a relatively more elastic demand than the agricultural producers, and it still generates enough revenue to cover costs. Although this result contrasts with the inverse-elasticity rule and the second-best pricing scheme, it may explain why it is observed that households pay a higher price for water consumption than the agricultural producers although the demand for tap water is usually predicted to be more elastic.

3.2 Analysis with Both Constraints

The corresponding Ramsey problem with both constraints becomes

$$\max_{<p_1(w),p_2(w)>} \Upsilon(\mathbf{P};I) \tag{3.3a}$$

$$\ni w_1(\mathbf{P};I) + w_2(\mathbf{P};I) = WD(\mathbf{P};I) \leq \bar{w}, \tag{3.3b}$$

$$p_1 w_1(\mathbf{P};I) + p_2 w_2(\mathbf{P};I) \geq \Phi\left(WD(\mathbf{P};I)\right) + \tau, \tag{3.3c}$$

$$w_1(\mathbf{P};I),\ w_2(\mathbf{P};I) \geq 0, \tag{3.3d}$$

$$p_f = p_2/\kappa, \tag{3.3e}$$

$$f(\mathbf{P};I) = \kappa w_2(\mathbf{P};I). \tag{3.3f}$$

Let μ and λ be the Lagrange multipliers on the revenue and the resource constraints, respectively. Using the households' budget constraint, the revenue constraint (3.3c) can be written as:

$$WD(\mathbf{P};I) \leq \Phi^{-1}\left(I - \tau - y(\mathbf{P};I)\right) \tag{3.4}$$

where the function $\Phi^{-1}(.)$ is the inverse of the cost function[6].

The FOCs lead to the following necessary condition:

[6]One can find the inverse of a cost function since for any cost level, there is a corresponding production level and also not two different production levels can lead to the same cost level.

$$1 = \frac{\frac{1}{w_1}\left[\mu\frac{\partial\Phi^{-1}(I-\tau-y)}{\partial p_1} - (\lambda+\mu)\frac{\partial WD}{\partial p_1}\right]}{\frac{1}{w_2}\left[\mu\frac{\partial\Phi^{-1}(I-\tau-y)}{\partial p_2} - (\lambda+\mu)\frac{\partial WD}{\partial p_2}\right]}$$

where the FOCs declare that both the numerator and the denominator in the right-hand-side (henceforth, RHS) equal $-\delta$, which is the negative of the marginal value of income.

Although it is unclear from the FOCs which constraint will bind, if not both, one can divide the solution into three regions:

3.2.1 Region 1: No Water Scarcity

Assume that the water supply is abundant ($\bar{w} \to +\infty$). In this case, one can ignore the resource constraint. The solution to the problem is unique, and the revenue constraint is binding, while the resource constraint is irrelevant. Without loss of generality, denote this solution by $\{p_1^{**}, p_2^{**}\}$, so $\{WD^{**}, y^{**}\}$. As long as the water supply is above the total withdrawals at the optimum; i.e., $\bar{w}^{**}$ equals WD^{**}, the solution stays the same, so do the prices. This is also the solution to the static Ramsey problem with no resource constraint; so the inverse-elasticity rule should apply.

3.2.2 Region 2: Water Scarcity

Assume that the water supply is quite low. In this case, the solution for prices $\{p_1^{**}, p_2^{**}\}$ in region 1 does not satisfy the resource constraint, so the water prices should adjust to make the resource constraint hold. Denote the new optimum by $\left\{p_1^{'}(\bar{w}), p_2^{'}(\bar{w})\right\}$, and so $\{WD'(\bar{w}), y'(\bar{w})\}$, which would depend on the water supply. In this new solution, the total withdrawals are lower. As long as the profits are above zero, the revenue constraint should become slack in this new optimum. As the water supply gets enormously scarce ($w \to 0$), the resource constraint will be a much stronger factor, and the price changes will be more drastic. It is noteworthy that the solution is derived according to the FOCs presented in equation (3.2), so water prices may exceed one another, depending on the parameter values.

3.2.3 Region 3

It is possible that both constraints bind at the optimum for a connected interval of $\bar{w}$. The lower and upper bounds of this middle region, $[\bar{w}^{*}, \bar{w}^{**}]$, can be found in the following way:

$$
\begin{aligned}
\bar{w}^* =& \max\left\{\bar{w} \mid WD(\tilde{\mathbf{P}}) \leq \Phi^{-1}\left(I - \tau - y(\tilde{\mathbf{P}})\right); \tilde{\mathbf{P}} = \operatorname{argmax} \Upsilon(\mathbf{P}) \ni WD(\mathbf{P}) \leq \bar{w}\right\} \\
\bar{w}^{**} =& \min\left\{\bar{w} \mid \bar{w} \geq WD(\mathbf{P}^{**}); \mathbf{P}^{**} = \operatorname{argmax} \Upsilon(\mathbf{P}) \ni WD(\mathbf{P}) \leq \Phi^{-1}\left(I - \tau - y(\mathbf{P})\right)\right\}
\end{aligned}
$$

In words, $\bar{w}^*$ is maximum of $\bar{w}$s such that the value for which the solution to the optimization problem with only the resource constraint delivers a positive profit for the government. Meanwhile, $\bar{w}^{**}$ is equals to the optimal total withdrawals for the optimization problem with only the revenue constraint.

3.3 Numerical Example

I assume the Stone–Geary function for preferences of consumers[7]:

$$
\begin{aligned}
U(w_1, f, y) =& \pi_1 \log\left(w_1 - \underline{w}_1\right) + \pi_2 \log\left(f - \underline{f}\right) + (1 - \pi_1 - \pi_2)\log(y), \\
& \underline{w}_1,\ \underline{f} \geq 0, \\
& \pi_1,\ \pi_2 \in [0, 1].
\end{aligned}
$$

where π_1 and π_2 denote the marginal budget shares of tap water, and food, respectively. The parameters $\underline{w}_1$ and $\underline{f}$ represent the subsistence level consumption of tap water and food. One can view the drinking water use under the subsistence level, so the two different uses of water for households can be distinguished in this way. I assume that consumers need to consume some food for survival, and that the composite good does not a subsistence level. Given the functional form, the Marshallian demand for tap water is:

$$
w_1 = (1 - \pi_1)\underline{w}_1 + \pi_1 \frac{(I - p_1 \underline{w}_1 - p_f \underline{f})}{p_1}.
$$

The demand consists of two components: the subsistence level $\underline{w}_1$, and the price-responsive component. The price-elasticity of demand for tap water and food are always inelastic in their own prices:

[7] Stone–Geary function is used in estimating the demand for tap water in the water literature; see Gaudin, Griffin, and Sickles [7]

$$\epsilon_{w_1,p_1} = \frac{\pi_1(I - p_1\underline{w}_1 - p_f\underline{f})}{(1-\pi_1)\underline{w}_1\ p_1 + \pi_1(I - p_1\underline{w}_1 - p_f\underline{f})} \leq 1,$$
$$\epsilon_{f,p_f} = \frac{\pi_2(I - p_1\underline{w}_1 - p_f\underline{f})}{(1-\pi_2)\underline{f}\ p_f + \pi_2(I - p_1\underline{w}_1 - p_f\underline{f})} \leq 1.$$

In addition to the preferences, I assume the following cost function for supplying water:

$$\Phi(WD) = \theta_1\ WD^{\theta_2};\ \theta_1 > 0 \text{ and } \theta_2 \geq 1$$

I assume the parameter κ equals one for the remainder of the paper to simplify the computations and the notation. In this way, the demand for food equals the demand for irrigation water, and the price of food equals the price of irrigation water. The per-period utility function simplifies to:

$$\begin{aligned} U(w_1, w_2, y) =& \pi_1 \log(w_1 - \underline{w}_1) + \pi_2 \log(w_2 - \underline{w}_2) + (1 - \pi_1 - \pi_2)\log(y), \\ & \underline{w}_1,\ \underline{w}_2 \geq 0, \\ & \pi_1,\ \pi_2 \in [0, 1]. \end{aligned}$$

First, the parameters of the model $\{I, \underline{w}_1, \underline{w}_2, \pi_1, \pi_2, \kappa, \tau, \theta_1, \theta_2\}$ must be determined. There are also parameters for the state and control variables: $\{N_w, \underline{w}, \bar{w}\}$, where N_w is the number of grids for water stock, $\underline{w}$ and $\bar{w}$ are the lower and upper bounds for water stock, respectively. Although I do not use real data to determine these parameters, I believe the values for the parameters are reasonable for this analysis. I display the parameter values in table 1.

Parameters	$\underline{w}_1$	$\underline{w}_2$	π_1	π_2	I	θ_1	θ_2	N_w	$\underline{w}$	$\bar{w}$
Values	10	50	0.2	0.2	100	0.5	1	500	63	200

Table 1: Parameter Values

The fixed cost τ takes 16 equally-spaced values from 6 to 20. I set the marginal budget shares of food and tap water the same at 0.2, but the tap water has a lower subsistence level than food. This assumption makes the demand for tap water relatively more elastic with respect to its own price than that of the agricultural demand for water, at the same prices. This results from the first component in the denominator in the elasticity equations below:

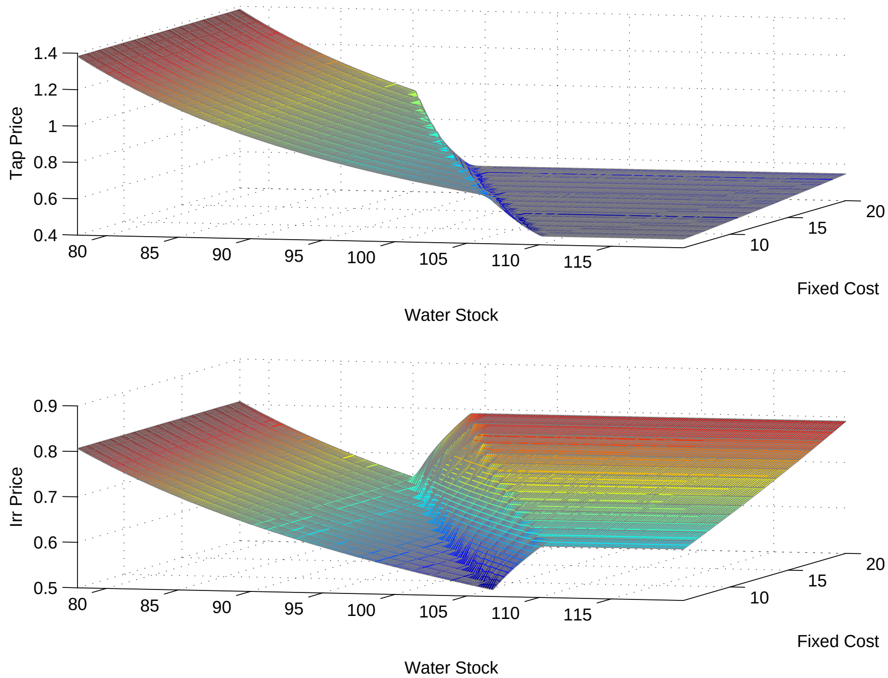

Figure 2: Static Problem with Revenue and Resource Constraints

$$\epsilon_{w_1,p_1} = \frac{\pi_1(I - p_1\underline{w}_1 - p_2\underline{w}_2)}{p_1\left[(1-\pi_1)\underline{w}_1 + \pi_1\frac{(I-p_1\underline{w}_1-p_2\underline{w}_2)}{p_1}\right]},$$
$$\epsilon_{w_2,p_2} = \frac{\pi_2(I - p_1\underline{w}_1 - p_2\underline{w}_2)}{p_2\left[(1-\pi_2)\underline{w}_2 + \pi_2\frac{(I-p_1\underline{w}_1-p_2\underline{w}_2)}{p_2}\right]}.$$

On figures 2 and 3, I illustrate the case when the government has both revenue and resource constraints. The two water prices, and the marginal cost of water (MC) are plotted for various values of available water supply and fixed cost. The three regions (R1, R2, and R3) are also displayed on figure 3. The tap price is decreasing in water stock. In region 1, where there is no water scarcity, the resource constraint does not play any significant role, and the tap price is increasing in fixed cost. In region 2, where water scarcity is severe, the revenue constraint is slack, and the tap price is constant in fixed cost. However, the fixed cost affects the lower and upper bounds of region 3: higher fixed cost will increase water price(s) and decrease total water withdrawals, so the upper bound goes down. Also, the lower bound decreases as profits go to zero more quickly. In region 3, the tap price decreases in fixed cost, as higher fixed cost increases the irrigation price and the tap price is reduced to allow more withdrawals. However, the irrigation price is not necessarily decreasing in water stock: in region 2, for low water stock, the irrigation price is decreasing in water stock, while

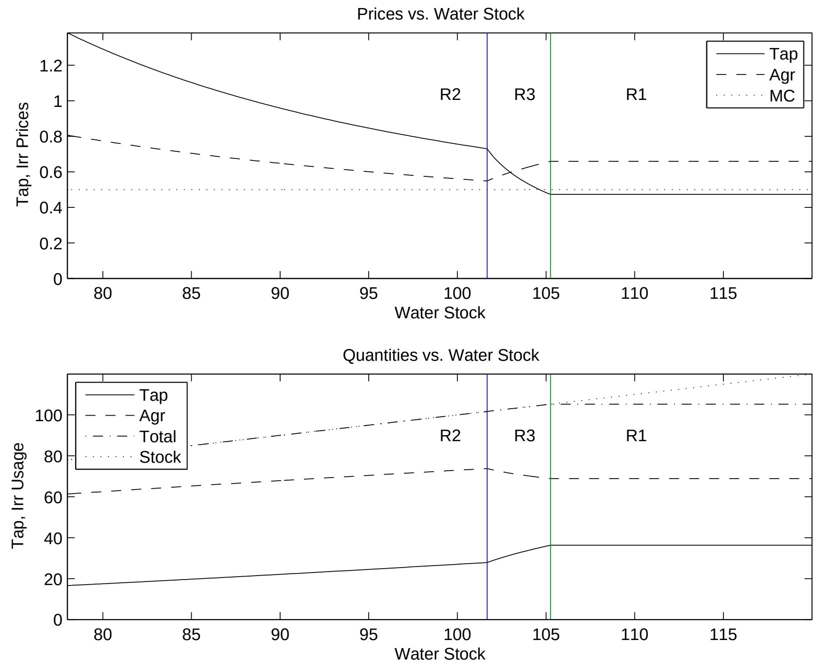

Figure 3: Static Problem with Revenue and Resource Constraints

in region 3, it rises as the effect of the revenue constraint increases.

I plotted the water prices and quantities (withdrawals) over water stock on figure 3. I set the fixed cost at ten. When there is no water scarcity (R1); i.e., the water stock is reasonably abundant, the resource constraint does not play any significant role, and the water stock in the reservoir is sufficient to meet the demand by both sectors. In this case, the revenue constraint binds; the solution is the unique and the same for any such levels of water supply. Moreover, the inverse-elasticity rule applies, so the price of irrigation water exceeds that of tap water. Note, too, that the price of tap water is less than the marginal cost of water, so the government makes losses from supplying water to households and profits from providing irrigation. The profits made from agricultural sector compensates these losses as well as the fixed cost.

When water scarcity is severe enough (R2), the revenue constraint becomes slack. In this case, withdrawals equal the water supply, which is below the volume of withdrawals the government would like to supply should there be no resource constraint. In other words, WD is strictly less than $(I - \tau - y)$ in this region. As water gets increasingly scarce, the price of tap water exceeds that of irrigation water. Another interesting result is that since there is water scarcity, both water prices are higher than the marginal cost of water. The government generates profits from supplying water to both sectors, which exceeds the fixed cost. Nonetheless, I assume that these profits do not return to the economy in this partial-equilibrium setup[8].

These results are important because they show that the price of a public good for a sector does not necessarily exceed that for another sector even though the demand by the first sector is more elastic than the other. In fact, with the introduction of a binding resource constraint, the problem may be independent of the revenue constraint altogether if the two demand functions are inelastic in their own prices. As a result, the prices may optimally deviate from the inverse-elasticity rule, which is the solution of a static Ramsey problem with no resource constraint.

4 Conclusion

In this paper, I have attempted to explain the price-discrimination problem of a local government in supplying water to multiple user-groups. To analyze the pricing scheme decision of the government, I constructed a dynamic partial-equilibrium model, but considered its static version, since the goal of my paper is to analyze whether prices are optimally different from each other. When the water supply in the reservoir is abundant enough, the government's budget constraint plays a more significant role in the determination of the water prices, as the reservoir has enough water stock for both user groups. Thus, one expects the inverse elasticity rule to apply: after accounting for the cross-price elasticities, more is charged for irrigation water, which has relatively less elastic demand than tap water. However, when the water supply is scarce enough, the stock of water in the reservoir becomes crucial, as there is not enough water for both user groups anymore. For this reason, the government increases the price of tap water, and households end up paying a higher price for water than the agricultural sector. This conclusion results from the fact that the costs involved in increasing the tap water price are less those involved in increasing the irrigation water price. As a result, the local government aims to collect as little revenue as possible from households so that the households can allocate more of their income for other commodities. Consequently, agricultural sector pays a lower price for water than households.

As simple as my model is, results may have interesting implications concerning government aid to agriculture in supplying water. The general understanding is that as water scarcity gets more severe, the price of irrigation water has to be raised substantially to decrease the volume of irrigation water. This idea stems from the fact that the demand for irrigation water is quite inelastic, as without water, there is no crop production. For this

[8]In many countries, the law states that the government cannot make any profits from supplying water to the different sectors. Thus, I did not model how profits are being used in this model. However, one may consider cases where part or all of the profits is being rebated to the households.

reason, governments may choose to subsidize agriculture through lower irrigation prices. However, as I have shown in this paper, this may not necessarily be the case. In fact, as water gets more scarce, the increase in the price of tap water may be more than that of the price of irrigation water. Consequently, the necessity of government aid to agriculture may be questionable.

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